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PLASTIFICATION PROCESS OF STEEL FRAMES THROUGH DISCRETIZATION OF PLASTIC ZONE

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Esad Mešić

Civil Engineering Faculty, University of Sarajevo, Bosnia and Herzegovina

Abstract. *In the paper there is propose of numerical model for nonlinear analysis of steel frames with use of localized nonlinearity. Main concept of model consists of discret ordering of series nonlinear finite elements – springs «zero length» in the areas of expected frames plastifikations. The frame elements between the sections with nonlinear elements behaved linear.On that way the forces in the elements of frames for the most degrees of freedom are linear in the relation of displacement, but nonlinear behaviour occurs in relatively small number degrees of freedom. This kind of model can be explicitly introduced in consideration of geometrical imperfection, backward stresses and str.*

Key words: *Steel Frames, Nonlinear analysis,Semi-rigid connections*

1. Analysis of Problem

In the last thirties years occurs different plastic analysis of steel structures. Basically, they can be divided into two groups:

- Model of localized plasticity – plastic hinges;
- Model of plastic zones.

The method of plastic hinges in general is in its use is very simple and it is based on a simple suppositions. Obtained solutions in global analyses represent the real behavior of structures with maximal ultimate loads. However, this method does not give reliable solutions for the cases in which the behavior of structure in conditionally with the behavior of individual elements, then when the higher level axial forces occurs in elements, and spread plasticity of cross sections and the plasticity on extents lengths. Also, this method is not recommended to use in systems with flexible connection.

The method of plastic zones is accurate method of elastic-plastic analysis of frame structures which takes in consideration extension as in cross sections as along elements. Also, taking in consideration change of position of neutral axis, with a concern on the size of elastic core (in function of progress of plastic areas), then different possibilities of dis-

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tribution residual stresses end effects large deflection. Basic lack of method in plastic zones is its relative complicity and complexity design for usual use in construction.
In this paper I propose the method for nonlinear of steel frames analysis which contains the main advantages which are described above in two methods, and the main characteristic are:

- Possibility of gradual plasticity of cross sections – gradual decrease of stiffness on the level of cross section;
- Possibility of extension of plastic zones on definite length - decrease of stiffness on the system level;
- Following redistribution forces at a cross section in system on any level of load;
- Taking in consideration influence of axial force on plasticity of cross section (M-N interaction curve);
- Modeling common type of connection beam-column (semi-rigid connection) based on experimental and numerical date;
- Involving in consideration an initial imperfection and backward tensions.

According to previous items this method is called **The Plastification process of steel frames through discretization of plastic zone.** This analyze method is represented with computation programme SAP2000n which contains possibility of insertion nonlinear elements ("link" elements) in discrete cross sections along span in system with linear ("frame") elements. Geometrical nonlinearities (large deformations) are considered with geometrical stiffness matrix. In the paper are established two models of elastic-plastic behavior of cross section (M-ϕ relationship). Integral line of failure is introduced which defined common influence bending moment and axial force on plasticity. The model of semi-rigid connection and its influence on behavior of steel frames is also presented.

2. EQUILIBRIUM EQUATION IN NONLINEAR AREA

In general case, the behavior of structures is not linear, but depends by relationship generalized forces - generalized displacement on the level of a pure deformation value (material nonlinearity) as on the level of displacement element as rigid body (geometrical nonlinearity). So, in common case the equilibrium equations two mentioned criteria will be carried. They are considered introducing relation where in the purpose of simplicity analysis elastic-plastic stiffness matrix $[K_{ep}]$ is separated by geometrical stiffness matrix $[K_g]$:

$\{F\} = ([K_{ep}] + [K_g]) \cdot \{u\}$, where are ($\{F\}$- forces, $\{u\}$-displacements).

2.1 Elastic-plastic stiffness matrix

Here we introduce the model of elastic-plastic behavior on the base of stiffness matrix of member with nonlinear elements – springs at the ends (in the rotation direction – transversal stiffness matrix, Fig. 1, and axial – matrix of axial stiffness, Fig. 2 for material nonlinearity cross section of member or flexibility of connections [3].

Fig. 1. Model of member with rotational semi-rigid connections on the ends (S_{rj}, S_{rk} – rotational stiffness)

Fig. 2. Model of member with axial flexibility on the ends (S_{aj}, S_{ak} – axial stiffness)

Superposition of suitable stiffness matrix for members with degrees of freedom, according to the Figure above, we are obtained:

$$[K_{ep}] = \frac{EI}{L(1+\beta_3)} \begin{bmatrix} \alpha_0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & \frac{12+\beta_1+\beta_2}{L^2} & \frac{6+\beta_2}{L} & 0 & -\frac{12+\beta_1+\beta_2}{L^2} & \frac{6+\beta_1}{L} \\ 0 & \frac{6+\beta_2}{L} & 4+\beta_2 & 0 & -\frac{6+\beta_2}{L} & 2 \\ -\alpha_0 & 0 & 0 & \alpha_0 & 0 & 0 \\ 0 & -\frac{12+\beta_1+\beta_2}{L^2} & -\frac{6+\beta_2}{L} & 0 & \frac{12+\beta_1+\beta_2}{L^2} & -\frac{6+\beta_1}{L} \\ 0 & \frac{6+\beta_1}{L} & 2 & 0 & -\frac{6+\beta_1}{L} & 4+\beta_1 \end{bmatrix}$$

Where are: $\alpha_0 = \frac{A^*(1+\beta_3)}{L}$; $A^* = \frac{A}{1+\frac{EA}{L}\left(\frac{1}{S_{a1}}+\frac{1}{S_{a2}}\right)}$;

$\beta_1 = \frac{12EI}{S_{rj}L}$; $\beta_2 = \frac{12EI}{S_{rk}L}$; $\beta_3 = \frac{\beta_1}{3} + \frac{\beta_2}{3} + \frac{\beta_1\beta_2}{12}$.

2.2 Geometrical stiffness matrix

In this paper geometrical nonlinearity is applied which including large displacement on simplicity way, so called P-Δ method. The essence is in increasing bending moment caused by axial force because of displacement from the basic load. Iteration analyze is using for determined axial force for P-Δ analyze. Preliminary, linear analysis used, and every next iteration is using with solving a new system of equations in a new position, until realization of convergence criteria for forces and displacements.

3. Integral Curve of Yielding. Interaction M-N.

In consideration that from an axial forces and a bending moment occurs normal stresses, relation between M and N is very easy to establish. For different cross sections it is possible plotted integral curves of yielding variation relationships between M and N – interaction curves. On the Fig. 3 are describe curves of yielding for different relationships between dimension of flanges and webs (bending about a strong axis) [1].

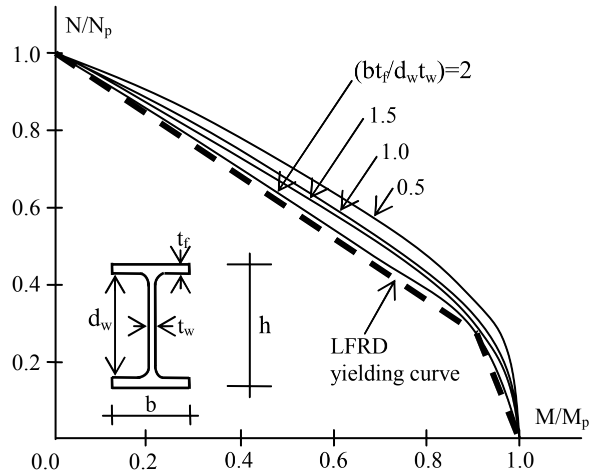

Fig. 3. Interaction curves of yielding M-N for different relationships (bt_f/d_wt_w) and suitable interaction curves according to The American standards for steel structures (LFRD-AISC).

Lower boundary curve is curve of yielding according LFRD-AISC – American standards for steel structures and presents approximation on the safe side for every "I" shape. It is taken here as secure approximation for any cross-section.

The conditions of yielding are given by equation:

$$\frac{N}{N_y} + \frac{8}{9}\frac{M}{M_p} = 1 \quad for \quad \frac{N}{N_y} \geq 0.2 \text{ and}$$

$$\frac{N}{2N_y} + \frac{M}{M_p} = 1 \quad for \quad \frac{N}{N_y} < 0.2$$

Where are: N_y – axial carrying capacity of element without buckling, M_p– plastic caring capacity for the case of pure bending, until M and N are cross section forces in general case obtained according to Second-Order Analysis.

The influence of shear force on plastification can be neglected with the assumption that condition $h/L \leq 1$ at all elements is satisfied.

In the paper are used two models of behavior (moment-rotation M–ϕ) nonlinear elements (Fig. 4., a) and b)):

- Bilinear model of elastic-plastic behavior - Model "a" (a);
- Exponential model of elastic-plastic behavior – Model "b" (b).

The first model is formed on the base of consideration Kempa [8], except it is a little bit modified because of adaptability LRFD line of yielding [1]. According to this model relaxation is express for value of moment:

$$M_{y,N} = 1.125 \cdot M_p \cdot \left(1 - \frac{N}{N_y}\right) \cdot \left(0.9 - 0.25\frac{N}{N_y}\right) \qquad \text{for} \qquad \frac{N}{N_y} \geq 0.2$$

$$M_{y,N} = M_p \left(1 - 0.5\frac{N}{N_y}\right) \cdot \left(0.9 - 0.25\frac{N}{N_y}\right) \qquad \text{for} \qquad \frac{N}{N_y} < 0.2$$

where is

$$M_{p,N} = 1.125 \cdot M_p \cdot \left(1 - \frac{N}{N_y}\right) \qquad \text{for} \qquad \frac{N}{N_y} \geq 0.2$$

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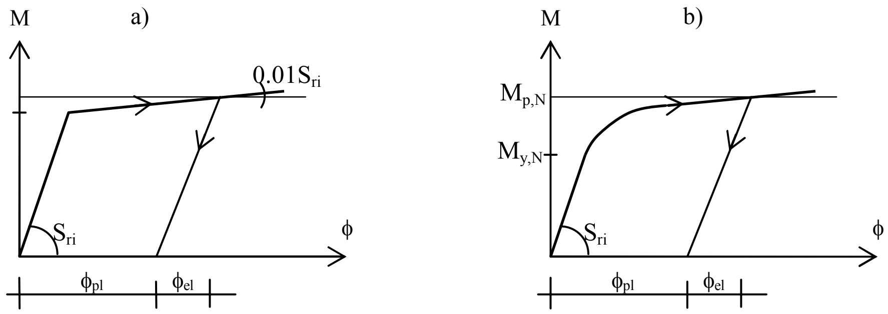

Fig. 4. Relationship moment-rotation (M–ϕ) nonlinear elements

The second model of behavior M–ϕ is suggested by Liewa and Chena [2] and gives possibilities of uniform changes of stiffness in cross section, and taking in consideration (implicitly) structural imperfections, which reduce stiffness.

4. MODEL OF FLEXIBLE CONNECTION

Specific importance at the frames have the connections beam – column. Flexibility of these connections has significant influence on distribution of section forces and frame deformations. Relationship M–ϕ can be obtained on experimental way for different kind of connections. Modeling of semi-rigid connections is possible with rotational springs at the end of the beam (Fig.1.) [3].

5. PLASTIFICATIN PROCESS OF STEEL FRAMES THROUGH DISCRETIZATION PLASTIC ZONES

Method of discretization of plastic zones represent the method in which occurs complex mechanism connections between systems of nonlinear springs – LINK elements and linear elements – FRAME elements. In this nonlinear analyze are used link elements type "Plastic 1", zero-length (integral part of programme SAP2000n) and can be located between the end joint of the structural elements or between structural elements and supports. Nonlinear analyze as "Time-history" analyze consisted of solving the following dynamic equilibrium equations:

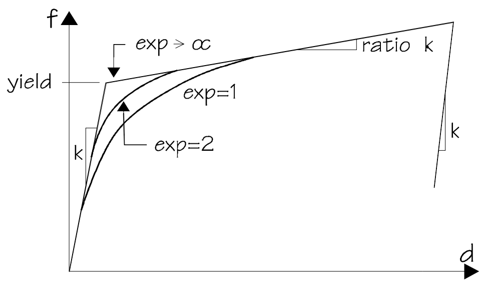

Fig. 5. Parameters definition of element Plastic1

$f = ratio \cdot k \cdot d + (1 - ratio) yield \cdot z$
k – spring constant
f_y – yield force
ratio – ratio of post–yield stiffness

$$[m]\{\ddot{v}(t)\} + [c]\{\dot{v}(t)\} + ([k_0] + [k_d])\{v(t)\} = \{p(t)\}$$

$[k_0]$ – stiffness matrix of linear-elastic elements;
$[k_d]$ – stiffness matrix of nonlinear elements

If the nonlinear stiffness changes are transferred to the right-hand side of equations of motion, one obtains

$$[m]\{\ddot{v}(t)\} + [c]\{\dot{v}(t)\} + [k_0]\{v(t)\} = \{p(t)\} - [k_d]\{v(t)\}$$

So that nonlinear changes of stiffness perform as pseudo-forces $[F_{sd}(t)] = [k_d]\{v(t)\}$ on the right-hand side of equation [5]. Further procedure in solving former system equation of motion transforming by natural modes, on this way we got the system uncoupled equations. On this way the procedure is simplified in relation on standard incremental procedure step by step.

6. THE EXAMPLE

The use of discretization method plastic zones is showed on two-storey steel frame with flexible connection in joints (1) (Fig. 6).

LINK elements in the areas of negative bending moments, located on the small distance (~0.6h, h-depth of elements – profiles which is discritized), in the areas of positive moments on bigger distance (~1.0-2.0h) and in the areas of linear change of moment line on ~1.0h. LINK elements are located in the places of flexible connections.

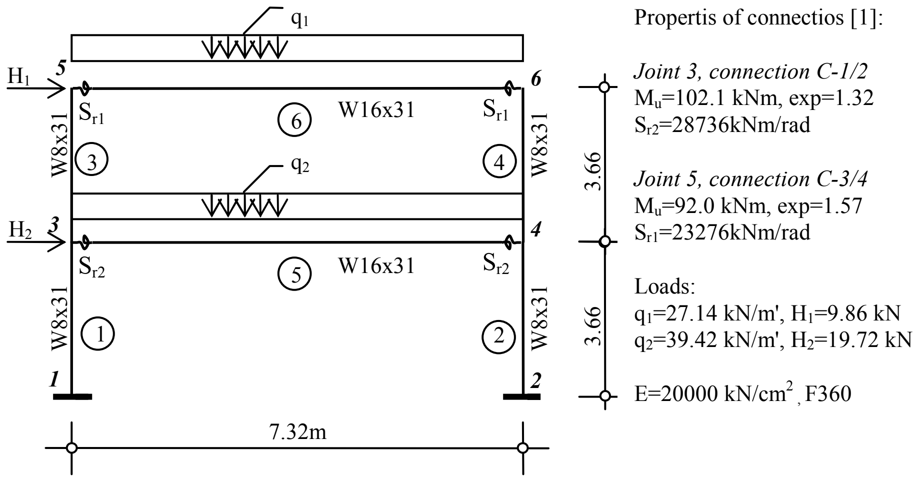

Fig. 6. Two-storey frame with flexible connections

Corresponding load combination for determine of ultimate load is λ(1.0DL+1.3WL) where DL is q_1 and q_2 load and WL load H_1 and H_2.

6.1 Results of analyze

Relationship M–ϕ - model "a" for member of frame, model "b" for connections:

The first yielding occurred in connections 4 and 6 on the load level λ=0.9. Plastic hinge is formed in the joint 4 for λ=1.05. After that plastification of beam 5 has happened, in the mid span, on length of 100 cm (5 LINK elements) at λ=1.25, what is also the ultimate load of frame.

According Chen (1), λ=1.25.

Relationship M–ϕ - model "b" for member of frame, model "b" for connections:

Plastification of frame has started with relaxation of connection in joint 4 at the load level λ=0.8, and continue with plasticity connection in joint 6 at λ=1.1. Like at the previous model, before ultimate limit state plastification occurs in the mid-span beam 5 on length of 125 cm (6 LINK elements). Ultimate load factor is λ=1.25.

According Chen (1), λ=1.259. Results obtained by this analyze are on the safe side, or no conservative inner satisfied boundary.

7. Conclusion

Suggested methodology for nonlinear analyze very complexly considered the problems of steel frames and can apply for practical needs. The methodology gives the possibility of including influences of axial forces on cross-section plastification. Also, taking in consideration simplify geometrical nonlinear analyze with elements imperfections as well as effects flexible connections. The method gives the possibility of getting section forces for any level of load up to ultimate load. Also, the method gives possibility of discovery "real" mechanism of crash, taking in consideration that it is the result of incomplete mechanism and large displacement (P-Δ).

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PRAĆENJE TOKA PLASTIFIKACIJE ČELIČNIH OKVIRA DISKRETIZACIJOM PLASTIČNIH ZONA

Esad Mešić

Predložena metodologija za nelinearnu analizu vrlo kompleksno tretira problematiku čeličnih okvira i primjenjiva je za praktične potrebe. Daje mogućnost uključivanja uticaja normalnih sila na plastifikaciju presjeka. Uključuje u razmatranje pojednostavljenu geometrijski nelinearnu analizu uz imperfekcije elemenata (strukturne i geometrijske) kao i efekte popustljivih veza. Metoda daje mogućnost otkrivanja „istinskog" mehanizma sloma, sobzirom da je on rezultat nepotpunog mehanizma i velikih pomjeranja (P-Δ), kao i dobivanje presječnih sila za bilo koji nivo opterećenja.