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SOME ASPECTS OF THE STIC SYSTEM STABILITY CALCULATION [1]

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Emra Bujar[1], Dragoslav Stojić[2]

[1]Department of Civil Engineering, University of Priština, Serbia
[2]Department of Civil Engineering and Achitecture, University of Niš, Serbia
E-mail: stojicd@gaf.ni.ac.rs

Abstract. *The problem presented in this paper has been treated intensively by various researchers. However, considering that by its nature the problem of structure stability belongs to the area of nonlinear analysis, it represents an inexhaustible source for investigations. Thus, we propose a simple method to determine the elements of geometrical matrix of member rigidity by energetic method through finite element method.*

Key words: *Stability, nonlinear analysis*

1. INTRODUCTION

Structures deform depending on the magnitude and the disposition of external forces, physical and mechanical characteristics of the material of which they were made, and their geometrical characteristics.

Under the influence of loads the geometry of a structure, or its elements, experiences changes such as dilatations and mutual displacements of cross-sections and points.

Determination of these changes, in the process of examination, enables the estimation of the strain condition, deformation and load carrying capacity.

The deformation of the structure elements or its particular parts could be defined through linear displacement of the points or the group of points. Tension or compression or both occurs that numerical shift (of points, rotation angles, cross section axes or any other plane and longitudinal deformation).

Design of engineering structures requires not only the strength calculation in order to get the complete picture of the structure safety, but also an analysis of the deformable system balance stability, that represents an essential task in civil engineering.

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[1] This paper is dedicated to our teacher Prof. PhD Milić Milićević, Department of Civil Engineering University of Niš, Serbia

Static method, deformation method, energetic method could be used for determining the critical forces of the linear elastic plane systems consisting of members which are influenced by coplanar longitudinal forces and transversal loading:

2. FORCES METHOD

In solving the stability problem using the force method (statical method), a given system that is n time statically indefinite is reduced to the so called basic system that is statically definite and geometrically unchangeable.

The system of the linear algebra equation is used for finding the unknown forces X_k ($k = 1,2,3,...n$). If the unknown values of X_k are unequal to zero, in respect to the observed bended element of the system, it is a critical condition in this case only if the determinant of the linear equation system is equal to zero.

$$D = \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2n} \\ . & . & \cdots & . \\ \delta_{n1} & \delta_{n2} & \cdots & \delta_{nn} \end{bmatrix} \tag{1}$$

where: $\delta_{ik} = \delta_{ik}^{0} \cdot \varphi_i(\nu)$

The critical forces or their critical parameter can be calculated from the equation (1). While observing the equation (1) as an equation on ν we can find all its values (of ν) that are equal to all critical magnitudes of knot loadings. Usually the calculation of the equation (1) is very difficult. To accomplish that, a certain value for ν is adopted, and then the function $\varphi_i(\nu)$ is calculated using the table [1] and only afterwards the value of the determinant is calculated.

3. DEFORMATION METHOD

This method yields the basic system by implanting certain "*brakes*" in the system. With these "*brakes*" we are stopping the rotations and linear displacements of knots. The rotation angles and linear movements of the knots are considered as unknown values Z_k , ($k = 1,2,3,...n$).

Since in this paper the static systems with such kind of knot loading are observed, that actually only pressures or tensions in some elements of the system are caused until the moment of losing the stability, the reactions in fictitious connections are equal to zero, so the whole system of finite equations will be homogeneous.

$$\begin{gathered} Z_1 \cdot r_{11} + Z_2 \cdot r_{12} + ... + Z_n \cdot r_{1n} = 0 \\ Z_1 \cdot r_{21} + Z_2 \cdot r_{22} + ... + Z_n \cdot r_{2n} = 0 \\ \cdots \\ Z_1 \cdot r_{n1} + Z_2 \cdot r_{n2} + ... + Z_n \cdot r_{nn} = 0 \end{gathered} \tag{2}$$

With the symbols $r_{ik} = \overset{0}{r_{ik}}$ $(i,k = 1,2,3,...n)$ the reactions in the basic system are labeled, which appears in an introduced i-th connection of the unit displacement $Z_k = 1$.

The loss of stability corresponds to the inequality of the unknown values to zero, which is possible only when the determinant, consisting of coefficients along with the unknown values equals to zero:

$$\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ . & . & \cdots & . \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix} = 0 \tag{3}$$

The equation (3) is equation of stability by the calculation of the deformation method.

4. ENERGETIC METHOD IN THE FEM (FINITE ELEMENT METHOD) FORM

In the calculation of the stability of the member systems by using the energetic method, as of the equation (4), which characterize the deformation of the system, movements and rotations of the knot system are adopted as Z_k, and also some of the member cross-sections.

$$y_{(x)} = f_{1(x)} \cdot Z_1 + f_{2(x)} \cdot Z_2 + ... + f_{n(x)} \cdot Z_n \tag{4}$$

If a certain form of the member deformation φ_i is given it is possible to avoid the application of the special transcendent function and to more simply define P_{kr} and the form of stability loss Z in an algebraic solution such as (7) or (8), which can be obtained from the system of equations (5).

$$\begin{gathered} (a_{11} - P \cdot b_{11}) \cdot Z_1 + (a_{12} - P \cdot b_{12}) \cdot Z_2 + \ldots + (a_{1n} - P \cdot b_{1n}) \cdot Z_n = 0 \\ \ldots \\ (a_{n1} - P \cdot b_{n1}) \cdot Z_1 + (a_{n2} - P \cdot b_{n2}) \cdot Z_2 + \ldots + (a_{nn} - P \cdot b_{nn}) \cdot Z_n = 0 \end{gathered} \tag{5}$$

where:

$$a_{ij} = \int_0^L E \cdot I_{(x)} \cdot \left(\frac{d^2 f_i}{dx^2}\right) \cdot \left(\frac{d^2 f_j}{dx^2}\right) \cdot dx \quad ; \quad b_{ij} = \int_0^L N_{(x)} \cdot \left(\frac{df_i}{dx}\right) \cdot \left(\frac{df_j}{dx}\right) \cdot dx \tag{6}$$

If the equation (5) is represented in a metrical form we have:

$$\{A - P \cdot B\} \cdot \{Z\} = \{0\} \tag{7}$$

Characteristic equation of the stability that will give us the value of the critical force P_{kr} will be:

$$[A - P \cdot B] = 0 \tag{8}$$

If the members are of the constant rigidity, it is possible to get for elements a_{ij} and b_{ij} (6) simple mechanical and geometrical interpolation as in the static problems.

The energy equation of the bended member in the Fig. 1 from the revolution of the left fixed end for a small angle Z,

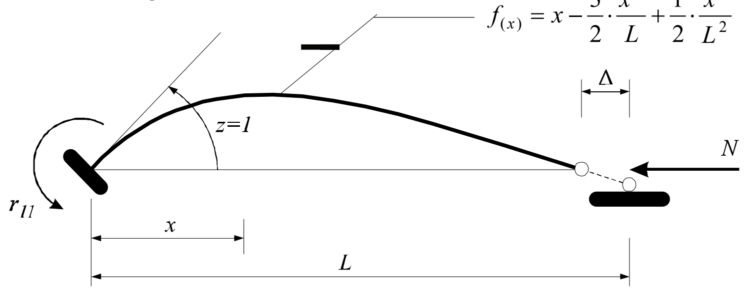

Fig. 1

is of the form:

$$E_e = \frac{1}{2} \cdot r_{11} \cdot Z^2 \tag{9}$$

For the member bending caused by the influence of the normal and transversal forces we have: r_{11} in $\varphi_1(\nu)$; so $r_{11} = r_{11}^{\ 0} \cdot \varphi_1(\nu)$; where,

$$r_{11}^{\ 0} = 3 \cdot \frac{E \cdot I}{L} \quad ; \quad \nu = L \cdot \sqrt{\frac{N}{E \cdot I}} \tag{10}$$

In this case the energy equation will be presented as:

$$E_e = U^0 + \Pi = U^0 + W \tag{11}$$

For the deflections of the rotations $Z = 1$ of the same function $f_{(x)}$, as in the static problems without the normal force N:

$$y_{(x)} = f_{(x)} = x - \frac{3}{2} \cdot \frac{x^2}{L} + \frac{1}{2} \cdot \frac{x^3}{L^2}$$

In accordance with (6):

$$U^0 = \frac{1}{2} \cdot r_{11}^{\ 2} \cdot Z^2$$

where:

$$r_{11}^{\ 0} = \int_0^L E \cdot I \cdot \left(\frac{d^2 f}{dx^2}\right)^2 \cdot dx = 3 \cdot \frac{E \cdot I}{L} .$$

$$W = N \cdot \Delta = \frac{1}{2} \cdot P \cdot \gamma_{11} \cdot Z^2$$

and:

$$\gamma_{11} = \int_0^L \overline{N} \cdot \left(\frac{df}{dx}\right)^2 \cdot dx = \frac{\overline{N} \cdot L}{5} \tag{12}$$

with: $\overline{N} = \frac{N}{P}$; P - basic loading parameter

The coefficient of the rigidity in the equation (9) can be presented:

$$r_{11} = r_{11}{}^{0} - P \cdot \gamma_{11} \tag{13}$$

where: $r_{11}{}^{0}$ – rigidity without consideration of influences of the normal and transversal forces.
γ_{11} – coefficient that is proportional to the movements

Coefficient γ_{11} will be called geometrical coefficient and it will be calculated from the equation (12) and (15). Equally for any other coefficient r_{ik} the deformation methods will be calculated by the equation:

$$r_{ik} = r_{ik}{}^{0} - P \cdot \gamma_{ik} \tag{14}$$

where:

$$r_{ik}{}^{0} = \sum_{i=1}^{n} E \cdot I \cdot \left(\frac{d^2 f_{ij}}{dx^2} \right) \cdot \left(\frac{d^2 f_{kj}}{dx^2} \right) \cdot dx$$

$$\gamma_{ik} = \sum_{i=1}^{n} \int_{0}^{L_j} \overline{N}_j \cdot \left(\frac{d\varphi_{ij}}{dx} \right) \cdot \left(\frac{d\varphi_{kj}}{dx} \right) \cdot dx \tag{15}$$

In these equations the coefficients on the members are multiplying and the very same are deforming in the basic system from the position $Z = 1$ for each one of the members when $E \cdot I = const.$ and $\overline{N}_j = const.$ Coefficients $r_{ik}{}^{0}$ and geometrical coefficients could be calculated for any of the members.

5. Numerical example

Now we will show the way of defining the rigidity matrix for the member with one fixed and one freely supported point Fig. 2.

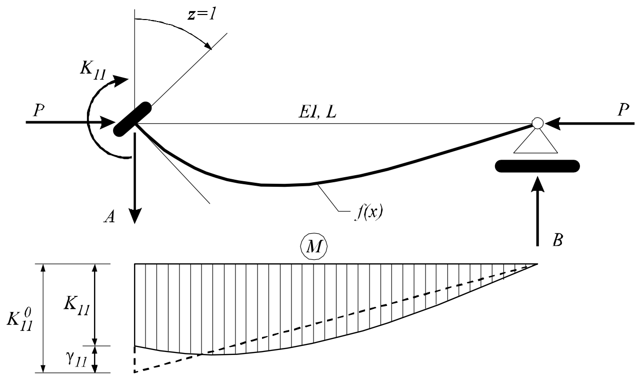

Fig. 2

$$f_{(x)} = x - \frac{3}{2} \cdot \frac{x^2}{L} + \frac{1}{2} \cdot \frac{x^3}{L^2}; \qquad \frac{df_{(x)}}{dx} = 1 - 3 \cdot \frac{x}{L} + \frac{3}{2} \cdot \left(\frac{x}{L}\right)^2$$

$$\frac{d^2 f_{(x)}}{dx^2} = \frac{3}{L} \cdot \left(\frac{x}{L} - 1\right); \quad r_{11}{}^0 = E \cdot I \cdot \int_0^L \left(\frac{d^2 f_{(x)}}{dx^2}\right)^2 \cdot dx = 3 \cdot \frac{E \cdot I}{L}$$

$$\gamma_{11} = \int_0^L P \cdot \left(\frac{df_{(x)}}{dx}\right)^2 \cdot dx = P \cdot \frac{L}{5}; \quad r_{ik} = r_{ik}{}^0 - \gamma_{ik}; \quad r_{11} = r_{11}{}^0 - \gamma_{11} = \frac{3 \cdot E \cdot I}{L} - \frac{P \cdot L}{5}$$

$$r_{11} = \frac{3 \cdot E \cdot I}{L} \cdot \left(1 - \frac{P \cdot L^2}{15 \cdot E \cdot I}\right)$$

With:

$$k = \frac{E \cdot I}{L} \quad ; \quad K_{11} = r_{11}$$

we have:

$$K_{11} = 3 \cdot k \cdot \left(1 - \frac{P \cdot L}{15 \cdot k}\right)$$

From the condition of balance we have:

$$A = -B = \left(\frac{3 \cdot E \cdot I}{L^2} - \frac{P}{5}\right)$$

or:

$$A = -B = \left(\frac{3 \cdot k}{L} - \frac{P}{5}\right)$$

or:

$$A = -B = k \cdot \left(\frac{3}{L} - \frac{P}{5 \cdot k}\right)$$

6. CONCLUSIONS

The goal of the paper is to determine the elements of geometrical matrix of member rigidity by energetic method through finite element method. A very simple method procedure for determination of relevant factors of this problem has been demonstrated.

REFERENCES

1. A. R. Ržanicin: *Stroitelnaja mehanika*, Moskva, 1982.
2. I. M. Rabinović: *Osnovi stroitelnoj mehaniki steržnevih sistem*, Moskva, 1960.
3. A. A. Umanskovo: *Sprovočnik projektirovščika promišlenih,žilih i občestvenih zdanij i sooruženij*, Moskva, 1973.
4. A. F. Smirnov, A.V. Aleksandrov, B. J. Lašenikov, N. N. Šapošnikov: *Stroitelnaja mehanika*, Moskva, 1984.
5. A. B. Aleksandrov, B. J. Lašenikov, N. N. Šapošnikov: *Stroitelnaja mehanika,* Moskva, 1983.
6. Milan Đurić: *Statika konstrukcija*. IRO Građevinska knjiga Beograd, 1983.
7. Miodrag Sekulović: *Metod konačnih elemenata*. IRO Građevinska knjiga Beograd, 1984.
8. Milić Milićević, Branko Popović, Slavko Zdravković: *Statika konstrukcija.* Građevinski fakultet Univerziteta u Nišu. Niš,1992.

OSVRT NA PRORAČUN STABILNOSTI LINIJSKIH SISTEMA

Emra Bujar, Dragoslav Stojić

Problem koji se tretira u ovom radu u svetu i kod nas je dosta razrađivan. Međutim, imajući u vidu da problemi stabilnosti konstrukcija, po svojoj prirodi spadaju u područje nelinearne analize sa komplikovanim algoritmom rešavanja, ovde se iznosi jedan sasvim jednostava postupak rešavanja tog problema.

U sistemu jednačina ravnoteža, geometrijska matrica krutosti je nepoznata, pošto je zavisna od naprezanja, koji su takođe nepoznate, pa se javlja problem nelinearnosti. U ovom radu izložen je jednostavan postupak određivanja članova geometrijske matrice krutosti štapa, energetskom metodom u obliku MKE (Metod konačnih elemenata).

Ključne reči: *stabilnost, nelinearna analiza*