

APPLICATION OF MINDLIN'S THEORY FOR ANALYSIS OF FOOTING PLATE BENDING BASED ON EXPERIMENTAL RESEARCH

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Abstract. *The assumptions and basic equations of the first-order shear deformation plate theory of Mindlin, as one which provides more accurate solutions compared to the classical theory, are briefly presented in this paper. Application of one analytical solution derived according to this theory is presented by use of example of stress-state and deformations calculation of the reinforced concrete footing, which has been the object of recent author's experimental research. Numerical results obtained by applied procedure, which refer to the elastic domain of material behavior, are compared with experimentally obtained data of deflections of footing plate midpoint.*

Key words: *Mindlin's plate bending theory, deflection, rotation of cross section, internal forces, footing, soil slenderness.*

1. INTRODUCTION

The exact solution of the plate bending problem means an analysis of it as a three-dimensional body, i.e. solving of the basic equations of Theory of elasticity. This way is too complex and it is not usually applied in design of plates, except in theoretical investigation. Simplification of the problem, more or less justified, is possible in many cases by introducing corresponding assumptions, first of all based on the fact that one dimension is considerably smaller than two others. Thus, the two-dimensional plate theories can be classified into two types: classical plate theory (Kirchhoff's thin plate theory), in which the transverse shear deformation effects are neglected, and shear deformation plate theories.

Neglecting the effect of transverse shear strains, γ_{zx} and γ_{zy} , leads to significant disagreement regarding the real behavior of thick plates, particularly in the cases when the loading is a result of high concentrated force, in the area near edges and around the hollows if diameter is not large compared with thickness of the plate. Designers meet the

problem of thick plate bending in praxis in the case of shelter, foundations of tall buildings, contra plate and so on.

The assumptions and basic equations of the first-order shear deformation plate theory of Mindlin, as one which provides more accurate solutions compared to the classical theory, are briefly presented in this paper. Application of one analytical solution derived according to this theory is presented by use of examples of stress-state and deformations calculation of the reinforced concrete footing, which has been the object of a recent author's experimental research. The obtained numerical results are sufficiently proximate to the experimental ones in the elastic domain of material behavior that is expected considering the introduced assumptions of the applied theory.

2. BASIC ASSUMPTIONS AND EQUATIONS OF MINDLIN'S PLATE BENDING THEORY

Mindlin's theory is derived based on the following assumptions:

- Straight lines normal to the xy -plane before deformation remain straight and with unchanged length, but not compulsory normal to the mid-surface after deformation (Fig.1);
- Deformation of mid-plane linear elements is neglected;
- Stress component σ_z is too small regarding the other two components, so it can be neglected;
- Material is homogenous, isotropic ideally and elastic.

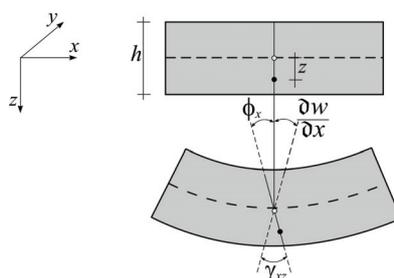


Fig. 1 Deformation of a plate cross section according to Mindlin's assumptions

It is assumed that displacement components (u , v) along the (x , y) coordinate directions are proportional to z , while transverse deflection w is independent of z , so that the displacement field looks like:

$$u = z\phi_x(x, y), \quad v = z\phi_y(x, y), \quad w = W(x, y), \quad (1)$$

where ϕ_x and ϕ_y denote rotations about the y and x axes, respectively, and $W(x, y)$ is deflection of the plate function. Transverse shear strain is assumed to be constant with respect to the thickness coordinate. In this first-order shear deformation theory, shear correction factor is introduced to correct discrepancy between the actual transverse shear force distributions and those computed using the kinematic relations (1). The shear correction factor depends not only on the geometric parameters, but also on the loading and boundary conditions of the plate.

Relations between internal forces and deformations are derived from full three-dimensional constitutive equations of Hook's law where six components of the strain are expressed by six stress components. The first assumption implies the vanishing of strain ε_z , while the remaining five equations are solved so that $\sigma_x, \sigma_y, \tau_{xy}, \tau_{xz}, \tau_{yz}$ are expressed as functions of $\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}, \sigma_z$. Having in mind definitions of internal forces in terms of stresses for plane-stress state and relations strains-displacements of three-dimensional theory, the following relations are established:

$$M_x = D \left(\frac{\partial \phi_x}{\partial x} + \nu \frac{\partial \phi_y}{\partial y} \right), \quad M_y = D \left(\frac{\partial \phi_y}{\partial y} + \nu \frac{\partial \phi_x}{\partial x} \right), \quad M_{xy} = \frac{1-\nu}{2} D \left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right), \quad (2)$$

$$T_x = k^2 h G \left(\phi_x + \frac{\partial W}{\partial x} \right), \quad T_y = k^2 h G \left(\phi_y + \frac{\partial W}{\partial y} \right), \quad (3)$$

where $G = \frac{E}{2(1+\nu)}$ is shear modulus and $D = \frac{Eh^3}{12(1-\nu^2)}$ is flexural rigidity. E denotes Young's modulus, ν Poisson's ratio, h thickness of the plate and k shear correction factor.

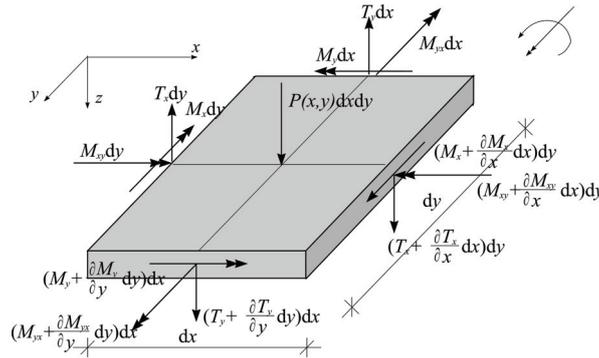


Fig. 2 Internal forces on a differential element of the rectangular plate in bending

Well known equations of equilibrium for the above element are:

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - T_x = 0, \quad \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - T_y = 0, \quad \frac{\partial T_x}{\partial x} + \frac{\partial T_y}{\partial y} + p = 0. \quad (4)$$

From the first and the second equation given in (4) system of two differential equations is obtained:

$$T_x = -D \frac{\partial}{\partial x} (\Delta W) + \frac{h^2}{12k} \Delta T_x - \frac{h^2}{12k} \frac{1+\nu}{1-\nu} \frac{\partial p}{\partial x},$$

$$T_y = -D \frac{\partial}{\partial y} (\Delta W) + \frac{h^2}{12k} \Delta T_y - \frac{h^2}{12k} \frac{1+\nu}{1-\nu} \frac{\partial p}{\partial y}. \quad (5)$$

Substituting T_x and T_y in the third equilibrium equation with functions given by Eq. (5) the differential equation is obtained in the form:

$$D\Delta\Delta W = p(x, y) - \frac{h^2}{6k(1-\nu)} \Delta p(x, y). \quad (6)$$

Introducing stress function, so that homogeneous parts of the solution of differential Eq.(5) are in the shape $T_{x,o} = \frac{\partial\varphi}{\partial y}$ and $T_{y,o} = \frac{\partial\varphi}{\partial x}$, it is obtained:

$$\Delta\varphi - \frac{12k}{h^2} \varphi = const = 0. \quad (7)$$

Equations (6) and (7) describe the problem of plate bending according to the Mindlin's theory. Equation (6) is the fourth-order biharmonic differential equation which solutions are deflections of the plate W , and it includes an additional term, regarding classical thin plate bending theory, as consequence of shear deformation. The elliptic differential equation (7) is used to define the stress function φ which influence on rotation of cross-sections and internal forces is significant only in the narrow zone near the edges. Coupling of basic unknowns W and φ is achieved through boundary conditions.

Now, the rotations ϕ_x and ϕ_y , as well as the expressions (2) and (3) for the force and moment resultants, in terms of basic unknowns W and φ , look like:

$$\phi_x = -\frac{\partial W}{\partial x} - \frac{h^2}{6k(1-\nu)} \left[\frac{\partial}{\partial x} (\Delta W) + \frac{1}{khG} \frac{\partial p}{\partial x} \right] + \frac{1}{khG} \frac{\partial\varphi}{\partial y}, \quad (8)$$

$$\phi_y = -\frac{\partial W}{\partial y} - \frac{h^2}{6k(1-\nu)} \left[\frac{\partial}{\partial y} (\Delta W) + \frac{1}{khG} \frac{\partial p}{\partial y} \right] - \frac{1}{khG} \frac{\partial\varphi}{\partial x}, \quad (9)$$

$$T_x = -D \frac{\partial}{\partial x} (\Delta W) - \frac{h^2}{6k(1-\nu)} \frac{\partial p}{\partial x} + \frac{\partial\varphi}{\partial y}, \quad T_y = -D \frac{\partial}{\partial y} (\Delta W) - \frac{h^2}{6k(1-\nu)} \frac{\partial p}{\partial y} - \frac{\partial\varphi}{\partial x}, \quad (10)$$

$$M_x = -D \left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2} \right) - \frac{h^2}{6k(1-\nu)} \left(\frac{\partial^2}{\partial x^2} + \nu \frac{\partial^2}{\partial y^2} \right) \left(D\Delta W + \frac{h^2}{6k(1-\nu)} p \right) + \frac{h^2}{6k} \frac{\partial^2\varphi}{\partial x\partial y} \quad (11)$$

$$M_y = -D \left(\frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2} \right) - \frac{h^2}{6k(1-\nu)} \left(\frac{\partial^2}{\partial y^2} + \nu \frac{\partial^2}{\partial x^2} \right) \left(D\Delta W + \frac{h^2}{6k(1-\nu)} p \right) - \frac{h^2}{6k} \frac{\partial^2\varphi}{\partial x\partial y} \quad (12)$$

$$M_{xy} = -D(1-\nu) \frac{\partial^2 W}{\partial x\partial y} - \frac{h^2}{6k} \frac{\partial^2}{\partial x\partial y} \left(D\Delta W + \frac{h^2}{6k(1-\nu)} p \right) + \frac{h^2}{12k} \left(\frac{\partial^2\varphi}{\partial y^2} - \frac{\partial^2\varphi}{\partial x^2} \right). \quad (13)$$

The common edge conditions for the Mindlin's plate theory are given below.

All of three static values can be defined along free unloaded edge, i.e.: $M_n = 0$, $M_{nt} = 0$, $T_n = 0$.

There are two kinds of simply supported edges. The first kind, which is referred to as the *hard* type simple support, requires $W = 0$, $M_n = 0$, $\phi_t = 0$. The effect of this boundary condition is that there are torsion moments M_{nt} along this simply supported edge. The second kind, commonly referred to as the *soft* type simple support, requires $W = 0$, $M_n = 0$, $MN_t = 0$.

When clamped edge condition is called *hard* that means $W = 0$, $\phi_n = 0$, $\phi_t = 0$, while *soft* clamped edge requires $W = 0$, $\phi_n = 0$, $M_{nt} = 0$.

3. MAURICE LÉVY SOLUTION OF THE MINDLIN THEORY EQUATIONS

One way to obtain the exact analytical solution of differential equation of transversally loaded plate is using Lévy's method of analysis. It can be applied for the bending problem of rectangular plates with two opposite edges simply supported while the other two edges are supported in an arbitrary manner. As the generalized displacement is expressed as series, the vertical load has to be expressed in the shape of series too. In this paper uniformly distributed loading on a rectangular, symmetrical regarding x -axis is considered Fig.3.

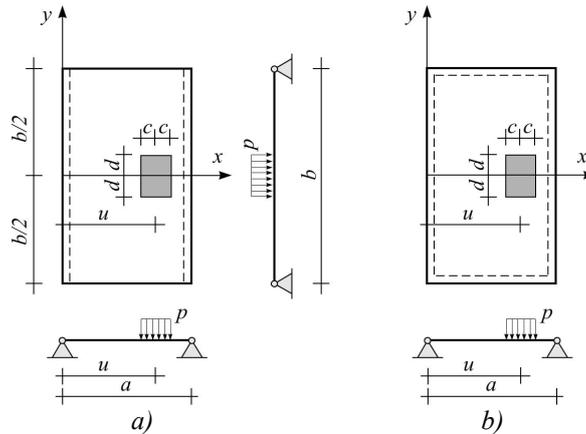


Fig. 3 Uniformly distributed loading on the area $2c \times 2d$: a) A strip with two opposite edges simply supported, b) All four edges of a plate are simply supported

At first, a strip with two opposite edges simply supported shown in Fig.3. is considered. In order to represent the load as continuous function of coordinates x and y , it is chosen as uniformly distributed along x direction, in width of $2c$, in the form of Fourier's series with period $L=2a$.

$$p(x) = \frac{4p}{\pi} \sum_{m=1,2,\dots} \frac{1}{m} \sin \frac{m\pi c}{a} \sin \frac{m\pi u}{a} \sin \frac{m\pi x}{a}. \quad (14)$$

Applying Fourier's integral formula [4], the loading $p(x)$ can be characterized as uniformly distributed in y direction, in width of $2d$:

$$p(x, y) = \frac{2p(x)}{\pi} \int_0^{\infty} \frac{\sin \alpha d}{\alpha} \cos \alpha y \, d\alpha, \quad (15)$$

which allowing for (14), can be expressed by the function

$$p(x, y) = \frac{2P}{\pi^2 cd} \sum_{m=1,2,\dots} \frac{1}{m} \sin \frac{m\pi c}{a} \sin \frac{m\pi u}{a} \sin \frac{m\pi x}{a} \int_0^{\infty} \frac{\sin \alpha d}{\alpha} \cos \alpha y \, d\alpha. \quad (16)$$

Solution of differential equation (6) is to be find in the form:

$$W = W_h + W_p, \quad (17)$$

where W_p is particular integral of Eq. (6) that depends on loading and satisfies boundary conditions only at the edges $x = 0$ and $x = a$, along which the plate is simply supported, while W_h is the solution of the homogenous differential equation which together with W_p fulfill all boundary conditions.

The solution W_p is to be in the form similar to the loading function (15) as follows:

$$W_p = \sum_m \sin \frac{m\pi x}{a} \int_0^{\infty} C(\alpha) \cos \alpha y \, d\alpha. \quad (18)$$

The requirement that this solution has to satisfy differential equation (6) yields to:

$$C(\alpha) = \frac{2P}{m\pi^2 Dcd} \frac{1 + \frac{h^2}{6k(1-\nu)} \left(\frac{\pi^2 m^2}{a^2} + \alpha^2 \right)}{\left[\frac{m^2 \pi^2}{a^2} + \alpha^2 \right]^2} \frac{\sin \alpha d}{\alpha} \sin \frac{m\pi c}{a} \sin \frac{m\pi u}{a}. \quad (19)$$

With this value for $C(\alpha)$, the expression (18) može can be written in the following form:

$$W_p = \frac{2P}{\pi^2 Dcd} \sum_m \frac{1}{m} \sin \frac{m\pi c}{a} \sin \frac{m\pi u}{a} \sin \frac{m\pi x}{a} \int_0^{\infty} \frac{1 + \frac{h^2}{6k(1-\nu)} \left(\frac{\pi^2 m^2}{a^2} + \alpha^2 \right)}{\left[\frac{m^2 \pi^2}{a^2} + \alpha^2 \right]^2} \frac{\sin \alpha d}{\alpha} \cos \alpha y \, d\alpha \quad (20)$$

Taking into account trigonometric relations:

$$\sin \alpha d \cos \alpha y = \frac{1}{2} [\sin \alpha (y + d) - \sin \alpha (y - d)],$$

integration according *Fourier's* integral formulas can be done, after what one obtains:

$$\int_0^{\infty} \frac{\alpha \sin \alpha x}{(\alpha^2 + k^2)^2} \, d\alpha = \frac{\pi x}{4k} e^{-kx}, \quad \int_0^{\infty} \frac{\sin \alpha x}{\alpha(\alpha^2 + k^2)^2} \, d\alpha = \frac{\pi}{4k^4} [2 - (2 + kx) e^{-kx}]. \quad (21)$$

If we denote: $\mu = \frac{m\pi}{a}$, $\gamma_m = \frac{\mu^2 h^2}{6k(1-\nu)}$ and $Q = \frac{P}{2Dacd} \sin \mu c \sin \mu u$, particular solution looks like following:

$$\bar{W}_p = \sum_m \frac{Q_m}{\mu^5} \{2 + [\mu y Sh \mu y - (2 + \mu d) Ch \mu y] e^{-\mu d} + 2\gamma_m (1 - e^{-\mu d} Ch \mu y)\} \sin \mu x, \quad (22)$$

for $0 \leq y \leq d$,

$$W_p = \sum_m \frac{Q_m}{\mu^5} [(2 + \mu y) Sh \mu d - \mu d Ch \mu d + 2\gamma_m Sh \mu d] e^{-\mu y} \sin \mu x, \quad (23)$$

for $d \leq y$.

The solution W_h is assumed in the form:

$$W_h = \sum_m Y_m(y) \sin \frac{m\pi x}{a}. \quad (24)$$

By substituting corresponding derivatives of the function (24) in homogenous differential equation $D\Delta\Delta W_h = 0$, after solving the obtained fourth-order differential equation with constant coefficients, one finds:

$$W_h = \sum_m [(A_m + \mu y B_m) Ch \mu y + (C_m + \mu y D_m) Sh \mu y] \sin \mu x. \quad (25)$$

If both edges $y = const$ are supported at the same way, rectangular Cartesian coordinate system, as shown in Figure 3., with its origin at the mid-left side of the plate, is adopted. Then W_h is even function of variable y , and the solution can be written as follows:

$$W_h = \sum_m (A_m Ch \mu y + \mu y D_m Sh \mu y) \sin \mu x. \quad (26)$$

Finally, the deflection functions at the loaded and unloaded parth respectively, have the form:

$$\begin{aligned} \bar{W} = \sum_m \frac{Q_m}{\mu^5} \{2 + 2\gamma_m + [\mu y Sh \mu y - (2 + \mu d - 2\gamma_m) Ch \mu y] e^{-\mu d}\} \sin \mu x + \\ + \sum_m (A_m Ch \mu y + \mu y D_m Sh \mu y) \sin \mu x \end{aligned}, \quad \text{for } 0 \leq y \leq d, \quad (27)$$

$$\begin{aligned} W = \sum_m \frac{Q_m}{\mu^5} [(2 + \mu y + 2\gamma_m) Sh \mu d - \mu d Ch \mu d] e^{-\mu y} \sin \mu x + \\ + \sum_m (A_m Ch \mu y + \mu y D_m Sh \mu y) \sin \mu x \end{aligned}, \quad \text{for } d \leq y. \quad (28)$$

Constants A_m and D_m can be evaluated from the boundary conditions at the edges $y = \pm b/2$.

The solution of elliptic differential equation:

$$\Delta\varphi - \frac{12k}{h^2}\varphi = \text{const} = 0, \quad (29)$$

can be obtained by method of variable separation in the form:

$$\varphi = (E_m Ch\eta y + F_m Sh\eta y) \cos qx + (G_m Ch\eta y + H_m Sh\eta y) \sin qx, \quad (30)$$

where it is denoted: $\eta = \sqrt{q^2 + 12k/h^2}$.

From the boundary conditions along the edges $x = 0$ and $x = a$, where $W = 0$ and $T_y = 0$, it is obtained $G_m = H_m = 0$ and $q = \mu$. Now, the expression (30) can be written in the form of series

$$\varphi = \sum_m (E_m Ch\eta y + F_m Sh\eta y) \cos qx. \quad (31)$$

Unknown constants E_m i F_m are determined from boundary conditions at the edges $y = \text{const}$. In the case of simetrical boundary conditions, adopting coordinate system shown in Figure 3., it has to be $E_m = 0$.

3.1 Simply supported all four edges of the plate

The boundary conditions for the plate with all four edges simply supported (Figure 3,b) in Mindlin's theory are at the edges $y = \pm b/2$:

$$W = 0, \phi_x = 0, M_y = 0. \quad (32)$$

Substituting (28), (16), (31) into (8) leads to the following expression for rotation ϕ_x :

$$\phi_x = \sum_m \left\{ \begin{array}{l} \left[A_m Ch\mu y + \mu y \quad D_m Sh\mu y + \right. \\ \left. -\mu \left[\frac{Q_m}{\mu^5} ((2 + \mu y) Sh\mu d - \mu d Ch\mu d) e^{-\mu y} \right] \right] \\ \left. -2\mu \gamma_m D_m Ch\mu y + \frac{1}{khG} \eta F_m Ch \quad \eta y \right\} \cos \mu x, \quad (33)$$

while from (12) expression for bending moment M_y is obtained in the form:

$$M_y = -D(1-\nu) \sum_m \mu^2 \left\{ \begin{array}{l} \left[A_m Ch \quad \mu y + D_m \left(\frac{2}{1-\nu} Ch\mu y + \mu y Sh\mu y \right) + \right. \\ \left. + \frac{Q_m}{\mu^5} \left[\left(\mu y - \frac{2\nu}{1-\nu} \right) Sh\mu d - \mu d Ch\mu d \right] e^{-\mu y} + \right. \\ \left. + 2\gamma_m D_m Ch\mu y - \frac{\eta}{\mu khG} F_m Ch\eta y \right] \end{array} \right\} \sin \mu x. \quad (34)$$

Based on the conditions (32), taking into account (28), (33) i (34), the system of three linear equations with unknowns A_m , D_m i F_m , can be formed. Solving this system of equations yields to the following terms for calculation of the constants:

$$A_m = \frac{1}{Ch \frac{\mu b}{2}} \left\{ \begin{array}{l} -\frac{Q_m}{\mu^5} \left[\left(2 + \frac{\mu b}{2} + 2\gamma_m \right) Sh\mu d - \mu d Ch\mu d \right] e^{-\frac{\mu b}{2}} \\ -\frac{\mu b}{2} D_m Sh \frac{\mu b}{2} \end{array} \right\}, \quad (35)$$

$$D_m = \frac{Q_m}{\mu^5 Ch \frac{\mu b}{2}} e^{-\frac{\mu b}{2}} Sh(\mu d), \quad (36)$$

$$F_m = 0, \quad (37)$$

With these exact values, the deflection functions (27) and (28), cross-section rotations (8) and (9), transversal forces (10), bending moments (11) and (12), as well as torsion moment (13), of the considered plate are exactly defined. In order to calculate the values of these functions in chosen points of the plate with coordinates (x,y) , computer programm MIND2 is made and included in [2].

4. BRIEF DESCRIPTION OF THE EXPERIMENTAL RESEARCH

For illustration of the design presented above, an example of the footing which has been tested in the frame of the recent author's experimental research, [9]. The goal of the research was determination of behavior of reinforced concrete footings supported on a deformable subgrade soil. The program of experimental research included production of a model *in situ*, i.e. preparation of the bed of prescribed geomechanical characteristics and production of test elements – foundation footings of appropriate dimensions and defined characteristics of concrete and reinforcement. As may be seen from the Figure 5, the experiment required construction of a steel frame intended for reception of the reactive force of the hydraulic jack used to load foundations. The frame is the steel space structure consisting of the bottom and space frame.

The test element – foundation footing was 85x85cm at the base, statical height of 17cm, Figure 4 a). Compressive strength of concrete used for footing was $f_{c,cube} = 38.37\text{MPa}$ (cube 150mm). Diameter of applied reinforcement was Ø8mm with reinforcement percentage approximately 0.4%.

A pit of a 4x5m layout was excavated, to the depth of 3m, and then the previously prepared steel frame was lowered to the bottom of the pit. The excavated material was substituted by the river aggregate with fractions 0-32 mm. Elasticity modulus of subgrade soil was approximately 40MPa.

The remaining space between the soil and frame of about 0.9m was used to install the foundation footing, hydraulic jack and necessary equipment for measuring the applied force. The hydraulic jack loaded the footing with vertical centric force. During loading on

the footing, apart from the other parameters, the reactive compression at the footing was registered, and its parabolic distribution in the cross-section (through the mid point) is obtained as in Figure 4 (b).

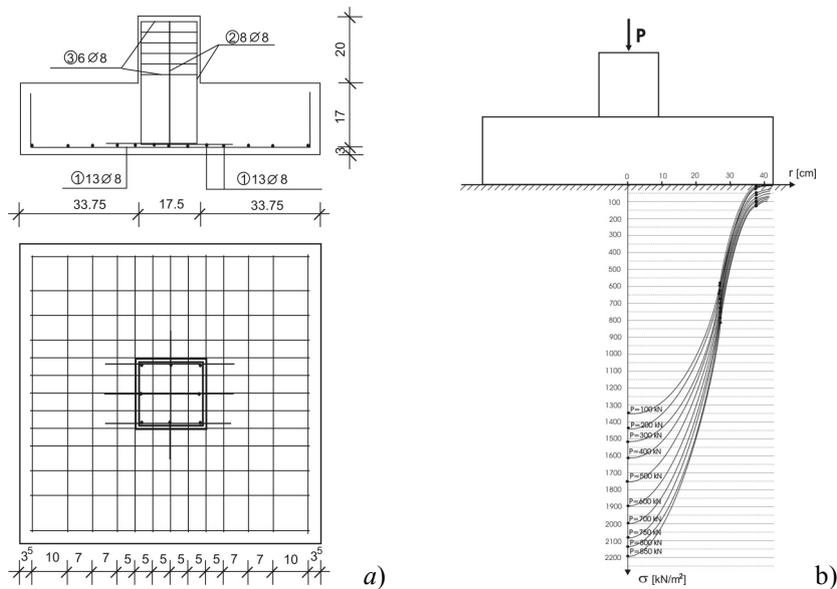


Fig. 4. a) Tested footing, b) Registered reactive compression of the soil

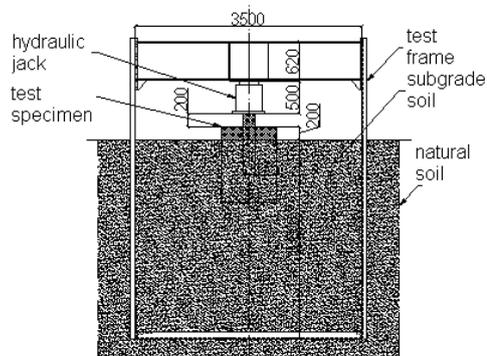


Fig. 5 Scheme of the experiment



Fig. 6 Disposition of the experiment in situ

5. AN EXAMPLE FOR THE APPLICATION OF DERIVED ANALYTICAL SOLUTIONS FOR CALCULATION OF INFLUENCES OF THE FOOTING

Very often ratio between smaller edge length of a footing is higher than 1/10, i.e. the plate is thick, and loading is usually of great intensity at small surface, that requires using of some shear deformation plate theory for calculation of stresses and strains in such

structural element. In this paper Mindlin's theory is chosen for the application by use of analytical solution derived above. As mathematical model of the footing, the rectangular plate simply supported along all four edges, loaded by uniform load at the surface $2d \times 2c$ is adopted.

For illustration of the design presented above, an example of the experimentally investigated footing from Fig. 4 a), whose static scheme with prescribed loading and corresponding influences is shown in Fig. 7, is presented. The loading represents the column effect on the footing plate, as well as the influence of the soil. Instead of usually uniform distribution of the reactive compression of the soil, here the influence of the soil is taken into account by approximation of registered compressions, based on experimental data, as it is shown in Fig. 7.

Deflections, rotations and internal forces in chosen points of the plate are calculated and some of these values are shown in the Fig. 7. as diagrams for the section α .

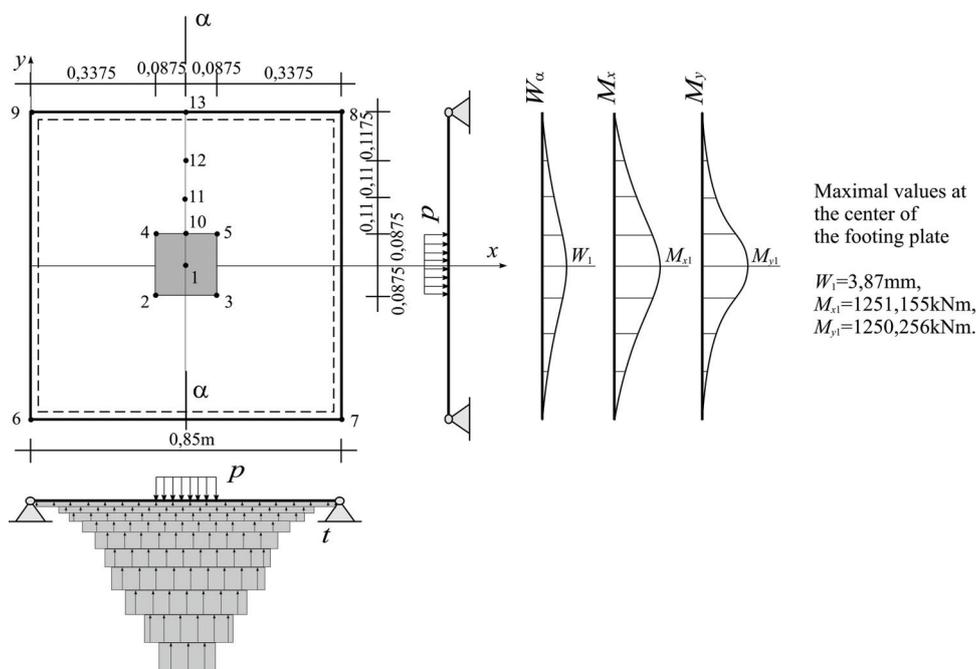


Fig. 7. Statically scheme of the footing with loading and corresponding influences

5.1. Comparison of analytical and experimental results

Deflection of the centre of the footing plate, as significant one, is chosen for comparison. Ratios displacement of the footing centre - applied force calculated with Young's modulus of elasticity according to Regulation BAB87, according EC2, as well as based on the experimental data, are shown in Fig. 8.

Influence of reinforced concrete properties is included in the design through corresponding values of Young's modulus of elasticity E and Poisson's coefficient ν . Values for

E are calculated based on measured compressive strength of cube 15 cm. For ν it is adopted value 0,2, while correction factor of Mindlin's theory is 0,833.

Influences of real footing plate greatly depend on soil rigidity, mechanical properties and type of soil. This influence is included by reactive compression of the soil as it is described above. In presented calculation values are obtained as differences between influence of the carrying structure transmitted by the column and influences of reactive soil compressions.

Because of rough approximation of boundary conditions in applied mathematical model near the edges, there are higher differences regarding the measured values, while footing centre deflection for service loading is close to experimentally obtained values.

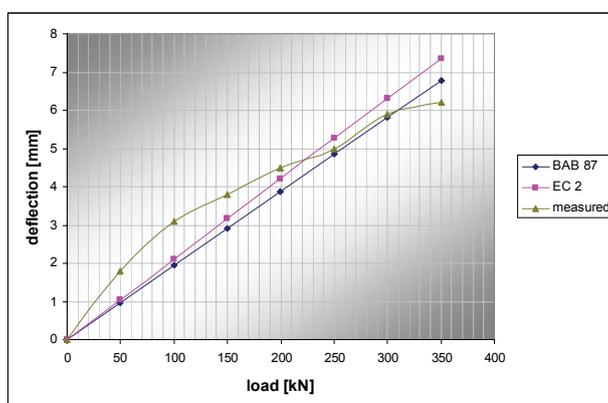


Fig. 8 Relation force-displacement of the mid point of the footing

6. CONCLUSION

Complex behavior of footings as part of construction which is in interaction both with soil and carrying structure is caused by many parameters, which are difficult to involve in numerical and analytical design methods. Application of contemporary software for structural design requires many input data which are difficult to be defined, so that they have to be determined experimentally. In addition the computational cost of such design is high. Having in mind all these facts, simple design of stresses and strains of reinforced concrete footing such as presented in this paper gives satisfactory results for exploitation loadings and could be successfully applied.

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PRIMENA MINDLINOVE TEORIJE ZA ANALIZU SAVIJANJA TEMELJNE PLOČE NA OSNOVU EKSPERIMENTALNIH ISTRAŽIVANJA

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U radu su sažeto prikazane osnovne pretpostavke i jednačine Mindlin-ove teorije, kao jedne od tačnijih teorija savijanja ploča u odnosu na klasičnu Kirhofovnu teoriju, koja uzima u obzir uticaj deformacije smicanja. Primena ove teorije prikazana je na primeru proračuna stanja naprezanja i deformacije usled savijanja plitkog armiranobetonskog temelja koji je bio i predmet eksperimentalnog ispitivanja autora rada. Uz kratak opis sprovedenog eksperimenta, priložen je i dobijeni dijagram raspodele merenih pritisaka tla na temeljnu ploču koji je korišćen u proračunu. Rezultati dobijeni primenjenim postupkom proračuna, koji se odnose na domen elastičnog ponašanja materijala, upoređeni su sa eksperimentalno dobijenim o ugibu težišta temeljne ploče.

Ključne reči: *Mindlinova teorija savijanja ploča, ugib, rotacija poprečnog preseka, unutrašnje sile, temelj samac, sleganje tla.*