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NONLINEAR SEISMIC ANALYSIS OF ASYMMETRIC IN PLAN BUILDINGS

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Abstract. *In many buildings the centres of resistance do not coincide with the centres of mass. As a consequence, lateral base motion during an earthquake gives rise to torsional vibration of the structure. The inelastic seismic behaviour of asymmetric-plan buildings is considered using the histories of base shear and torque. The procedure to construct the base shear and torque (BST) surface of the system with an arbitrary number of resisting elements in the direction of asymmetry and of ground motion is proposed. The factors that determine the shape of the BST surface are the strength eccentricity, lateral and torsional capacity of the system and planwise distribution of strength. The BST surface contains most of information necessary to describe the inelastic properties of a system. However, the inelastic deformation cannot be computed unless a nonlinear static or dynamic analysis is performed.*

Key words: *rural development program, spatial planning, regional policy.*

1. INTRODUCTION

The behaviour of buildings during earthquake will be satisfactory only if all measures are taken to provide a favourable failure mechanism. A special account must be taken so that torsional effects do not endanger or preclude global ductile behaviour of the structure. Building with asymmetric distribution of stiffness and strength in plan undergo coupled lateral and torsional motions during earthquakes. Asymmetric buildings are especially vulnerable during strong earthquakes. Because of torsion, the seismic demands of asymmetric buildings increase above those required by just translational deformation. It is well known that the larger the eccentricity between the centre of rigidity (stiffness) and the centre of mass, the larger the torsional effects. An important aspect of the inelastic behaviour of asymmetric structures is the considerations of the degree of control over inelastic twist. One of the design aims should be to restrain the system against unrestricted inelastic twist.

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An asymmetric-plan building consisting of rigid diaphragms with lumped storey masses is analysed. Lateral resistance of the building is provided by elasto-plastic structural elements (Fig. 1) located along resisting planes in the x– and y–directions. The resisting elements in the y–direction may have different stiffness and strengths, and may be arbitrarily located about the y–axis, creating an eccentricity e_{rx} between the centre of mass (CM) and the centre of rigidity (CR) of the building plan. On the other hand, the system is symmetric about the x–axis.

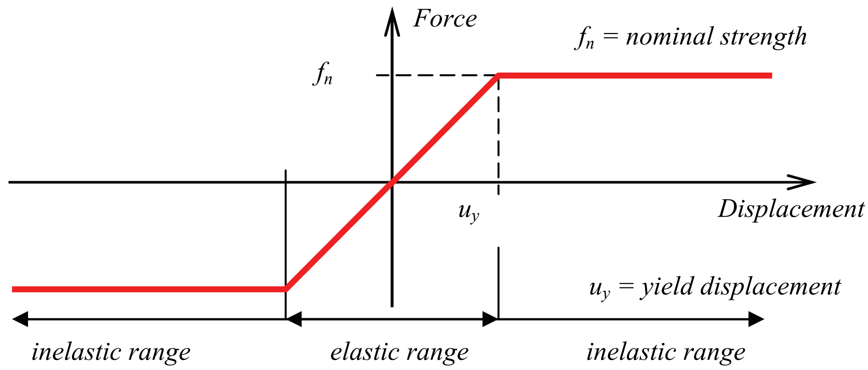


Fig. 1. Elasto – plastic force displacement relationship of resisting elements

Most of issues relevant to torsion are based on elastic structural response. In the structures, that remain elastic during an earthquake, torsional vibrations may cause significant additional displacements and forces in the lateral load resisting elements. However, the design of the majority of buildings relies on inelastic response. In that case torsional motion leads to additional displacement and ductility demands. Hence the relevance of current code recommendations, based on elastic torsional response, is open to questions.

2. Base shear and torque ultimate surface

The base shear and torque (BST) ultimate surface is the locus of the storey shear and torque combinations that applied statically onto the story produce a plastic mechanism. Therefore, no combination of shear and torque can go beyond this surface. Inelastic behaviour of the system is represented in this force space as motions along the surface. Notice that plastic deformations can occur even when there is no motion along the BST surface. Thus, the inelastic deformation cannot be computed from the BST surface unless a step-by-step static or dynamic analysis is performed.

2.1 Base shear torque relationships

The BST surface is assumed to divide the force space into two regions: the interior ant the exterior. The interior region containing combinations of the base shear force and torque representing elastic behaviour of the structure, while the exterior containing statically inadmissible base shear and torque combinations. Any point at the boundaries quantifies actions associated with the plastic (yielding) state and the kinematics of different collapse mechanisms in the system.

De la Llera and Chopra have been demonstrated that the BST surface is convex and it is point-symmetric with respect to the origin if the element yield displacements are the same under load reversals. In the paper [1] the expressions for co-ordinates (V, M_t) of the BST surface vertices for a building with only three resisting planes along the y–axis (with central plane passing through the centre of mass), and two resisting planes in the orthogonal direction are given. Here, the procedure to predict accurately the response of the system with an arbitrary number of resisting planes in y–direction (the direction of asymmetry and of ground motion) is proposed.

The BST surface is composed of linear branches and can be constructed knowing a finite number of points (Fig. 2). Point A corresponds to a purely translational mechanism of the system and implies that all resisting planes in the y-direction must yield. Therefore, the equilibrium in the system gives:

$$V_{nA} = V_{ny} = \sum_{k=1}^{n} f_{nyk}, \qquad M_{tA} = V_{ny} \cdot e_{vx}, \qquad e_{vx} = \frac{1}{V_{ny}} \sum_{k=1}^{n} f_{nyk} \cdot x_k \tag{1}$$

where V_{ny} is the lateral capacity of the structure in the y–direction, f_{nyk} is the lateral capacity (nominal strength) of the k-th resisting element in the y–direction, n is the number of resisting planes in the y–direction, e_{vx} is the strength eccentricity, and x_k is the distance of k-th resisting plane from the centre of mass.

The plastic mechanisms associated with branch A – 0 are generated by the rotation of the system about resisting plane 1, leaving the deformation of this element equal to the yield displacement u_{y1}. For this case, the equilibrium in the system gives:

$$V_n = V_{ny} = \sum_{k=1}^{n} f_{nyk}, \qquad M_t = V_{ny} \cdot e_{vx} + \lambda \cdot V_{nx} \cdot b, \qquad 0 \le \lambda \le 1 \tag{2}$$

where V_{nx} is the lateral capacity of the structure in the x–direction, b is the distance between resisting planes in the x–direction (Fig. 2), and λ is the parameter that includes the torque caused by deformation of resisting planes in the x–direction ($\lambda = 0$ for equilibrium of the system at point A, and $\lambda = 1$ for equilibrium at point 0). For the x–direction, the equal nominal strength of the both orthogonal elements is assumed ($f_{nx1} = f_{nx2} = V_{nx} / 2$).

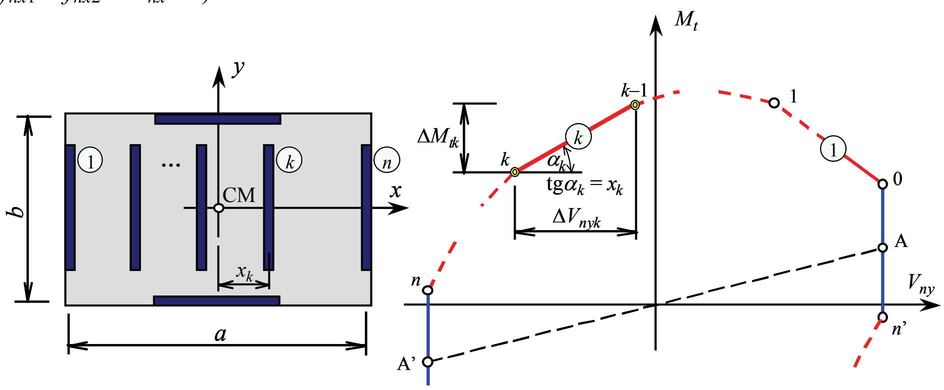


Fig. 2. The BST surface of the system with an arbitrary number of the resisting planes

All the plastic mechanisms associated with branch 0 – 1 have the same rotation, i.e. these collapse mechanisms are generated by the translation of the system, keeping the deformations of resisting plane 1 always in the elastic range (from $+u_{y1}$ to $-u_{y1}$). The equilibrium of the system at point 0 gives:

$$V_{n0} = V_{ny}, \qquad M_{t0} = V_{ny} \cdot e_{vx} + V_{nx} \cdot b \tag{3}$$

Because of the translation (from $+u_{y1}$ at point 0 to $-u_{y1}$ at point 1), the forces in resisting plane 1 are reduced, which in turn reduces the base shear V_{ny} and increase the base torque M_t resisted by the system:

$$V_{ny1} = V_{ny0} - 2f_{ny1}, \qquad M_{t1} = M_{t0} + 2f_{ny1} \cdot |\, x_1 \,| \tag{4}$$

The plastic mechanisms associated with next branch are a sequel of the previous mechanisms, keeping the deformations of one resisting plane always in the elastic range. Each branch of the BST ultimate surface is defined with two points: "k–1" and "k" (Fig. 2). The co-ordinates (V_{nk}, M_{tk}) of the point "k" are given by the following expressions:

$$V_{nyk} = V_{nyk-1} - \Delta V_{nyk}, \qquad M_{tk} = M_{tk-1} - \Delta M_{tk} \tag{5}$$

where ΔV_{nyk} and ΔM_{tk} are the increments of base shear and torque:

$$\Delta V_{nyk} = 2f_{nyk}, \qquad \Delta M_{tk} = 2f_{nyk} \cdot x_k \tag{6}$$

In this way, the co-ordinates of subsequent vertex (point "k") of considered branch can be obtained if the co-ordinates of preceding vertex (point "k–1") are known. Thus, the BST surface for the first and second quadrant can be computed using simple plastic analysis concept. Because the BST surface is point-symmetric with respect to the origin, the co-ordinates of points for the other two quadrants can be easily obtained.

The slope of a tangent to the BST surface is equal to the location of the element in the building plan ($\mathrm{tg}\alpha_k = \Delta M_{tk} / \Delta V_{nyk} = x_k$). This slope defines the centre of plastic rotation of the system. The BST surface has as many branches with finite slope as twice the number of resisting planes in the direction of ground motions. The first branch is associated with mechanisms that leave the leftmost resisting plane in the elastic range, the second branch the second farthest plane to the left, and so forth until we reach the rightmost resisting plane.

2.2 Properties of BST surface

Because the inelastic behaviour of a building is developed along the BST surface, its shape controls this behaviour [2]. Therefore, even without static or dynamic nonlinear analysis it is possible to compare the expected seismic performance of different structural configurations based on their BST surface. The factors that determine the shape of the BST surface and influence the inelastic behaviour are the strength eccentricity, lateral and torsional capacity of the system and planwise distribution of strength (Fig. 3 and Fig. 4).

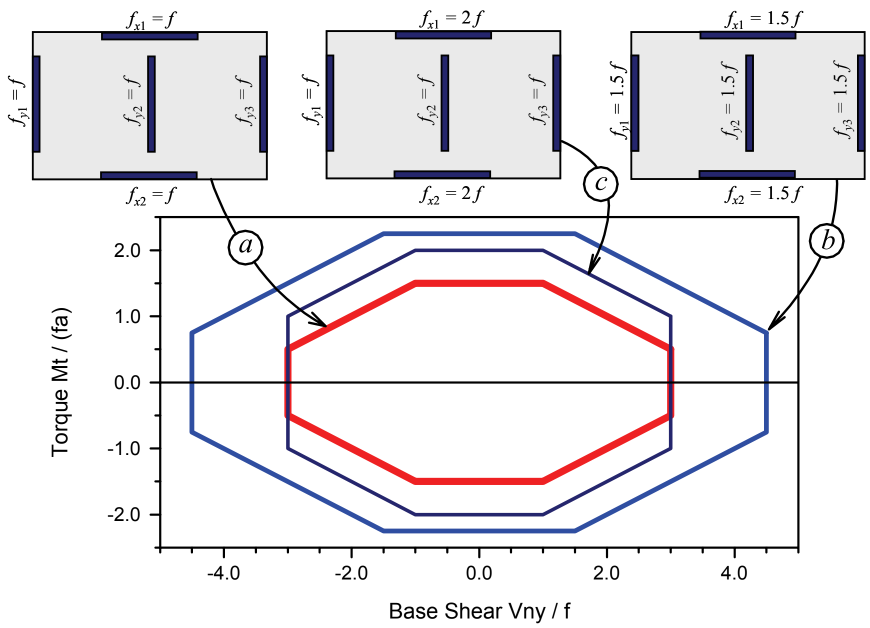


Fig. 3. Effects of different parameters on the BST surface: a) symmetric structure, b) global increase in strength, c) increase in strength of orthogonal resisting planes

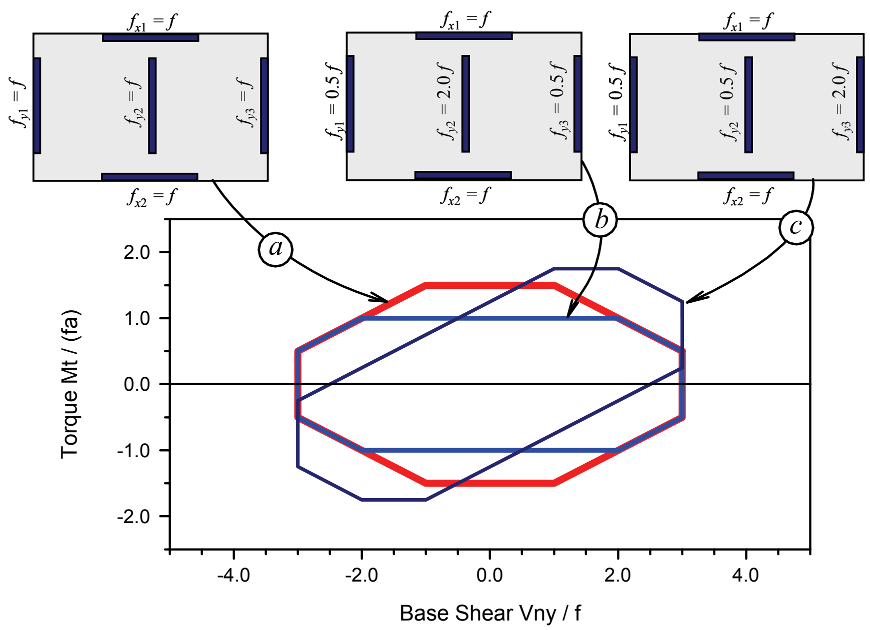


Fig. 4. Effects of different parameters on the BST surface: a) symmetric structure, b) planwise distribution of strength, c) strength asymmetry

Reference system is symmetric structure, which has plan aspect ratio $b / a = 1 / 2$ and five resisting planes, three in the *y*–direction and two in the *x*–direction, all with identical properties (Fig. 3a). A proportional increase (or decrease) in the strength of each resisting planes in both directions, produces an isotropic dilation (or contraction) of the BST ultimate surface (Fig. 3b). The increase in strength of resisting planes in the orthogonal direction produces a stretching of the BST surface along the base torque axis in the positive and negative directions (Fig. 3c). As a result of this increase in strength, the length of the constant base shear branch of the BST surface is also increased. It leads to a proportional increase in the resisted base torque for all regions of the BST surface.

Another factor that affects the shape of the BST ultimate surface is the distribution of strength along the building plan (Fig. 4b). It is apparent that increasing the strength of resisting plane 2 (as in building with a strong central core) relative to other planes has two effects: it reduces the torsional capacity of the system, and it produces a stretching of the constant torque branches of the BST surface associated with purely torsional mechanisms.

The most important effect of strength asymmetry is to skew and stretch the BST surface toward the first and third quadrants (Fig. 4c). Because the resisting plane 3 has the large strength (relative to other planes) it produces increase of the length of branch 3. The skewness and stretching of the BST surface could have important consequences on the inelastic seismic behaviour of such (asymmetric) systems. Any of these systems excited into inelastic range will present inelastic behaviour along the long branches. It implies that the strongest resisting plane will remain essentially elastic while the other planes will yield significantly. Thus, the lateral capacity of the system, provided mainly by the large strength of the strongest element, will never be developed.

It is well known that stiffness asymmetry is an important parameter controlling the elastic response of the structure. Stiffness asymmetry has significant influence on distribution of forces among resisting planes in the elastic range. It controls how base shear and torque combinations move inside the BST surface. However, the shape of the BST surface is independent of the stiffness asymmetry in the system. In inelastic response, the yield strength of the elements and their location are the ones determining distribution of forces among resisting planes.

3. Deformation Demands

When structural performance relies on ductile response during a major seismic event, the relationship between inelastic deformations of the system affected by translational and torsional actions should be considered. But, as a general rule, in current design practise this is not done. The primary aim in the seismic design of buildings should be to address displacements corresponding with performance criteria. Considerations relevant to the ultimate limit state are governed by displacement ductility capacities that can be reliably provided for lateral force–resisting elements. Global displacement ductility factors, specified in codes for typical structural systems, are inappropriate when the geometry of the components of the system is different. Instead of estimating the ductility capacity of a system, the designer should identify the critical element and, to protect that element, reduce the design ductility demand on the system.

In a recent study Paulay [3] deals with the seismic design for torsional response of ductile buildings, where buildings are classified into two categories: *torsionally re-*

strained and *torsionally unrestrained*. In a torsionally unrestrained building, all elements that resist torsion may be yielding during an earthquake, while in a torsionally restrained building the resisting planes orthogonal to the ground motion direction are still elastic. Fig. 5 illustrates an example of torsionally unrestrained structure used by Paulay to study torsional response [4].

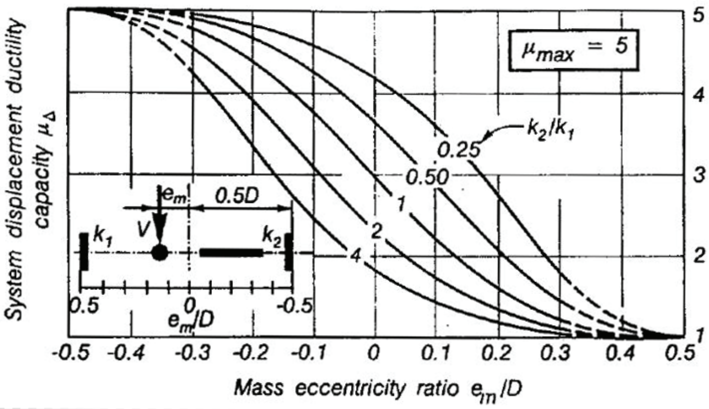


Fig. 5. Displacement ductility capacity of torsionally unrestrained system [3]

In the absence of eccentricities (in terms of both strength and stiffness), this structure is commonly referred as "torsionally balanced". Paulay suggests that a small excess of strength will lead to inelastic twisting of the system. In this case, if the ductility capacity of element μ_{max} is not to be exceeded, than the displacement ductility demand of the system μ_{Δ}, measured at the centre of mass, is to be severally restricted:

$$\mu_{\Delta} = \frac{\mu_{\max} - 1}{1 + \omega_0} + 1 \tag{7}$$

where ω_0 is structural parameter which depends on distribution of mass and stiffness in the building plan:

$$\omega_0 = \frac{k_2}{k_1}\left(\frac{0.5 + e_m / D}{0.5 - e_m / D}\right)^2 \tag{8}$$

The variation of μ_{Δ} is shown in Fig. 5. In this case, it is assumed that only element 2 will be subjected to inelastic deformation. When it is assumed that element 2 rather than element 1 remain elastic, the values given in Fig. 4 may be used with indexes 1 and 2 simply interchanged.

The assumptions (7) made by Paulay are highly conservative. Contrary to his assumption, the torsionally unbalanced building does not pivot about the stronger element during its dynamic response to the earthquake. To illustrate the foregoing points, the dynamic analysis of an unrestrained system with two resisting planes in *y*–direction (Fig. 4), subjected to the El Centro earthquake is performed [2]. To ensure the assumption made by Paulay, the strength of the element 2 is increased for 20% in relation to the strength of the element 1 (ω_0 = 1.20). The nonlinear dynamic analysis is performed using Newmark's method. An event–to–event solution strategy is used, where the structure properties are re-formed each time there is a nonlinear event (a change in stiffness). Results of analysis are shown in Fig. 6 and Fig. 7.

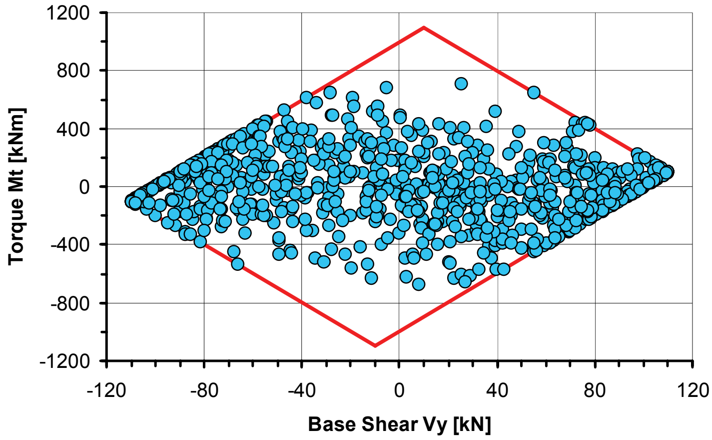


Fig. 6. Seismic response of torsionally unrestrained system: base shear and torque response histories

Results of analysis show that torsional motion produces in a significant increase in the displacement of the flexible edge. The maximum displacement demands of resisting planes 1 and 2, and the centre of mass are: $\max u_{1y}$ = 11.26 cm, $\max u_{2y}$ = 5.26 cm, and $\max u_{cmy}$ = 7.40 cm, respectively. The maximum ductility demand of the weaker resisting plane (element 1) is μ_{max} = 11.26. According to Paulay's equation (7), the ductility demand of the system, measured at the centre of mass, should be μ_Δ = 5.66, but the actual value is just μ_Δ = 7.40. Obviously, Paulay's estimate is highly conservative. This is easily explained by observing displacement history, which shows that both element yield and pivoting does not take place about the yield position of the strong element (Fig. 7).

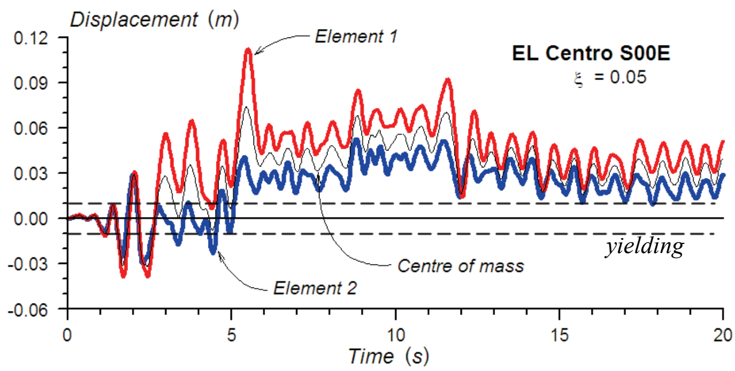
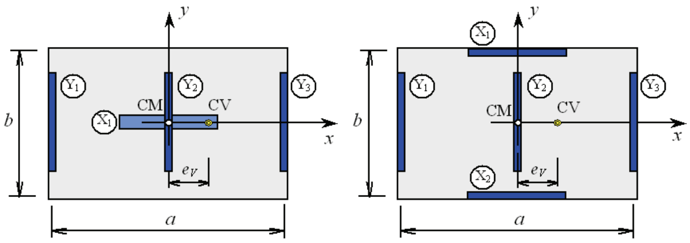


Fig. 7. Seismic response of torsionally unrestrained system: displacement time history

To study the response of torsionally restrained and unrestrained system to real seismic action, the two single-storey buildings with plan aspect ratio a / b of 2 and different structural configurations are considered [5]. The lateral capacity of the both systems is the same (Fig. 8), and the both system have equal normalized stiffness and strength eccentricity ($e_{vx} = e_{rx} = 0.125\ a$). The both systems were subjected to the N-S component of the El Centro ground motion in the y–direction. Results of analysis are shown in Fig. 9 and Fig. 10 for torsionally unrestrained system, and in Fig. 11 and Fig. 12 for restrained system.



$k = 4470.975$ kN/m
$k_{y1} = k_{y2} = k$
$k_{y3} = 2\ k$
$K_y = 4\ k,\ \ K_x = 2\ K_y / 3$
$f_{y1} = f_{y2} = f = 33.3$ kN
$f_{y3} = 2\ f$
$V_{ny} = 4\ f,\ V_{nx} = 2\ V_{ny} / 3$
$e_{vx} = e_{rx} = 0.125\ a$

Fig. 8. Characteristics of torsionally unrestrained and restrained system

For of torsionally unrestrained system results of analysis show that the most of the inelastic behaviour occurs along the two parallel branches with positive slope (Fig. 9). This implies that resisting plane 3, i.e. the strongest element in the y–direction, remains elastic in many instants during the response. As consequence, the instantaneous centre of plastic rotation is located in resisting plane 3 (the stiff edge of the building) during the most of the inelastic response. Because of that, the resisting plane 1 (the farthest plane from element 3) will experience a significant increase in displacements relative to the plane 3 due to the plan rotation (Fig. 10).

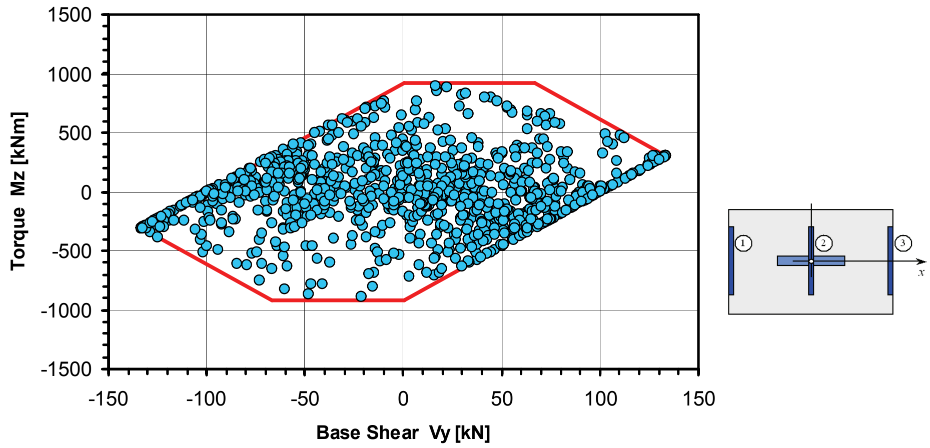


Fig. 9. Base shear and torque response histories of torsionally unrestrained system

The seismic response of the restrained system is substantially favourable, because a larger proportion of the inelastic behaviour migrates from the branches of the BST surface with positive slope to the constant base shear branches (Fig. 11). It implies that in this case (for restrained system) the numbers of plastic mechanisms that involve yielding of all y–direction planes were developed. Thus, more uniform displacement demands are expected for the resisting planes in these systems (Fig. 12).

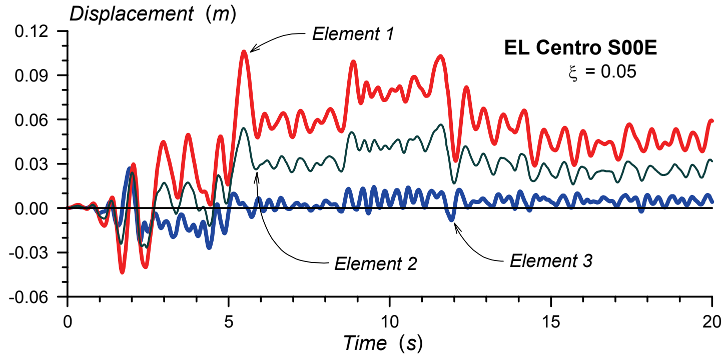


Fig. 10. Displacement time history of torsionally unrestrained system

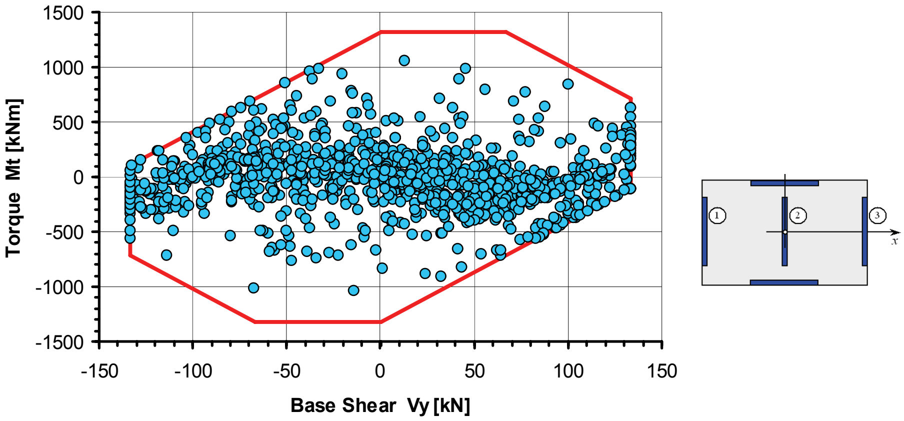


Fig. 11. Base shear and torque response histories of torsionally restrained system

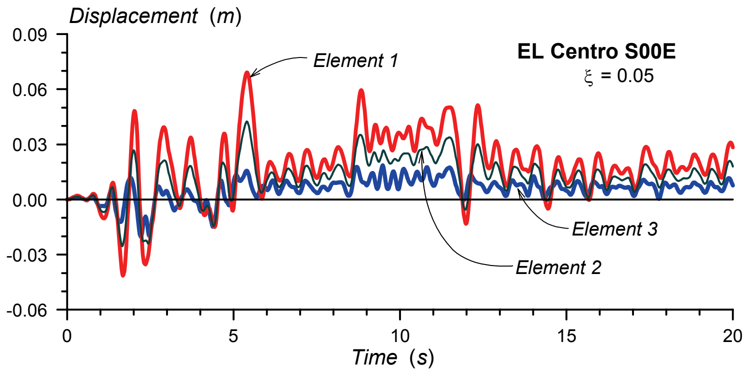


Fig. 12. Displacement time history of torsionally restrained system

4. CONCLUSIONS

The base shear and torque response histories, especially with the BST surface, may be a useful tool for conceptual seismic design of asymmetric-plan buildings. The factors that determine the shape of the BST surface and influence the inelastic behaviour are the strength eccentricity, lateral and torsional capacity of the system and planwise distribution of strength. Stiffness eccentricity does not affect the shape of the BST surface, but it controls where on this surface the system develops its inelastic behaviour.

The BST surface contains most of information necessary to describe the inelastic properties of a system. Its shape is directly related to the yielding mechanisms of the structure and, thus, controls the relative displacement demand among resisting planes. Inelastic behaviour of the system is represented in this force space as motions along the surface. However, the inelastic deformation cannot be computed from the BST surface unless a non-linear static or dynamic analysis is performed. Results of performed analysis show that seismic response of the restrained system is substantially favourable than unrestrained one. Also, in this case more uniform displacement demands are expected for the lateral load resisting planes.

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NELINEARNA SEIZMIČKA ANALIZA U OSNOVI NESIMETRIČNIH ZGRADA

Đorđe Lađinović

Kod mnogih zgrada centri nosivosti, krutosti i masa se ne poklapaju. To može dovesti do pojave torzionih vibracija u slučaju da zgrada bude izložena dejstvu zemljotresa. U radu je razmatrano neelastično ponašanje nesimetričnih zgrada prateći zavisnost između ukupne smičuće sile (BS) i momenta torzije (T) u osnovi zgrade. Predložen je postupak konstruisanja interakcionog BS–T dijagrama za sistem sa proizvoljnim brojem nosećih elemenata postavljenih u pravcu dejstva zemljotresa. Neelastični odgovor sistema i oblik interakcionog dijagrama zavisi od ekscentriciteta između centra masa i nosivosti, od kapaciteta torzione i poprečne nosivosti konstrukcije i distribucije nosivosti u osnovi zgrade. Interakcioni BS–T dijagram sadrži najveći deo informacija potrebnih da se opišu neelastične osobine posmatranog sistema, ali nelinearne deformacije ne mogu biti sračunate sve dok se nelinearna statička ili dinamička analiza ne sprovede.