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THE APPLIANCE OF INTERVAL CALCULUS IN ESTIMATION OF PLATE DEFLECTION BY SOLVING POISSON'S BISQUARED PARTIAL DIFFERENTIAL EQUATION

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Abstract. *The work analyzed the appliance of interval calculus conected to estimation of elements deflection done by reinforced concrete. The simply supported reinforced concrete plate was taken as an example. The plate was loaded by load per unit area. Knowing that there are several parametres that influence the plate deflection, we showed the calculus where the mentioned parametres were given at a certain interval. The results (the final values of deflection) we also got in the form of intervals so it was possible to follow the direct influence of a change of one or more entering parametres on deflection values by using one model and one caculus.*

Key words: *Reinforced concrete plate, deflection, interval calculus*

1. INTRODUCTION

While designing and calculating the structure elements, different parametres influence the final choice of systems, materials and dimensions of a main structure. Optimal and rational solution is often the result of numerous iterations. When solving such a complex problem it is advisable to have a good review of input parametres' influence on a final solution.

The work shows the problem of computing reinforced concrete simply supported square plate that is subjected to a load per unit area. There are parametres which directly influence the final values of plate deflection and the relationship between maximum and boundary deflection will depend on these parametres. By giving some input data in a form of closed interval $[x_{min}, x_{max}]$ we get the results in the same form, so it is possible to make certain conclusions connected to final adoption of this structure element.

We used the estimation of deflection by solving Poisson's bisquared partial differential equation (PDE) for the calculus of the model. The interval calculus is shown through certain numerical examples.

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2. Problem Definition

The simply supported square plate of side l that is subjected to a load q per unit area was given, as shown in Figure 1.

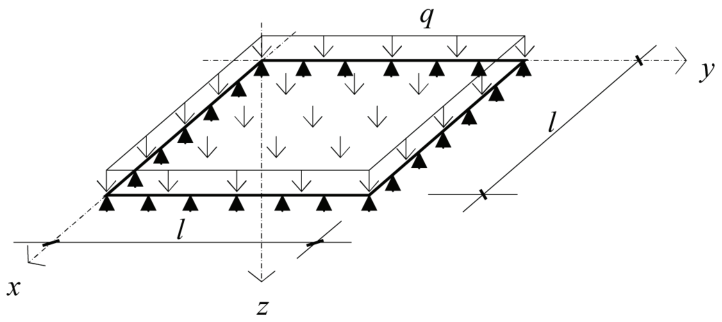

Fig. 1 Simply supported square plate

The deflection *w* in the *z*-direction is the solution of the biharmonic equation:

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \tag{1}$$

The boundary conditions along its four edges are:

$$w = 0, \quad \partial^2 v / \partial \eta^2 = 0 \tag{2}$$

where η denotes the normal to the boundary.

The flexural rigidity of the plate is given by:

$$d = \frac{E \cdot t^3}{12(1-\sigma^2)} \tag{3}$$

E - Young's modulus
t - plate thickness
σ - Poisson's ratio

3. Method of Solution and Marks

By introducing the variable $u = \nabla^2 w$, the problem amounts to solving Poisson's equation twice in succession:

$$\nabla^2 u = \frac{q}{d}, \text{ with } u = 0 \text{ along the four edges} \tag{4}$$

$$\nabla^2 w = u, \text{ with } w = 0 \text{ along the four edges} \tag{5}$$

For this purpose we will use the programme, named Poisson, that uses Gauss-Seidel method to approximate the solution of Poisson's equation (nehomogena Laplace's equation):

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \psi(x, y) \tag{6}$$

The finite-difference approximation of equation (6) is:

$$\frac{\phi_{i-1,j} - 2\phi_{i,j} + \phi_{i+1,j}}{(\Delta x)^2} + \frac{\phi_{i,j-1} - 2\phi_{i,j} + \phi_{i,j+1}}{(\Delta y)^2} = \psi_{i,j} \tag{7}$$

Thus, for $\Delta x = \Delta y$, the Gauss-Seidel method amounts to repeated application of:

$$\phi_{i,j} = \frac{1}{4}[\phi_{i-1,j} + \phi_{i+1,j} + \phi_{i,j-1} + \phi_{i,j+1} - (\Delta x)^2 \cdot \psi_{i,j}] \tag{8}$$

at every interior grid point.

The program is written for $it_{\max}$ applicatoins of (8) through all interior grid points.

Marks, used to assigned program writing, are shown at Table 1.

Table 1. Wolfram Mathematica® - List of principal variables

Program symbol	Definition	Unit of measurement
d	Flexural rigidity, *d*	kNm
e	Young's modulus, *E*	kN/m^2
i, j	Grid-point subscripts, *i*, *j*	
itmax	Number of Gauss-Seidel iterations, $it_{\max}$	
l	Lenght of side of square, *l*	m
n	Number of grid spacings along a side of the plate, *n*	
q	Load per unit area of the plate, *q*	kN/m^2
qoverd	Matrix with values qoverd=*q*/*d*, at each grid point	
sigma	Poisson's ratio, σ	
t	Plate thickness, *t*	m
u	Matrix of intermediate variable $u = \nabla^2 w$, at each grid point	
w	Matrix of downwards deflection *w*, at each grid point	
iter	Iteration counter, *it*	
phi, psi	Matrices of functions ϕ and ψ, occurring in Poisson's equation $\nabla^2\phi = \psi$	

4. ESTIMATION OF SQUARE PLATE DEFLECTION

4.1. Numerical example 1 (Wolfram Mathematica®)

Number of grid spacings along a side of the plate (n) was taken in order to have a better view of final results. Maximum number of iteration ($it_{\max}$) was chosen because for $it_{\max}\geq 25$ we get identical deflection values at certain grid points.

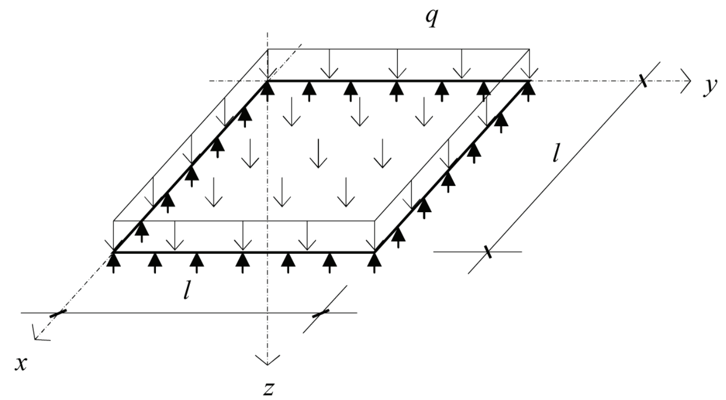

Fig. 2 Calculation model of simply supported square plate

▪ **Input data:**

n=4	- number of grid spacings along a side of the plate
$it_{\max}$=25	- number of Gauss-Seidel iterations
q=10.00 kN/m^2	- load per unit area
t=0.20 m	- plate thickness
l=6.00 m	- lenght of side of square
σ=0.20	- Poisson's ratio
E=3.15×10^7 kN/m^2	- Young's modulus (taken for MB30)

▪ **Main program:**

```
Clear all
Clear[n, u,w,qoverd,itmax]
(* Procedure Poisson *)

(* Number of grid spacings along a side of the plate *)
    n=4
(* Maximal number of iterations *)
    itmax= 25
(* Load per unit area of the plate *)
    q = 10.
(* Plate thickness *)
    t=0.2
    sigma=0.2
    l=6.0
    e=31500000.

  d=e  t^3/12/(1-sigma^2);
  rhs=q/d;
Print["rhs=", rhs]
```

```
np1=n+1
w=Table[0.,{np1},{np1}]
u=Table[0.,{np1},{np1}]
qoverd=Table[rhs,{np1},{np1}]

 Do[w[[i,j]]= 0. ; u[[i,j]]= 0. ; qoverd[[i,j]]= rhs,{i,1,np1},{j,1,np1}];
Print["Matrices w, u, qoverd"]
                      MatrixForm[w]
                      MatrixForm[u]
                      MatrixForm[qoverd]
(* Solving delsq(u) = q/d *)
phi=u;
psi=qoverd;
Do[
  Do[  phi[[i,j]] =(phi[[i-1,j]]  +  phi[[i +1,j]] +
       phi[[i,j-1]]  + phi[[i,j+1]] - (l/n)^2   psi[[i,j]] )/4     ,{i,2,n},
{j,2,n}],{iter, 1, itmax}];
u=phi;
qoverd=psi;

(* Solving delsq(w) = u *)
phi=w;
psi=u;
Do[
  Do[  phi[[i,j]] =(phi[[i-1,j]]  +  phi[[i +1,j]] +
       phi[[i,j-1]]  + phi[[i,j+1]] - (l/n)^2   psi[[i,j]] )/4 ,{i,2,n},
{j,2,n}],{iter, 1, itmax}];
u=psi;
w=phi;

Print["Matrices u  i  w = plate deflection "]
                         MatrixForm[w]
                         MatrixForm[u]
```

▪ **A part of computer output:**

```
Matrices u  i  w = plate deflection
{0., 0.,         0.,         0.,         0.},
{0., 0.00126562, 0.00173571, 0.00126562, 0.},
{0., 0.00173571, 0.00238661, 0.00173571, 0.},
{0., 0.00126562, 0.00173571, 0.00126562, 0.},
{0., 0.,         0.,         0.,         0.}
```

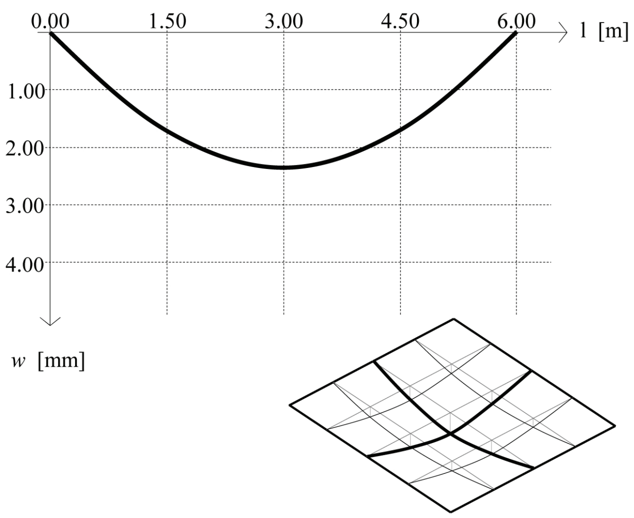

Fig. 3 Diagram of maximal plate deflections (Numerical example 1)

4.2. Discussion of results

The result in the matrics form even visually shows the expected symetry of deflection grid points of a reinforced concrete plate model. The boundary conditions were despected and by comparing some data to data of some user softvere packages, we concluded the coinciding of numerical values of deflection (w) at certain grid points. On the other hand, the accuracy of results depends on the number of grid spacings along a side of the plate (n) and number of iteration (itmax), applied to this model.

5. THE APPLIANCE OF INTERVAL CALCULUS IN ESTIMATION OF PLATE DEFLECTION

5.1. Numerical example 2 (Wolfram Mathematica®)

Calculation model was taken over from a Numerical example 1. One input data (plate load) was given at a certain interval.

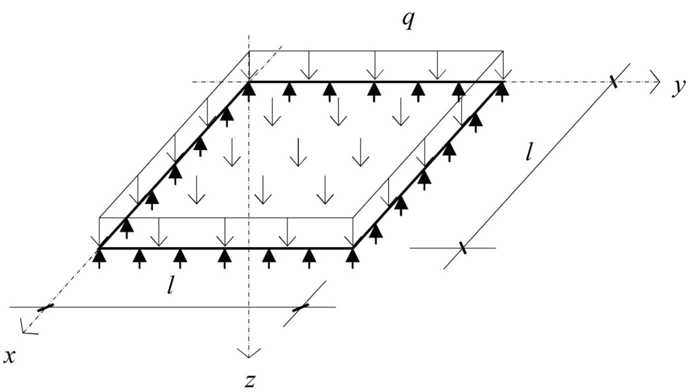

Fig. 4 Calculation model of simply supported square plate

▪ **Input data:**

n=4	- number of grid spacings along a side of the plate
it_{max}=25	- number of Gauss-Seidel iterations
q=7.50÷12.50 kN/m^2	**- load per unit area (given at intervals)**
t=0.20 m	- plate thickness
l=6.00 m	- lenght of side of square
σ=0.20	- Poisson's ratio
E=3.15×10^7 kN/m^2	- Young's modulus (taken for MB30)

▪ Main program:

```
Clear all
Clear[n, u,w,qoverd,itmax]
(* Procedure Poisson *)

(* Number of grid spacings along a side of the plate *)
    n=4
(* Maximal number of iterations *)
    itmax= 25
(* Load per unit area of the plate *)
    q= Interval[{7.50,12.50}]
(* Plate thickness *)
    t=0.2
     sigma=0.2
    l=6.0
    e=31500000.

  d=e  t^3/12/(1-sigma^2);
  rhs=q/d;
Print["rhs=", rhs]
np1=n+1
w=Table[0.,{np1},{np1}]
u=Table[0.,{np1},{np1}]
qoverd=Table[rhs,{np1},{np1}]

 Do[w[[i,j]]= 0. ; u[[i,j]]= 0. ; qoverd[[i,j]]= rhs,{i,1,np1},{j,1,np1}];
Print["Matrices w, u, qoverd"]
                        MatrixForm[w]
                        MatrixForm[u]
                        MatrixForm[qoverd]
(* Solving delsq(u) = q/d *)
phi=u;
psi=qoverd;
Do[
  Do[     phi[[i,j]] =(phi[[i-1,j]]  +  phi[[i +1,j]] +
        phi[[i,j-1]]  + phi[[i,j+1]] - (l/n)^2   psi[[i,j]] )/4 ,{i,2,n},
{j,2,n}],{iter, 1, itmax}];
u=phi;
qoverd=psi;

(* Solving delsq(w) = u *)
phi=w;
psi=u;
Do[
  Do[     phi[[i,j]] =(phi[[i-1,j]]  +  phi[[i +1,j]] +
        phi[[i,j-1]]  + phi[[i,j+1]] - (l/n)^2   psi[[i,j]] )/4 ,{i,2,n},
{j,2,n}],{iter, 1, itmax}];
u=psi;
w=phi;

Print["Matrices u  i  w = plate deflection "]
                        MatrixForm[w]
                        MatrixForm[u]
```

▪ A part of computer output:

```
Matrices u  i  w = plate deflection

{0.,  0.,                                 0.,                                0.,                                0.},
{0., Interval[{0.000949219,0.00158203}], Interval[{0.00130179,0.00216964}], Interval[{0.000949219,0.00158203}], 0.},
{0., Interval[{0.00130179,0.00216964}],  Interval[{0.00178996,0.00298326}], Interval[{0.00130179,0.00216964}],  0.},
{0., Interval[{0.000949219,0.00158203}], Interval[{0.00130179,0.00216964}], Interval[{0.000949219,0.00158203}], 0.},
{0.,  0.,                                 0.,                                0.,                                0.}
```

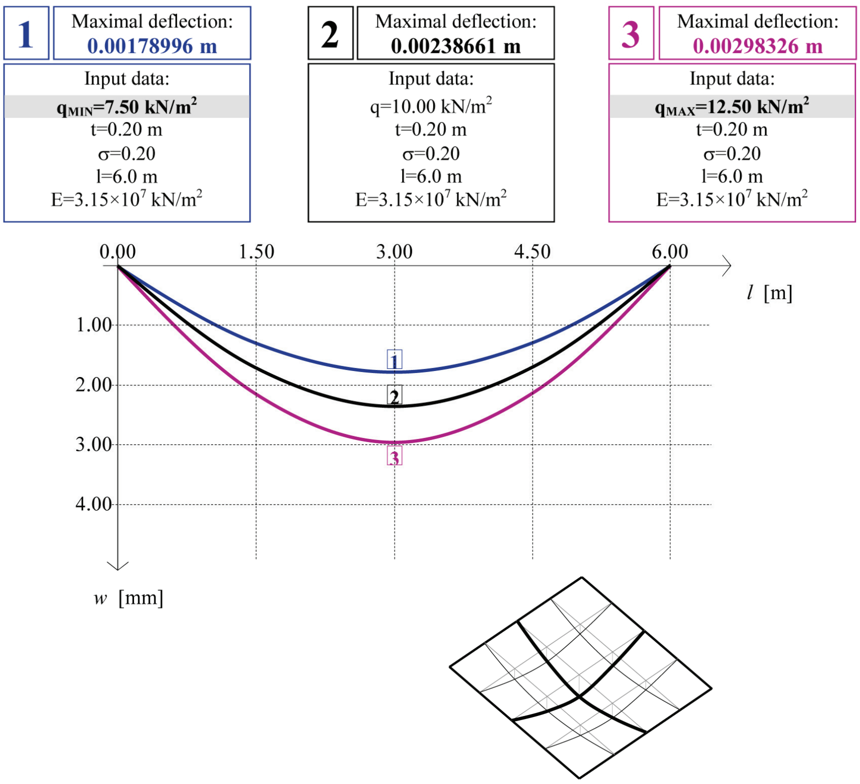

Fig. 5 Diagram of maximal plate deflections (Numerical example 2)

5.2. Numerical example 3 (Wolfram Mathematica®)

Calculation model was taken over from a Numerical example 1. Two input data (plate load and plate thickness) were given at a certain interval.

Comment: only final results are shown in this example.

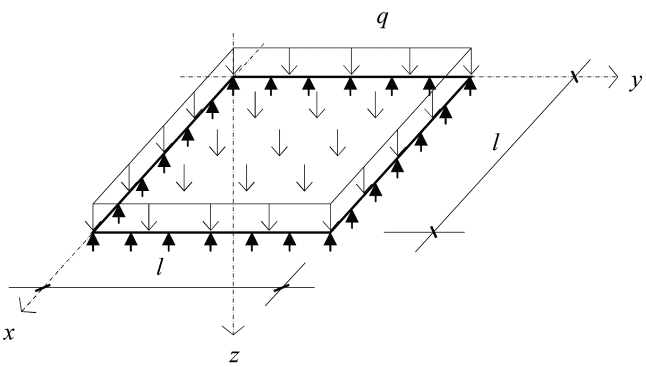

Fig. 6 Calculation model of simply supported square plate

▪ Input data:

n=4	- number of grid spacings along a side of the plate
it_{max}=25	- number of Gauss-Seidel iterations
q=7.50÷12.50 kN/m^2	**- load per unit area (given at intervals)**
t=0.15÷0.25 m	**- plate thickness (given at intervals)**
l=6.00 m	- lenght of side of square
σ=0.20	- Poisson's ratio
E=3.15×10^7 kN/m^2	- Young's modulus (taken for MB30)

▪ A part of computer output:

```
Matrices u  i  w = plate deflection

{0.,        0.,                                0.,                              0.,                                  0.},
{0., Interval[{0.000486,0.00375}],      Interval[{0.000666514,0.00514286}], Interval[{0.000486,0.00375}],       0.},
{0., Interval[{0.000666514,0.00514286}], Interval[{0.000916457,0.00707143}], Interval[{0.000666514,0.00514286}], 0.},
{0., Interval[{0.000486,0.00375}],      Interval[{0.000666514,0.00514286}], Interval[{0.000486,0.00375}],       0.},
{0.,        0.,                                0.,                              0.,                                  0.}
```

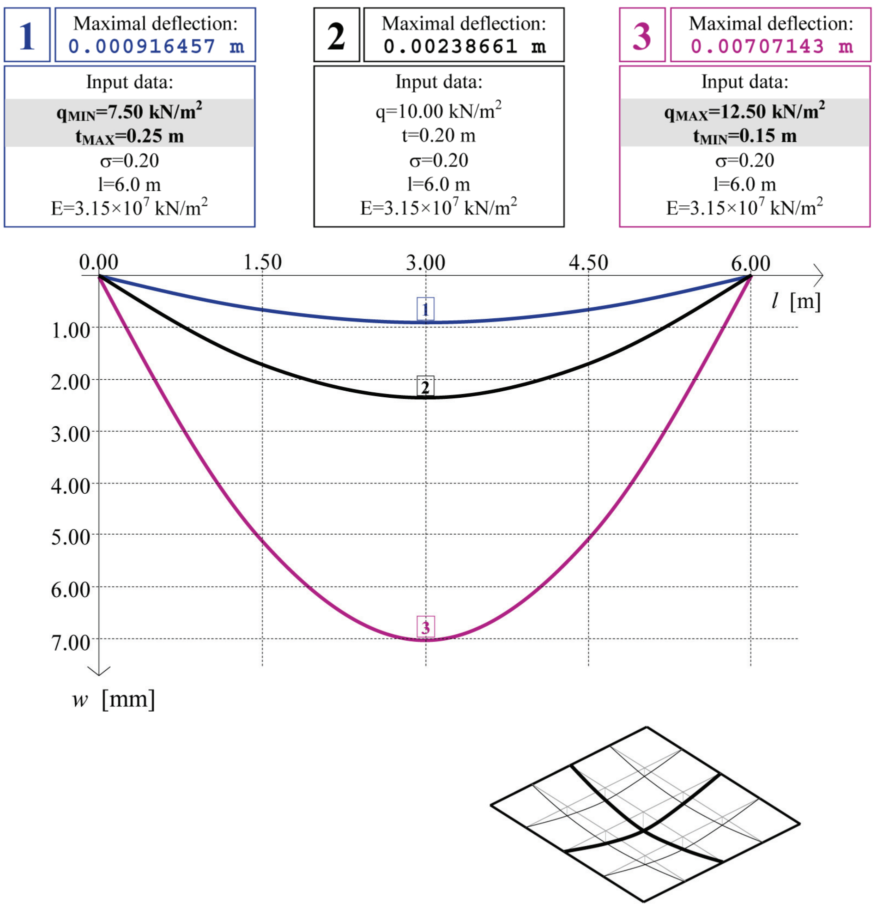

Fig. 7 Diagram of maximal plate deflections (Numerical example 3)

5.3. Numerical example 4 (Wolfram Mathematica®)

Calculation model was taken over from a Numerical example 1. Three input data (plate load, plate thickness and lenght of side of square) were given at a certain interval.

Comment: only final results are shown in this example.

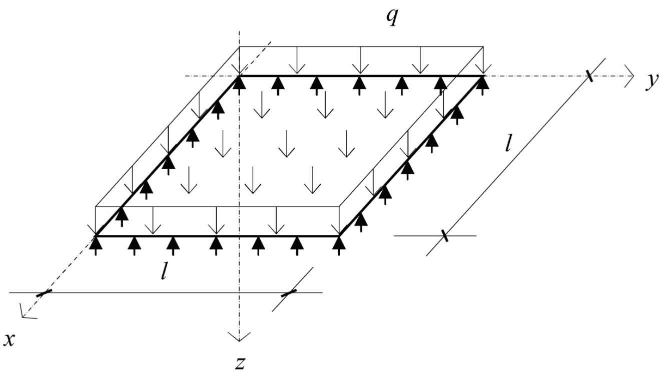

Fig. 8 Calculation model of simply supported square plate

▪ **Input data:**

n=4	- number of grid spacings along a side of the plate
it_{max}=25	- number of Gauss-Seidel iterations
q=7.50÷12.50 kN/m^2	**- load per unit area (given at intervals)**
t=0.15÷0.25 m	**- plate thickness (given at intervals)**
l=5.75÷6.25 m	**- lenght of side of square (given at intervals)**
σ=0.20	- Poisson's ratio
E=3.15×10^7 kN/m^2	- Young's modulus (taken for MB30)

▪ **A part of computer output:**

```
Matrices u  i  w = plate deflection

{0.,        0.,                              0.,                              0.,                0.},
{0., Interval[{0.000409923,0.00441516}], Interval[{0.000562181,0.00605507}],  Interval[{0.000409923,0.00441516}], 0.},
{0., Interval[{0.000562181,0.00605507}], Interval[{0.000772998,0.00832573}],  Interval[{0.000562181,0.00605507}], 0.},
{0., Interval[{0.000409923,0.00441516}], Interval[{0.000562181,0.00605507}],  Interval[{0.000409923,0.00441516}], 0.},
{0.,        0.,                              0.,                              0.,                0.}
```

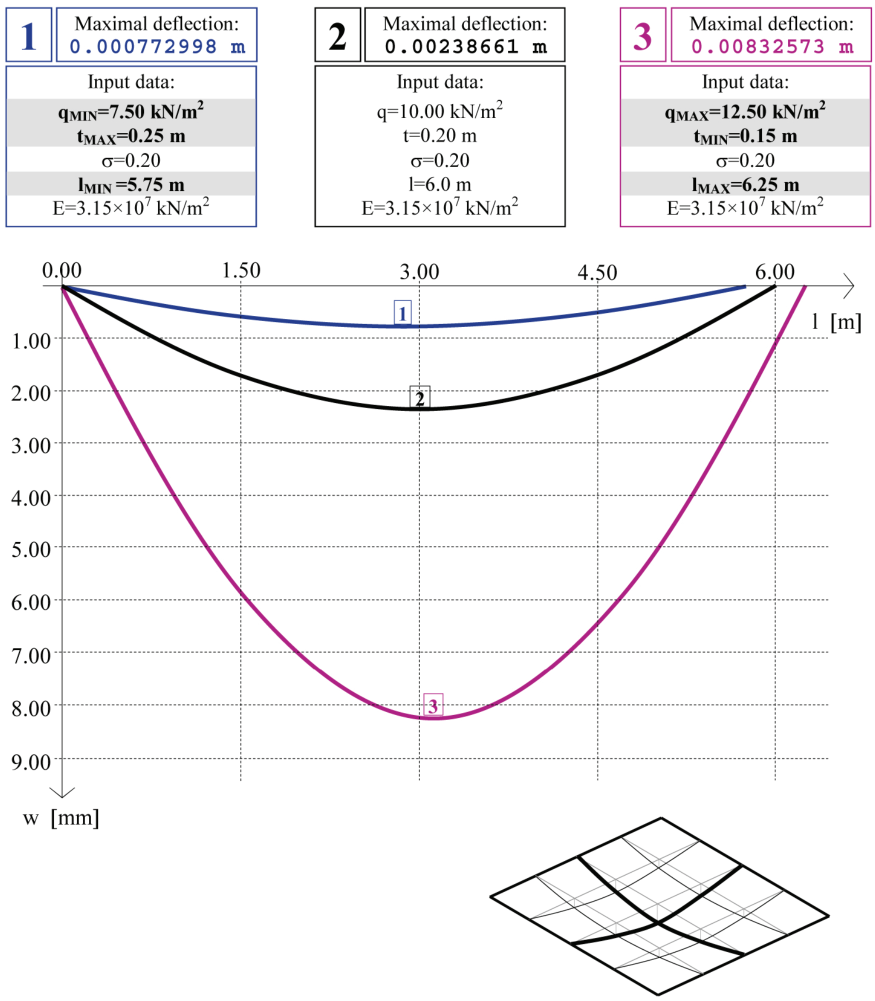

Fig. 9 Diagram of maximal plate deflections (Numerical example 4)

5.4. Numerical example 5 (Wolfram Mathematica®)

Calculation model was taken over from a Numerical example 1. Four input data (plate load, plate thickness, lenght of side of square and Young's modulus) were given at a certain interval.

Comment: only final results are shown in this example.

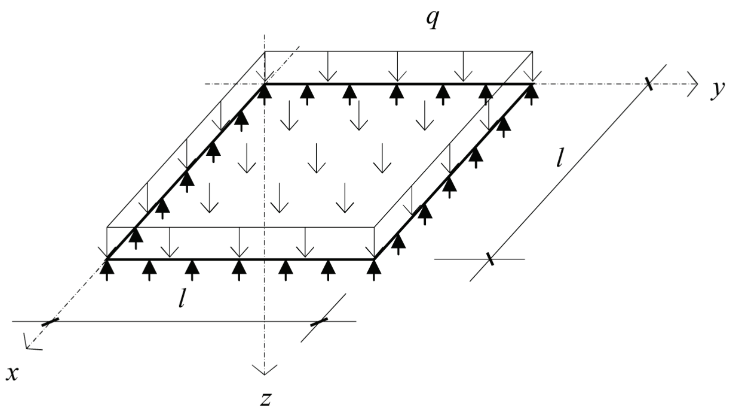

Fig. 10 Calculation model of simply supported square plate

▪ **Input data:**

n=4	- number of grid spacings along a side of the plate
$it_{\max}$=25	- number of Gauss-Seidel iterations
q=7.50÷12.50 kN/m^2	**- load per unit area (given at intervals)**
t=0.15÷0.25 m	**- plate thickness (given at intervals)**
l=5.75÷6.25 m	**- lenght of side of square (given at intervals)**
σ=0.20	- Poisson's ratio
E=2.85÷3.40 ×10^7 kN/m^2	**- Young's modulus (given at intervals, taken for MB20÷MB40)**

▪ **A part of computer output:**

```
Matrices u  i  w = plate deflection

{0.,         0.,                            0.,                            0.,                            0.},
{0., Interval[{0.000379782,0.00487991}], Interval[{0.000520844,0.00669245}], Interval[{0.000379782,0.00487991}], 0.},
{0., Interval[{0.000520844,0.00669245}], Interval[{0.00071616,0.00920212}],  Interval[{0.000520844,0.00669245}], 0.},
{0., Interval[{0.000379782,0.00487991}], Interval[{0.000520844,0.00669245}], Interval[{0.000379782,0.00487991}], 0.},
{0.,         0.,                            0.,                            0.,                            0.}
```

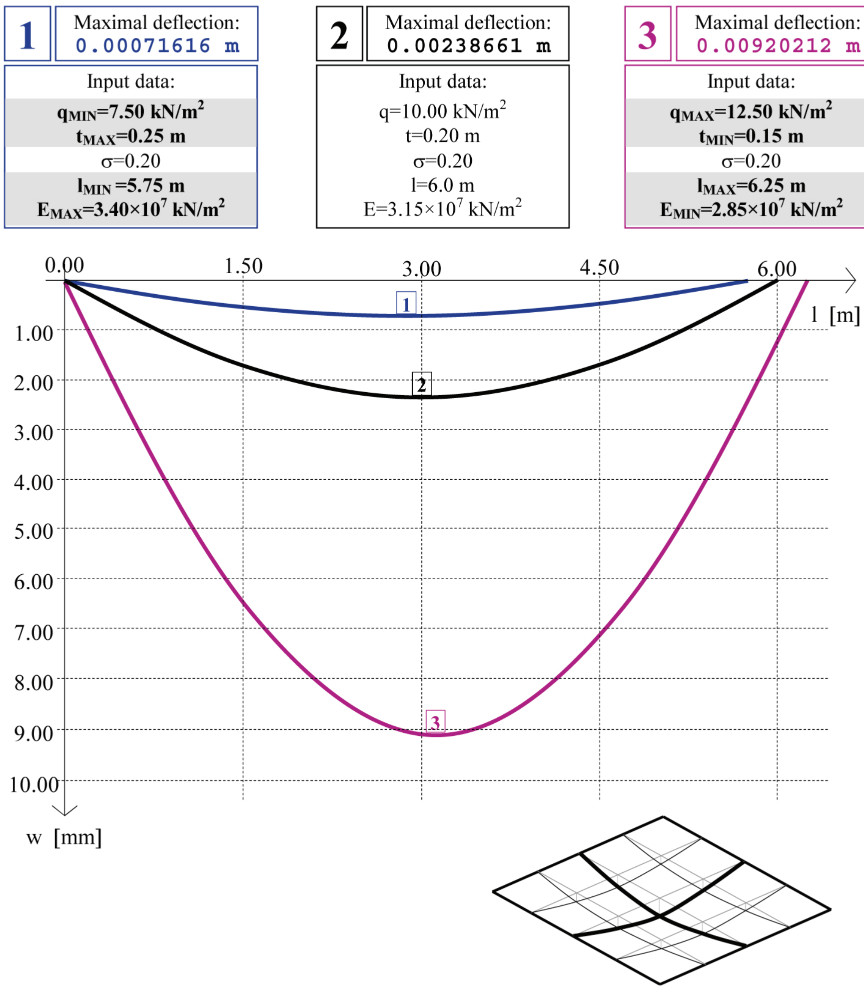

Fig. 11 Diagram of maximal plate deflections (Numerical example 5)

5.5. Discussion of results

Interval calculus can be used when calculating deflection of a reinforced concrete plate, because the final results in a form of closed interval can give better review of some input data influences on a maximum deflection. The results can be compared to a boundary deflection and then make conclusions connected to taking of optimal and rational problem solution. The advantage of such a calculus is that we can see the influences of different input parametres going on in one computing model.

Moreover, the given method for problem solving could be easily applied to a case of unequaly load per unit area by simple entering of suitable local values into matrics qoverd.

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PRIMENA INTERVALNOG RAČUNA PRI PRORAČUNU UGIBA PLOČE REŠAVANJEM POISSON-OVE BIKVADRATNE PARCIJALNE DIFERENCIJALNE JEDNAČINE

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U radu je analizirana primena intervalnog računa vezana za problematiku proračuna ugiba nosača izvedenog armiranim betonom. Za primer je uzeta armirano betonska ploča jednakih raspona, slobodno oslonjena na sve četiri strane, opterećena jednako raspodeljenim opterećenjem. Obzirom da na ugib nosača utiče više parametara, prikazan je proračun u kome su pomenuti parametri zadati u određenom opsegu (intervalu). Rezultati, tj. konačne vrednosti ugiba takođe se dobijaju u obliku intervala, te je moguće pratiti direktan uticaj promene jednog ili više ulaznih parametara na vrednosti ugiba, koristeći jedan model i jedan proračun.