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SPATIAL INTERPOLATION OF MEAN MONTHLY FLOW SERIES BY NONLINEAR CORRELATION MODEL APPLIED IN THE IBAR RIVER BASIN

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Abstract. *When providing hydrologic indicators needed for hydro power plant design, knowledge of mean monthly flows allows for estimation of majority of indicators. The paper deals with a problem of obtaining mean monthly flows at the location where flow observation data do not exist. The closest gauge station from the observation network is selected as an analogue location/basin. Since the selected gauge station observation period is rather short, the extension of the data set is achieved by generating synthetic mean monthly flow series by the nonlinear correlation method explained within the paper. Attention is paid to finding the best fit for correlation within the observation period. Four segmentation types of the correlation curve of transformed values are studied. Observed data are used for estimation of applied model quality i.e. spatial interpolation of surrounding gauge stations to the analogue station. At the selected hydro power plant location, synthetic series of mean monthly flow is calculated by scaling series from the analogue gauge station, using basin area ratio as scaling coefficient. Apart from mean annual flow and annual flow distribution, based on the synthetic mean monthly series, daily flow duration curve and maintenance flow were calculated.*

Key words: *Nonlinear correlation, non-gauged basins, synthetic mean monthly flows*

1. INTRODUCTION

Renewable energy source in small rivers in Serbia is estimated to 10% of the total renewable energy per annum. Two decades ago, a large study of potential locations for small hydro power plants (SHPP) in Serbia was completed [2]. More than 850 potential locations were roughly studied. Basic data related to hydrology, geology, sedimentation, power production potential, accessibility, etc. were provided for each location. The ex-

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ample shown in this paper refers to SHPP #617 Krona on the Vidrenjak river [2] recalculated hydrologic indicators within recent EC funded feasibility study of 15 potential SHPP locations [1].

There are five key indicators emanating from hydrologic analyses when it comes to hydro power plant design: 1) flood flow of a certain return period (usually 1000 years), 2) maintenance flow, 3) mean annual flow, 4) annual variation (distribution) of monthly flows, and 5) daily flow duration curve.

Locations of majority of hydraulic structures do not match gauge stations of hydrological observation network. Such locations are known as non-gauged locations. Three separate extensive regional analyses could provide estimates of indicators listed 1- 3, while all but one (indicator 1) could be obtained when knowing mean monthly flows at the location. This is the reason why generating mean monthly flow series at the non-gauged location is focus of this paper.

2. NONLINEAR CORRELATION MODEL

Mathematical model used for spatial interpolation of mean monthly flows from gauged to non-gauged river location from the region is an objective method of smoothing and normalization of multiple variable nonlinear correlation [3]. Smoothing and normalization is applied on the observed data x_{ji} where $j = 0,1,2,...L$ denotes geographic position of the observed variable, while $i = 0,1,2,...N_j$ denotes temporal position – location of the observed variable.

The first transformation is replacement of observed values

$$x_{j1}, x_{j2}, ..., x_{jN_j}, j = 0,1,2,...,l \tag{1}$$

with its empirical probability of non-exceedance

$$p_{ji} = p_j(x_{ji}) = \frac{m(x_{ji}) - 0,25}{N_j + 0,5}, m(x_{ji}) = 1,2,..., N_j \tag{2}$$

where $m(x_{ji}) \equiv m_{ji}$ is rank of observed value x_{ji} in the increasing – order series.

The second transformation is replacement of empirical probabilities p_{ji} with quantiles $u_{ji} = F\,[P_{ji}]$ of normalized normal distribution

$$u_{j1}, u_{j2}, ..., u_{jN_j}, j = 0,1,2,...,l, \tag{3}$$

that are estimated according to $p = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u_p} e^{-\frac{u^2}{2}} du = \Phi(u_p).$

The contribution of each of the variables $x_1, x_2, ..., x_l$ in the multiple linear regression equation

$$\tilde{x}_0 = a_0 + a_{01}x_1 + ... + a_{0l}x_l, \tag{4$_1$}$$

as well as in the multiple normalized regression

$$\tilde{u}_0 = \alpha_{01}u_1(x_1) + ... + \alpha_{0l}u_l(x_l) \tag{4$_2$}$$

Is estimated from the consequences of the theorem of the multiple correlation [3] among normalized variables $u_1,u_2,...,u_l,u_{l+1} \equiv x_0$ that represent monotonous functions of the initial variables:

$$u_j = F[p_j(x_j)] = u_j(x_j),\ j = 0,1,2,...,l. \tag{5}$$

In equations (4_1) and (4_2) squared general multiple correlation coefficient is obtained by the following formulae respectively

$$r^2_{x_0\tilde{x}_0} \equiv R_0^2 = \sum_{j=1}^{l} r_{0j} a_{0j} \frac{\sigma_j}{\sigma_0}, \tag{6_1}$$

$$r^2_{u_0\tilde{u}_0} \equiv \dot{R}_0^2 = \sum_{j=1}^{l} \dot{r}_{0j} \alpha_{0j}, \tag{6_2}$$

where $r_{0j} \equiv r_{x_0x_j}$ and $\dot{r}_{0j} \equiv r_{u_0u_j}$ represent linear correlation coefficients respectively, among x_0,x_j and u_0,u_j, while a_{0j} and α_{0j} stand for regression coefficients in equations (4_1) and (4_2), obtained by the least square method. $\sigma_j \equiv \sigma_{x_j}$, $\sigma_0 \equiv \sigma_{x_0}$ are standard deviations of initial variables.

Each formula member in (6_1) and (6_2)

$$\Delta(x_j) = r_{0j} a_{0j} \frac{\sigma_j}{\sigma_0}, \tag{7_1}$$

$$\dot{\Delta}(x_j) = \dot{r}_{0j} \alpha_{0j}, \tag{7_2}$$

is characterized by contribution of j-th argument of x_j, respectively in the equations (4_1) and (4_2). The following expression is always valid

$$\dot{\Delta}(x_j) \geq \Delta(x_j), \dot{R}^2{}_0 \geq R_0^2, \tag{8}$$

while equality stands for the case of $u_j = u_j(x_j)$ being straight lines. The bigger deviation of curves $u_j = u_j(x_j)$ from straight lines, the better input variables normalization effect.

In the case of absence of observed data $x_{ji} = x_{0i}$ at the location $j = 0$, the authors [7] propose the following model:

$$X(U_0) = \sum_{j=1}^{L} \delta_{0j}(X_j) \cdot X_j, \tag{9}$$

δ_{0j} being weight of variable X_j.

Core of presented results in this paper is estimation of the best segmentation type for $X(U_0)$ curve, based on the short observation period. Calculated parameters of nonlinear correlation equation were applied to longer time period in order to obtain synthetic mean monthly flow series.

Four segmentation types of the order series of observed values $x(u_{0i})$ $i = 1,2,...,N$ were examined:

1. 5 segments (4 linear equations for:
   $$m(x_{0i}) \equiv m_{0i} = (1,\frac{N_0}{5}),(\frac{N_0}{5},\frac{2N_0}{5}),(\frac{2N_0}{5},\frac{3N_0}{5}),(\frac{3N_0}{5},\frac{4N_0}{5})$$
   and exponential equation for $m(x_{0i}) \equiv m_{0i} = (\frac{4N_0}{5}, N_0)$.

2. 3 segments (2 linear equations for: $m(x_{0i}) \equiv m_{0i} = (1, \frac{N_0}{3}), (\frac{N_0}{3}, \frac{2N_0}{3})$,

   and exponential equation for $m(x_{0i}) \equiv m_{0i} = (\frac{2N_0}{3}, N_0)$.

3. 2 segments (exponential equations for both $m(x_{0i}) \equiv m_{0i} = (1, \frac{N_0}{2})$

   and $m(x_{0i}) \equiv m_{0i} = (\frac{N_0}{2}, N_0)$.

4. 1 segment (exponential equation for the entire order series $m(x_{0i}) \equiv m_{0i} = (1, N_0)$)

Among these types, 1 segment was selected as the best fit, using maintenance of statistical parameters in the observed series, as a selection criterion [6].

3. Studied Area and Input Data

The river of Vidrenjak is left tributary of the Ibar river. Its confluence into the Ibar river is 14 km upstream from the Gazivode reservoir.

The drainage basin of the Vidrenjak river stretches on the slopes of the Jarut mountain in the North, Hum mountain in the Southeast, and it borders with the wide plateau of Pešter in the Northeast. The area center is a small town of Tutin. Prevailing climate is sub-mountainous.

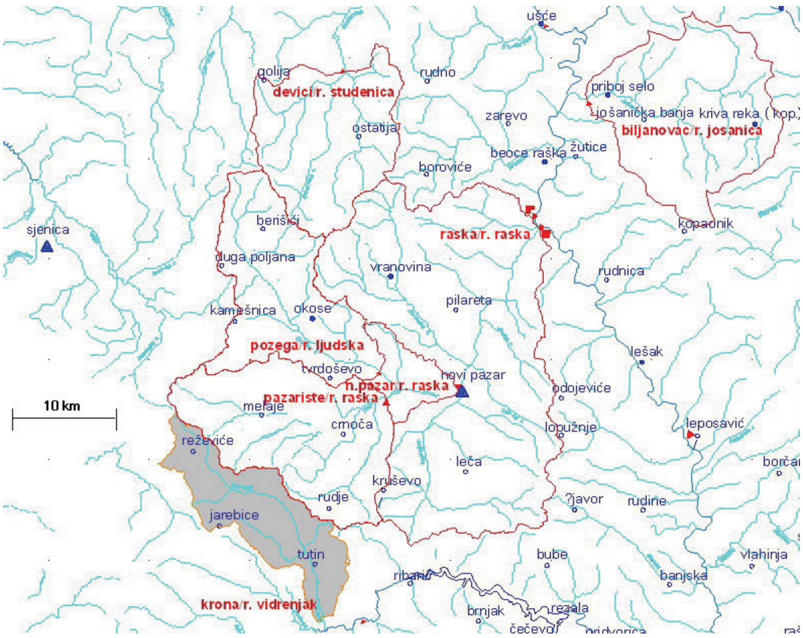

Fig. 1 Position of the drainage basin of SHPP #617 Krona dam, GS within the Ibar river drainage basin (Raska/Raska, Josanica/Biljanovac, Studenica/Devici, Ljudska/Pozega), precipitation GS, and meteorological stations from the Republic Hydro-meteorological Service of Serbia (RHMZS) observation network.

There were none of flow observations of the Vidrenjak river ever. Consequently, characteristic flows were calculated by the means of **methods for non-gauged basins.**

In order to **start** hydrological estimations and calculations and perform analysis afterwards, data records on hydro-meteorological variables are necessary. Flow, precipitation and meteorological data at the surrounding GS (Figure 1) were provided from Republic Hydro-meteorological Service of Serbia (RHMZS). Due to observation period and position, GS data were used for flow and meteorological parameter estimations for SHPP dam location drainage basin and drainage basins in the vicinity, as follows:

Table 1 Data on GS (observation period available, drainage basin area/altitude) from the RHMZS observation network used for characteristic flow estimations at SHPP # 617 Krona dam location. Key: P- precipitation (monthly sum), PM- precipitation maxima (daily), T- air temperature (monthly mean), HU- air humidity (monthly mean), VP- vapor pressure (monthly mean).

Flow data	Meteorological data
R. Studenica/ Devici (1964-2007) A= 191 km^2	Novi Pazar 545 masl (1965-2007); P, PM, T, HU, VP
R. Josanica/ Biljanovac (1956-2007) A= 265 km^2	Sjenica 1038 masl (1961-2007); P, PM, T, HU, VP
R. Lopatnica/ Bogutovac (1960-2007) A= 116 km^2	Ivanjica 465 masl (1961-2007); P, PM, T, HU, VP
R. Raska/Raska (1948-2007) A= 1036 km^2	Okose 1070 masl (1961-2006) incomplete P, PM
R. Raska/ N. Pazar (1963-1976, 1982-1983, 1985-2003, 2005-2007) A= 472 km^2	Melaje 980 masl (1961-2006) incomplete; P, PM
R. Raska/ Pazariste (1975-1978, 1981-1982, 1988-1992) A= 278 km^2	Vranovina 700 masl (1961-2006) P, PM
R. Ljudska / Pozega (1999-2001, 2002 incomplete record, 2007) A= 194 km^2	Lopuznje 940 masl (1961-2006) P, PM
	Rezevice 1195 masl (1961-2006) incomplete P, PM
	Tutin 850 masl (1961-2006) incomplete P, PM

4. MEAN MONTHLY FLOWS

GS Pozega/r. Ljudska is selected as analogue drainage basin for SHPP Krona/r. Vidrenjak basin due to similar basin area, geographic position, and other factors that affect runoff formation. Synthetic series of mean monthly flows at GS Pozega/r. Ljudska, are generated by the means of nonlinear multiple correlation. There were several models examined:

1. **Models *Q* = *f*(*Q*)** where flow was correlated to observed flows at GS in the vicinity or with the similar morphologic and climate characteristics (Table 1, left column). Input data set is defined as

   $$x_{ji}(Q_{ji}) = \frac{Q_{ji}}{A_j} = q_{ji} \text{ [l/s/km}^2\text{]}.$$

2. **Models *Q* = *f*(*P*, *T*, *HU*, *VP*)** where flow was correlated to mean monthly values of meteorological parameters in the basins (**Error! Reference source not found.**, right column):
   a. **Precipitation** data taken as mean of the precipitation observed at GS in the basins. Data completed from observed data at GS with the highest linear correlation coefficient.
   b. **Temperature** data calculated as mean from observed data at GS Novi Pazar and Sjenica, adjusted due to mean basin altitude.
   c. **Vapour pressure** data for basins estimated in the same manner as for mean monthly temperature.
   d. **Air humidity** data were taken as observed data at GS Sjenica, due to insignificant variation compared to GS Novi Pazar data.

Final model for spatial data interpolation was selected among models from the group of Models Q=f(Q). Model quality was estimated according to flow observation period of GS Pozega/r. Ljudska (1999-2002). Within selected model, mean monthly flows from the following GS were efficient judged by model mean square error criterion: GS Devici/r. Studenica, GS Biljanovac/r. Josanica, and GS Raska/r. Raska

Within the selected model, four segmentation types of the U(Xo)=f(X(Qo)) correlation equation curve were examined. Exponential type (one segment) was adopted as the best fit. Selection criterion was maintenance of the original series statistics for observed period at GS Pozega/r. Ljudska, compared to synthetic series statistics for the whole spatial data interpolation period (1964-2007), presented in Table 2.

Table 2 Original and synthetic series statistics for mean monthly flow at GS Pozega/r. Ljudska.

Statistics	Original series/ Observed (1999-2002)	Synthetic series (1964-2007) (1)	Number of segments of $U(Xo) = f(X(Qo))$ curve			
			5	3	2	1
avg	1.036	0.972	1.514	1.511	1.510	1.489
Sx	0.923	0.820	1.357	1.376	1.413	1.346
Cv	1.12	1.19	1.12	1.10	1.07	1.11
Cs	1.51	2.21	1.73	1.86	2.01	2.05

Fitting $U(Xo) = f(X(Qo))$ correlation by 5 and 3 curve segments as well as by one unique curve is given in graphic form in the Figure 2 - Figure 4.

Upon selection of the best fit for the $U(Xo) = f(X(Qo))$, synthetic series was produced.

GS Devici/r. Studenica was found to have the highest nonlinear correlation coefficient of mean monthly flows within the observation period of GS Pozega/r. Ljudska, and the highest weight coefficient in the nonlinear correlation model (Table 3). Correlation coefficient of the model as a whole was R=0.888. Matching observed and synthetic mean monthly flows in the period 1999-2002 is shown in Figure 5.

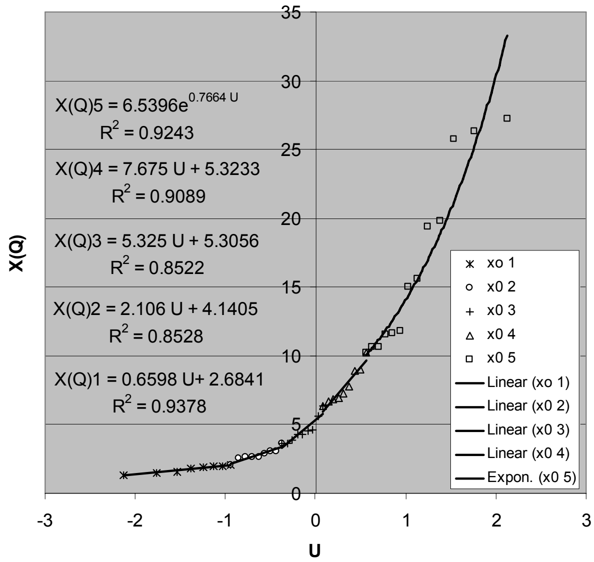

Fig. 2 Five segments fit for the $U(Xo) = f\,(X(Qo))$ correlation.

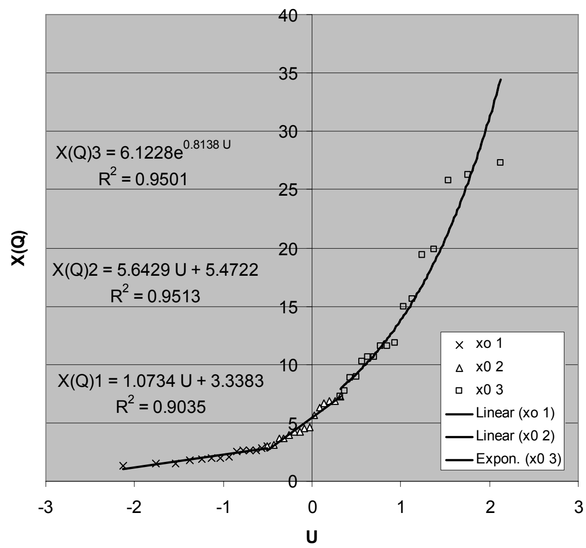

Fig. 3 Three segments fit for the $U(Xo) = f\,(X(Qo))$ correlation.

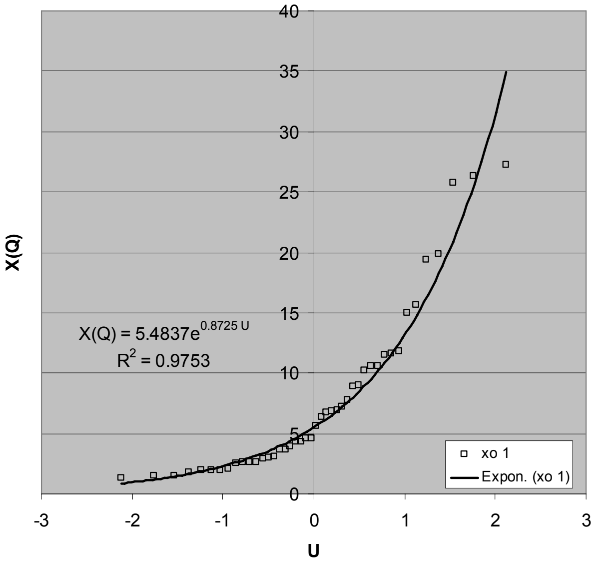

Fig. 4 Unique fit for the $U(Xo) = f(X(Qo))$ correlation – one segment.

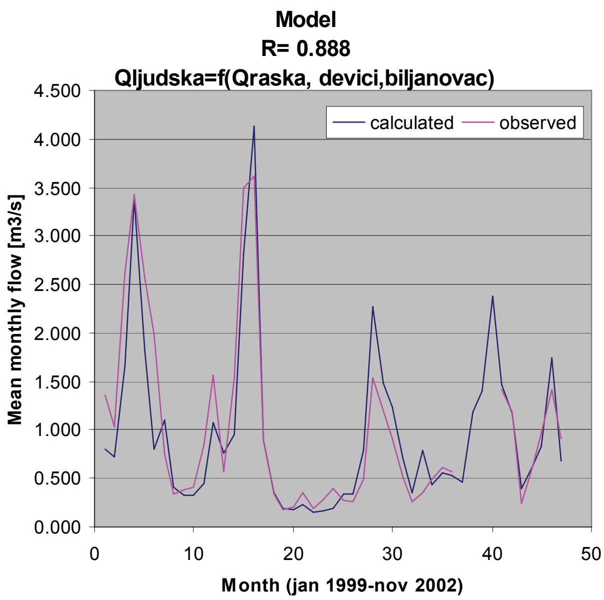

Fig. 5 Modelling results of mean monthly flow series for observation period of GS Pozega/r. Ljudska: January 1999-November 2002 (incomplete flow observation record for the year 2002).

Table 3 Spatial interpolation of mean monthly flows at GS Pozega/r. Ljudska nonlinear correlation model parameters.

GS Pozega/r. Ljudska	Nonlinear correlation coefficient	Weight coefficient in the model equation
GS Raska/r. Raska	0.797	0.302
GS Biljanovac/r. Josanica	0.805	0.125
GS Devici/r. Studenica	**0.868**	**0.573**

Finally, synthetic series of mean monthly flows at Krona/r. Vidrenjak was produced taking GS Pozega/r. Ljudska as analogue drainage basin i.e. multiplying values of synthetic series of mean monthly flows by drainage basin area ratio $K=133km^2/194\ km^2=0.686$.

4.1 Mean annual flow

Since GS Devici/r. Studenica observation period was 1964-2007, which is the period of available synthetic mean monthly flow series, base period for mean annual flow calculation is 1976-2005 (two full flow cycles: 1976-1990 and 1991-2005).

Mean annual flow estimation is based on models/methods that take into consideration meteorological parameters (precipitation and temperature), morphology characteristics of the basin (mean altitude), and iso-lines of runoff yield (**Error! Reference source not found.**).

Table 4 Calculated values of mean annual flow [m^3/s] for Krona/r. Vidrenjak.

Model	Period	1976-1990	1991-2005	1976-2005
Precipitation temperature	Pavg [mm]	655.0	541.0	598.0
	Tavg [°C]	5.5	6.1	5.8
Langbain [4]	Q=f(P,T)	0.848	0.563	0.698
Regional[8][1]	Q=f(Alt.)	1.59	1.59	1.59
	Q=f(P, Alt.)	1.04	0.702	0.861
Isolines[3][2]	Q	1.73	1.73	1.73
Synthetic series	Q	1.019	0.981	**1.000**

Comparing results for mean annual flow for the period 1976-2005, it is clear that value obtained from the synthetic mean monthly flow series is within range of values estimated by the other methods.

4.2. Maintenance flow

Minimum – maintenance flow is taken as greater value of Q95% and (10-15)% of mean annual flow - the expression valid for rivers in natural flow. Synthetic mean monthly minima series at Krona/r. Vidrenjak probability analysis was done for 1964-2007 period.

[1] Regression based on the 1961-1990 period.
[2] Annual runoff yield map iso-lines based on the 1946-1990 period.

Table 5 SHPP #617 Krona elements for estimating maintenance flow (Q [m^3/s]).

Location	Qavg.	(0.1÷0.15)Qavg	Qmin95%	Qmaint.
SHPP #617 Krona/r. Vidrenjak	1.000	0.100÷0.150	0.142	**0.142**

4.3 daily flow duration curve

There are two daily flow duration curves calculated for SHPP #617 Krona dam location (Figure 6, Table 6):

1. Valid – based on scaling coefficient K(T)=Qavg.monthly(T)/Qdaily(T) ratio at GS Devici/r. Studenica. K(T) was applied for each characteristic duration (T) on synthetic mean monthly flow duration curve calculated for Krona/r. Vidrenjak.
2. Preliminary – based on type 3 curve from SHPP Cadastre [2], multiplied by mean annual flow.

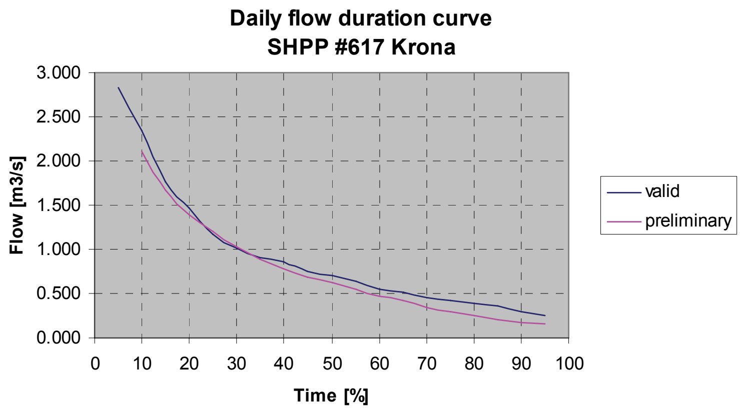

Fig. 6 Daily flow duration curves (1976-2005) SHPP #617 Krona.

Table 6 Daily flow duration curve coordinates at SHPP #617 Krona dam location

#617 Krona	**Duration curves Q[m^3/s]**	
T(%)	**Valid**	preliminary
10	2.344	2.115
20	1.466	1.387
30	1.009	1.032
40	0.856	0.779
50	0.697	0.618
60	0.552	0.472
70	0.456	0.343
80	0.387	0.247
90	0.298	0.177

4.4 Annual variation of flow

Annual variation of mean monthly flow at SHPP #617 Krona dam location is given in 5. . Distribution is based on the synthetic mean monthly flow series at Krona/r. Vidrenjak for 1976-2005 period.

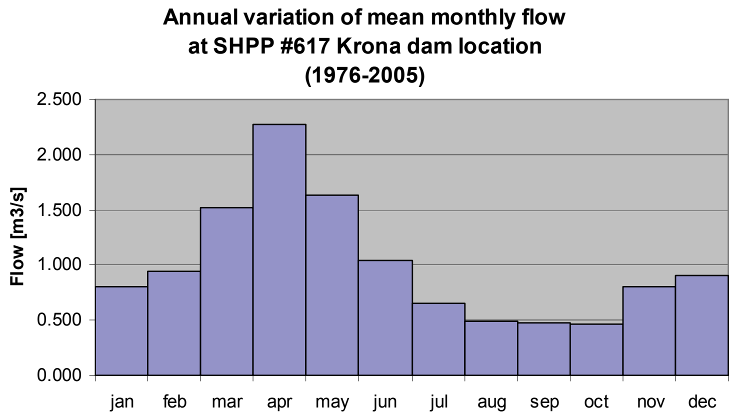

Fig. 7 Annual variation of mean monthly flow at SHPP #617 Vidrenjak dam location.

5. Conclusion

The results of application of nonlinear correlation for synthetic mean monthly flow generation in the non-gauged basin are within range of mean annual flow values estimated by other methodologies. It is shown that synthetic series matches observed values not only by series statistics, but by chronology as well (Fig.). It would further improve model if spatial correlation coefficients could be applied [7]. For such application an extensive regional analyses of mean monthly flow series statistics should be undertaken, spatial correlation function estimated, and checked for homogeneity within created regions.

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PROSTORNA INTERPOLACIJA PROSEČNIH MESEČNIH PROTICAJA MODELOM NELINEARNE KORELACIJE PRIMENJE U SLIVU REKE IBAR

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Kod obezbedjivanja hidroloških pokazatelja od značaja za projektovanje hidroelektrana, poznavanje niza srednjemesečnih proticaja na odabranoj lokaciji omogućava proračun većine traženih pokazatelja. U radu je tretiran problem dobijanja srednjemesečnih proticaja na lokaciji u slivu reke Ibar na kojoj nisu vršena merenja proticaja. Kao profil analog, odabrana je najbliža osmatračka stanica iz mreže Hidrometeorološkog zavoda Srbije, koja zadovoljava fizičko-geografseke i klimatološke uslove za izbor. Kako na ovoj stanici postoji skroman obim merenja, metodom nelinearne korelacije opisanom u radu, izvršeno je proširenje perioda osmatranja dobijanjem sintetičke serije mesečnih proticaja u profilu.

Posebna pažnja posvećena je pronalaženju najbolje veze u(x) u periodu osmatranja. Veza izmedju transformisanhi podataka osmatranja ispitana za 4 tipa segmentacije krive. Postojeći podaci osmatranja poslužili su za procenu kvaliteta primenjenih modela, odn. prostorne interpolacije podataka osmatranja sa okolnih osmatračkih stanica na stanicu analog.

Na odabranoj lokaciji, sintetička serija srednje mesečnih proticaja dobijena je skaliranjem niza srednjemesečnih proticaja u odnosu na površinu sliva analoga. Na osnovu ovog sintetičkog niza dobijene su osim prosečnih godišnjih proticaja i unutargodišnje raspodele oticaja, kriva trajanja dnevnih proticaja i garantovani proticaj.

Ključne reči: *nelinearna korelacija, neizučeni slivovi, sintetičke serije, srednjemesečni proticaji.*