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DETERMINATION OF THE CHARACTERISTIC PARAMETERS IN THE SPECIAL COLLINEAR SPACE IN THE GENERAL CASE

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Abstract. *The projective space consists of the finitely and infinitely distant elements. The special collinear spaces in the general case, are set with five pairs of biunivocally associated points, so the quadrangle in the first space obtained by the three principal and one penetration point of the remaining two through the plane of the first three identical or similar to the associated quadrangle obtained in the same way in the second space. In order to associate two special collinear spaces, it is necessary to determine the following characteristic parameters: vanishing planes, space axes (principal normal lines), foci (apexes of the associated identical bundles of straight lines) and directrix plane (associated identical fields of points). The paper is based on constructive processing of the special collinear spaces in the general case. The structural methods which are used are Descriptive Geometry (a pair of Monge's projections) and Projective geometry.*

Key words: *special collinear spaces, vanishing planes, axes, foci and directrix planes of space*

1. INTRODUCTION

"In the most general case, the general collinear spaces do not have identical fields, and thus no foci in these field. If the special collinear spaces should have foci, than a group of five points that set these spaces will have to be set in a way that the four points in the first field, obtained by the three basic lines and the penetration point of the remaining two through the plane of the first three is identical or similar to the associated quadrangle obtained in the same way in the second plane" /1/. In the most general case the general collinear spaces are set with five pairs of biunivocally associated points, taken arbitrarily. Hence, the collinear spaces are divided into two basic cases:

1. General collinear (further GC) spaces - without foci and directrix planes
2. Special collinear (further SC)spaces - with foci and directrix planes

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2. Special Collinear Space – With Foci and Directrix Planes

Two SC spaces θ^1 are θ^2 set by five pairs of biunivocally associated points, $\mathbf{A_1B_1C_1D_1E_1}$ and $\mathbf{A_2B_2C_2D_2E_2}$, so that the quadrangle $\mathbf{A_1C_1D_1P_1}$ in the first space is obtained from the three basic points $\mathbf{A_1C_1D_1}$ and the penetration point $\mathbf{P_1}$ connecting the remaining two points $\mathbf{B_1E_1}$ through the plane of the first three $\mathbf{A_1C_1D_1}$, identical or similar to the associated quadrangle obtained in the same way in the second space. In the space θ^2 the associated quadrangle are $\mathbf{A_2C_2D_2P_2}$, and they are identical to the quadrangle $\mathbf{A_1C_1D_1P_1}$ in the space θ^1. The point $\mathbf{P_2}$ is the penetration point of the straight line $\mathbf{B_2E_2}$ through the plane $\mathbf{A_2C_2D_2}$.

The four associated points $\mathbf{A_1C_1D_1P_1}$ and $\mathbf{A_2C_2D_2P_2}$, which are forming the identical quadrangle, set a pair of associated identical fields of points (directrix planes), $\mathbf{K_1}$ and $\mathbf{K_2}$, in the fields θ^1 and θ^2.

Association of two SC spaces is enabled through the following characteristic parameters: vanishing plane, space axis (principal normal line) of the space, foci (apexes of the associated identical bundles of straight lines) and directrix planes (associated identical fields of points) where one pair, $\mathbf{K_1}$ and $\mathbf{K_2}$, has already been set in advance.

The structural procedure for determination of the characteristic parameters in the SC spaces in the general case is significantly simpler than the GS spaces.

2.1. Determination of the vanishing planes in the SC spaces in the general case

Each projective space has one infinitely distant plane. The fictitious plane of the first space is associated to the vanishing plane of the second plane and vice versa. The structural procedure for determination of the vanishing planes comprises that the vanishing points should be determined on the associated sequence of points by establishing the cross ratio on the sequences.

The vanishing plane $\mathbf{N_1}$ (fig.1) in the space θ^1 associated to the infinitely distant plane $\mathbf{N_2^\infty}$ of the space θ^2, was determined by the vanishing points $\mathbf{T_1}$, $\mathbf{H_1}$ and $\mathbf{L_1}$ on the sequence of points ($\mathbf{A_1B_1}$), ($\mathbf{C_1D_1}$) and ($\mathbf{C_1B_1}$). The vanishing point $\mathbf{M_2}$ (fig.1) in the space θ^2 associated to the infinitely distant plane $\mathbf{M_1^\infty}$ of the space θ^1, is determined by the vanishing points $\mathbf{R_2}$, $\mathbf{U_2}$ and $\mathbf{Q_2}$ on the sequence of points ($\mathbf{A_2B_2}$), ($\mathbf{C_2D_2}$) and ($\mathbf{C_2B_2}$).

In order to determine the vanishing points on the associated sequences of points, it is necessary to fine yet another associated point on each sequence, because in this way it is possible to establish the cross ratio. The constructive procedure for the determination of vanishing points, figures of fictitious points on the associated sequences of points is to bring the sequences into the perspective position by coinciding one associated pair of points.

The straight line $\mathbf{A_1B_1}$ penetrates the plane $\mathbf{C_1D_1E_1}$ in the point $\mathbf{S_1}$ of the space θ^1, and the $\mathbf{S_2}$, which is a penetration point of the straight line $\mathbf{A_2B_2}$ through the plane $\mathbf{C_2D_2E_2}$ in the space θ^2 is associated to it. The infinitely distant point $\mathbf{R_1^\infty}$ on the sequence ($\mathbf{A_1B_1}$)

has associated the $\mathbf{R_2}$ point on the sequence ($\mathbf{A_2B_2}$), so the vanishing point $\mathbf{R_2}$ is determined from the relationship $\lambda=(\mathbf{A_1B_1S_1R_1}^{\infty})=(\mathbf{A_2B_2S_2R_2})$. In the same manner the vanishing points $\mathbf{U_2}$ and $\mathbf{Q_2}$ on the sequences ($\mathbf{C_2D_2}$) and ($\mathbf{C_2B_2}$) are determined. The point $\mathbf{I_1}$ is the penetration of the straight line $\mathbf{C_1D_1}$ through the plane $\mathbf{A_1B_1E_1}$ in the space $\theta^{\mathbf{1}}$, and the point associated to it $\mathbf{I_2}$ is the penetration point of the straight line $\mathbf{C_2D_2}$ through the plane $\mathbf{A_2B_2E_2}$ in the space θ^2. Since the sequences ($\mathbf{C_1D_1}$)=($\mathbf{C_2D_2}$) are identical, the vanishing point $\mathbf{U_2}$ in the sequence ($\mathbf{C_2D_2}$) is the fictitious point, which is a result of the simple relationship $\sigma=(\mathbf{C_1D_1I_1})=(\mathbf{C_2D_2I_2})$, because $\mathbf{U_1}^{\infty}=\mathbf{U_2}^{\infty}$. The penetration of the straight line $\mathbf{C_1B_1}$ through the plane $\mathbf{A_1D_1E_1}$ is the point $\mathbf{F_1}$ in the space $\theta^{\mathbf{1}}$, which has the associated point $\mathbf{F_2}$, which is the penetration point of the straight line $\mathbf{C_2B_2}$ through the plane $\mathbf{A_2D_2E_2}$ in the space θ^2. The point $\mathbf{Q_2}$ in the sequence ($\mathbf{C_2B_2}$), has associated the fictitious point $\mathbf{Q_1}^{\infty}$ in the sequence ($\mathbf{C_1B_1}$) and it is determined from the relationship $\lambda=(\mathbf{C_1B_1F_1Q_1}^{\infty})=(\mathbf{C_2B_2F_2Q_2})$. The straight line $\mathbf{A_1C_1}$ penetrates through the plane $\mathbf{B_1D_1E_1}$ in the point $\mathbf{J_1}$ of the space $\theta^{\mathbf{1}}$, and the associated straight line $\mathbf{A_2C_2}$ penetrates the plane $\mathbf{B_2D_2E_2}$ in the point $\mathbf{J_2}$ of the space θ^2. Since the sequences ($\mathbf{A_1C_1}$)=($\mathbf{A_2C_2}$) are identical, the vanishing point $\mathbf{V_2}$ in the sequence ($\mathbf{A_2C_2}$) is the fictitious point, which results from the simple relationship $\sigma=(\mathbf{A_1C_1J_1})=(\mathbf{A_2C_2J_2})$, because $\mathbf{V_1}^{\infty}=\mathbf{V_2}^{\infty}$.

The vanishing plane $\mathbf{N_1}$ ($\mathbf{T_1H_1L_1}$) in the space $\theta^{\mathbf{1}}$, which is associated to the infinitely distant plane of the spaceθ^2, is determined by the vanishing points: $\mathbf{T_1}$ in the sequence ($\mathbf{A_1B_1}$), $\mathbf{H_1}$ in the sequence ($\mathbf{C_1D_1}$) and $\mathbf{L_1}$ in the sequence ($\mathbf{C_1B_1}$). The vanishing point $\mathbf{T_1}$, which is associated to the fictitious point $\mathbf{T_2}^{\infty}$ in the sequence ($\mathbf{A_2B_2}$), is determined with the cross ratio $\lambda=(\mathbf{A_1B_1S_1T_1})=(\mathbf{A_2B_2S_2T_2}^{\infty})$. The point $\mathbf{H_1}$, which is associated to the fictitious point $\mathbf{H_2}^{\infty}$ in the sequence ($\mathbf{C_2D_2}$), is an infinitely distant point, because the sequences ($\mathbf{C_1D_1}$)=($\mathbf{C_2D_2}$) are identical and it is determined from the simple relationship $\sigma=(\mathbf{C_1D_1I_1})=(\mathbf{C_2D_2I_2})$, because $\mathbf{H_1}^{\infty}=\mathbf{H_2}^{\infty}$. The vanishing point $\mathbf{L_1}$, which is associated to the fictitious point $\mathbf{L_2}^{\infty}$ in the sequence ($\mathbf{C_2B_2}$), is determined with the cross ratio $\lambda=(\mathbf{C_1B_1F_1L_1})=(\mathbf{C_2B_2F_2L_2}^{\infty})$.

The vanishing planes (fig. 1), $\mathbf{N_1}$ in the space $\theta^{\mathbf{1}}$ and $\mathbf{M_2}$ in the space θ^2, are presented in the pair of Monge's projections with the triangles $\mathbf{T_1H_1}^{\infty}\mathbf{L_1}$ and $\mathbf{R_2U_2}^{\infty}\mathbf{Q_2}$. For the sake of the simpler construction, it has been adopted the associated quadrangle $\mathbf{A_1C_1D_1P_1}$ and $\mathbf{A_2C_2D_2P_2}$ are in the horizontal plane, so the vanishing planes $\mathbf{N_1}$ and $\mathbf{M_2}$ in the first projection are seen in their true size, and in the second projection are seen as straight lines. Such principle needs not be put in practice to achieve results.

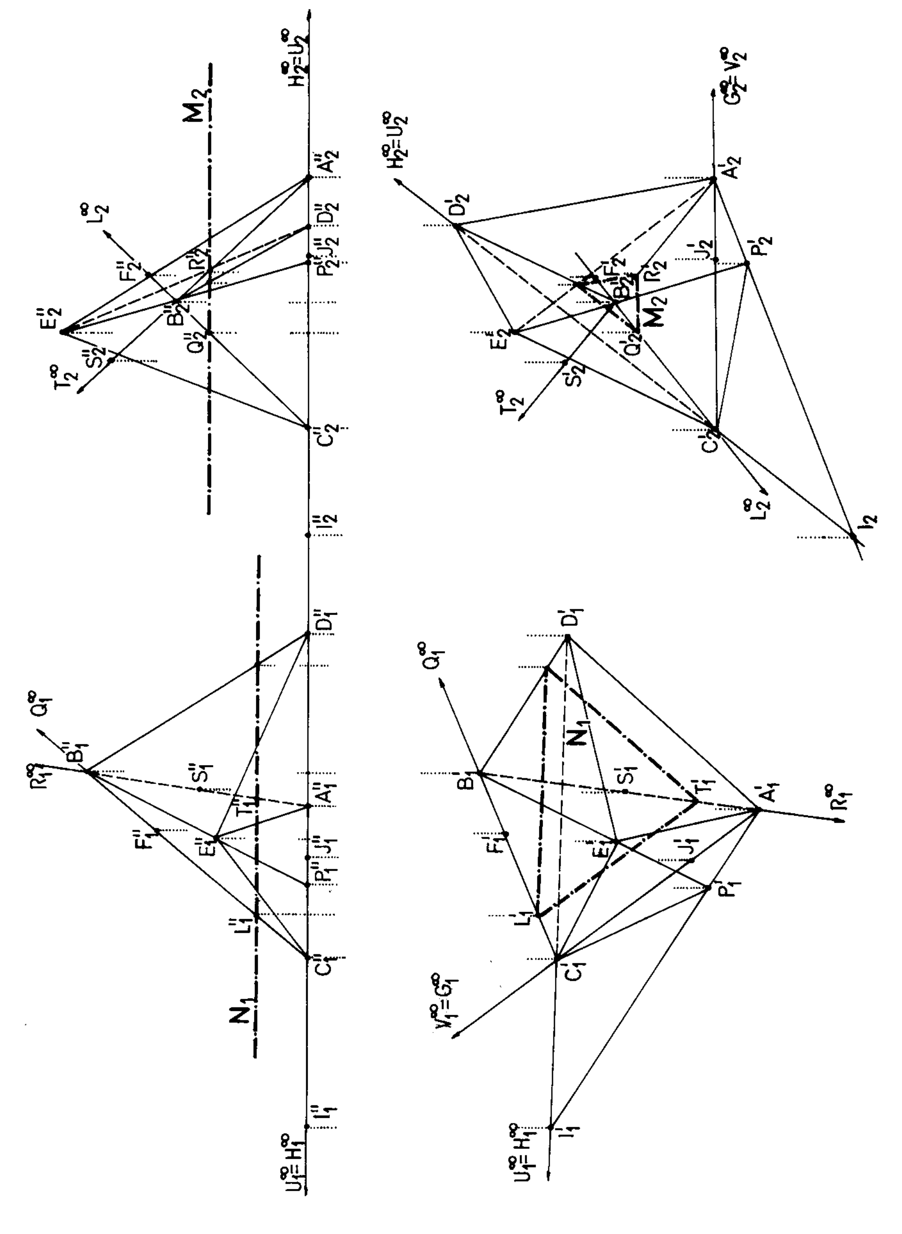

Fig. 1. the vanishing plane in SC spaces in the general case

2.2. Determination of the axes (principal normal lines) in the SC spaces in the general case

To the infinitely distant point $\mathbf{W_1}^{\infty}$, of the normal line $\mathbf{n_1}$ on the vanishing plane $\mathbf{N_1}$ in the space θ^1, the finitely distant point $\mathbf{W_2}$ is associated in the vanish plane $\mathbf{M_2}$ of the space θ^2. The point $\mathbf{W_2}$ is called the center of the space θ^2 and it is a point through which a straight line, $\mathbf{o_2}$, runs and it is normal to the vanishing plane $\mathbf{M_2}$ and is called the axis (principal normal) of the space θ^2.

To the fictitious point $\mathbf{X_2}^{\infty}$, normal line $\mathbf{m_2}$ on the vanishing plane $\mathbf{M_2}$ in the space θ^2, the finitely distant point $\mathbf{X_1}$ is associated in the vanishing plane $\mathbf{N_1}$ of the space θ^1. The point $\mathbf{X_1}$ is called the center of the space θ^1 and it is a point through which a straight line, $\mathbf{o_1}$,runs, which is normal to the vanishing plane $\mathbf{N_1}$ and it is called the axis (principal normal) of the space θ^1.

These two straight lines, $\mathbf{o_1}$ and $\mathbf{o_2}$, are the only straight lines which are normal to the corresponding vanishing planes prior to and after the mapping and which are projectively associated and are called the space axes.

2.2.1. Constructive procedure for determination of centers in SC spaces in the general case

Through the points $\mathbf{E_1}$ and $\mathbf{B_1}$, of the space θ^1, (fig.2), the normal lines $\mathbf{n_1}^{\mathbf{E}} \perp \mathbf{N_1}$ and $\mathbf{n_1}^{\mathbf{B}} \perp \mathbf{N_1}$, are set, whose fictitious points is $\mathbf{W_1}^{\infty}$. These normal lines penetrate the plane $\mathbf{A_1C_1D_1}$ in the points $\mathbf{]_1}$ and $\mathbf{[_1}$.

The straight line $\mathbf{n_2}^{\mathbf{E}}$=$\mathbf{E_2]_2}$, which is associated to the normal line $\mathbf{n_1}^{\mathbf{E}}$=$\mathbf{E_1]_1}$, has been determined in the following way. Connecting line of the points $\mathbf{A_1]_1}$ intersects the sequence ($\mathbf{C_1D_1}$) in the point $\mathbf{I_1}$, and the connecting line of the points $\mathbf{D_1]_1}$ intersects the sequence ($\mathbf{A_1C_1}$) in the point $\mathbf{II_1}$. Since the sequences ($\mathbf{C_1D_1}$)=($\mathbf{C_2D_2}$) and ($\mathbf{A_1C_1}$)=($\mathbf{A_2C_2}$) are identical, from the simple relationship σ=($\mathbf{C_1D_1I_1}$)=($\mathbf{C_2D_2I_2}$), has been determined the point $\mathbf{I_2}$ in the sequence ($\mathbf{C_2D_2}$), and from the simple relationship σ=($\mathbf{A_1C_1II_1}$)=($\mathbf{A_2C_2II_2}$), the point $\mathbf{II_2}$ is determined in the sequence ($\mathbf{A_2C_2}$). The connecting lines $\mathbf{A_2I_2}$ and $\mathbf{D_2II_2}$ intersect in the point $\mathbf{]_2}$. The connecting line of the points $\mathbf{n_2}^{\mathbf{E}}$=$\mathbf{E_2]_2}$ in the space θ^2, which is associated to the normal line $\mathbf{n_1}^{\mathbf{E}}$ in the space θ^1, penetrates through the vanishing plane $\mathbf{M_2}$ in the point $\mathbf{W_2}$, which is associated to the fictitious point $\mathbf{W_1}^{\infty}$, of the normal $\mathbf{n_1}^{\mathbf{E}}$. The point $\mathbf{W_2}$ is the center and the straight line $\mathbf{o_2}$, runs through it, and it is normal to the vanishing plane $\mathbf{M_2}$ and is called the axis of the space θ^2.

In the same way the straight line $\mathbf{n_2}^{\mathbf{B}}$=$\mathbf{B_2[_2}$, in the space θ^2,is determined, and it is associated to the normal $\mathbf{n_1}^{\mathbf{B}}$=$\mathbf{B_1[_1}$ in the space θ^1, by through the simple relationship on the associated identical sequences ($\mathbf{C_1D_1}$)=($\mathbf{C_2D_2}$) and ($\mathbf{A_1C_1}$)=($\mathbf{A_2C_2}$). The penetration of $\mathbf{n_2}^{\mathbf{B}}$=$\mathbf{B_2[_2}$ through the vanishing plane $\mathbf{M_2}$ is the point $\mathbf{W_2}$, center of the space θ^2, which is obtained by the previous procedure.

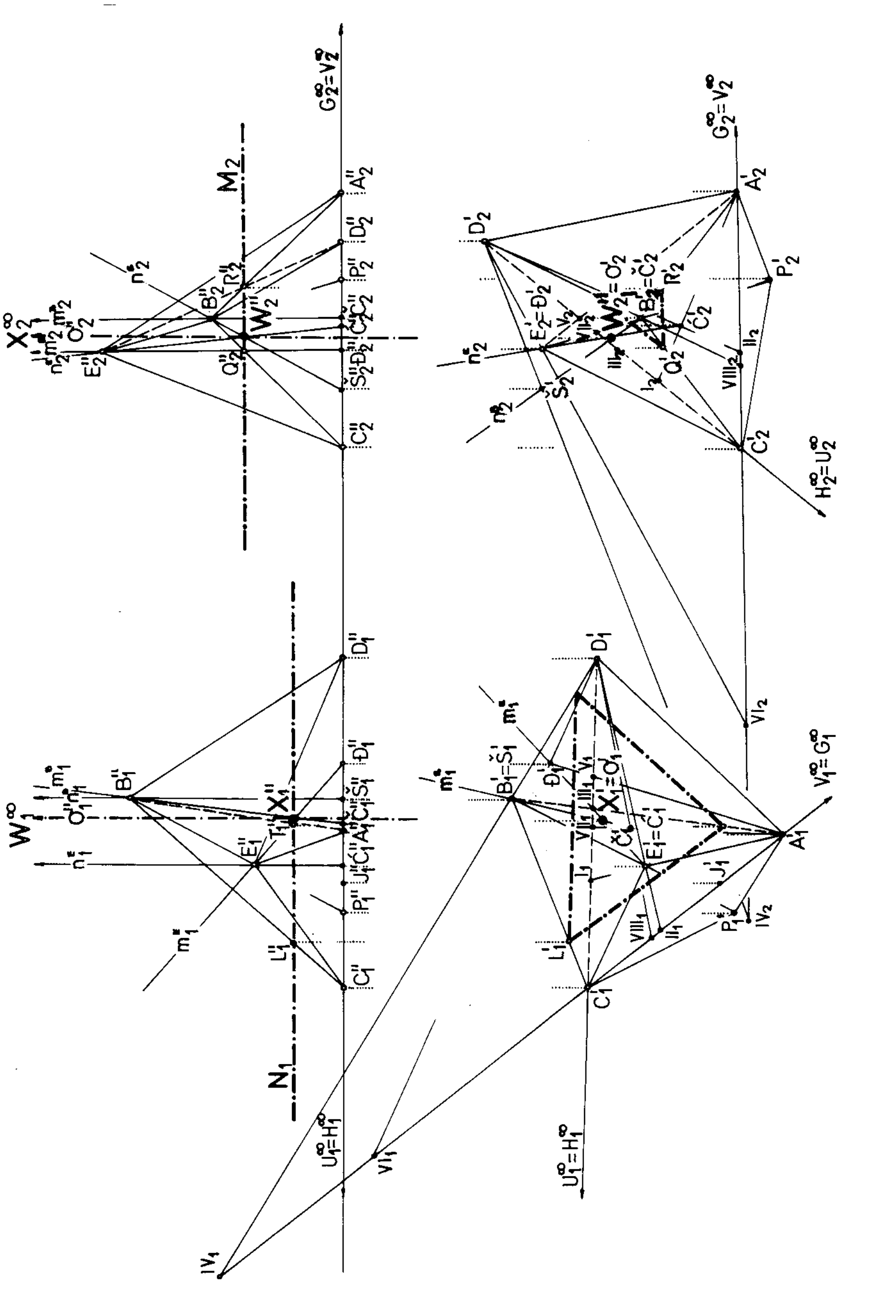

Fig. 2. The axes and centers in SC spaces in the general case

Through the points $\mathbf{E_2}$ and $\mathbf{B_2}$, of the space θ^2, (fig.2), the straight lines $\mathbf{m_2^E} \perp \mathbf{M_2}$ and $\mathbf{m_2^B} \perp \mathbf{M_2}$, are set, whose fictitious point is $\mathbf{X_2^\infty}$. These normal lines penetrate the plane $\mathbf{A_2C_2D_2}$ in the points $\backslash_2$ and $\wedge_2$.

The straight line $\mathbf{m_1^E}$=$\mathbf{E_1}\backslash_1$, which is associated to the normal $\mathbf{m_2^E}$=$\mathbf{E_2}\backslash_2$, is determined by the following procedure. The connection line of the points $\mathbf{A_2}\backslash_2$ intersects the sequence ($\mathbf{C_2D_2}$) in the point $\mathbf{V_2}$, and the connecting line of the points $\mathbf{D_2}\backslash_2$ intersects the sequence ($\mathbf{A_2C_2}$) in the point $\mathbf{VI_2}$. Since the sequences ($\mathbf{C_2D_2}$)=($\mathbf{C_1D_1}$) and ($\mathbf{A_2C_2}$)= ($\mathbf{A_1C_1}$) are identical, from the simple relationship $\mathbf{C_2D_2V_2}$)=($\mathbf{C_1D_1V_1}$) the point $\mathbf{V_1}$ in the sequence ($\mathbf{C_1D_1}$) is determined, and from the simple relationship σ=($\mathbf{A_2C_2VI_2}$)=($\mathbf{A_1C_1VI_1}$) the point $\mathbf{VI_1}$ in the sequence ($\mathbf{A_1C_1}$) is determined. The connecting lines of the points $\mathbf{A_1V_1}$ and $\mathbf{D_1VI_1}$ intersect in the point $\backslash_1$. The connecting line of the points $\mathbf{m_1^E}$=$\mathbf{E_1}\backslash_1$ in the space θ^1, which is associated to the normal line $\mathbf{m_2^E}$ in the space θ^2, penetrates the vanishing plane $\mathbf{N_1}$ in the point $\mathbf{X_1}$, which is associated to the fictitious point $\mathbf{X_2^\infty}$, of the normal $\mathbf{m_2^E}$. The point $\mathbf{X_1}$ is the center, and straight line $\mathbf{o_1}$, runs to it, and $\mathbf{o_1}$ is normal to the vanishing plane $\mathbf{N_1}$ and is called the axis of space θ^1.

In the same way the straight line $\mathbf{m_1^B}$=$\mathbf{B_1}\wedge_1$ is determined , in the space θ^1, which is associated to the normal $\mathbf{m_2^B}$=$\mathbf{B_2}\wedge_2$ in the space θ^2, by the simple relationship in the associated identical sequences ($\mathbf{C_2D_2}$)=($\mathbf{C_1D_1}$) and ($\mathbf{A_2C_2}$)=($\mathbf{A_1C_1}$). Penetration of $\mathbf{m_1^B}$=$\mathbf{B_1}\wedge_1$ through the vanishing plane $\mathbf{N_1}$ is the point $\mathbf{X_1}$, center of space θ^1, which is obtained by the previous procedure.

Through the points $\mathbf{X_1}$, of the space θ^1 and $\mathbf{W_2}$, of the space θ^2, run two only straight lines $\mathbf{o_1}$ and $\mathbf{o_2}$, which are prior to, and after the mapping, normal to the corresponding vanishing planes and are called the axes of space.

2.3. Determination of directrix planes (associated identical fields of points) in SC spaces in the general case

Foci and directrix planes are the invariants of the SC spaces and they enable mapping in the spaces without bringing them into a perspective position. The directrix planes are associated identical fields of points and occur in pairs and they are symmetrical in respect to the vanishing planes and are parallel to them. The procedure for determination of the directrix planes is based on the determination of the identical circular intersections of the associated rotating cylinders in one and rotating cones in other space.

If through any point and axis $\mathbf{o_1}$ of the space θ^1, a rotating cylinder ψ_1, is set, and whose radius is $\mathbf{r_1}$, it will, in the space θ^2, have associated a rotating cone ψ_2, whose apex is $\mathbf{W_2}$, axis $\mathbf{o_2}$, and it runs through the associated point and vice verse. All the planes of space θ^1, parallel to the vanishing plane $\mathbf{N_1}$ intersect cylinder ψ_1 across the circumferences $\Sigma\mathbf{t_1}$, of radius $\mathbf{r_1}$. These planes parallel to the vanishing plane $\mathbf{N_1}$ in the space θ^1, are associated to the vanishing planes parallel to the vanishing plane $\mathbf{M_2}$ in the space θ^2, intersecting the cone ψ_2, across the circumferences $\Sigma\mathbf{t_2}$. Only two planes from the set of circumferences $\Sigma\mathbf{t_2}$ will intersect the cone ψ_2, across the circumferences of the radius $\mathbf{r_1}$ and vice versa. These planes are called the directrix planes and the fields of points are identical to them.

2.3.1. Constructive procedure for determination of the directrix planes in SC spaces in the general case

Through the points $\mathbf{B_1}$ and $\mathbf{E_1}$ of the space θ^1, (fig.3), the planes β_1 and ε_1, are set, parallel to the vanishing plane $\mathbf{N_1}$. The axis of the space, $\mathbf{o_1}$, penetrates these planes in the points $\overline{\mathbf{B}_1}$ and $\overline{\mathbf{E}_1}$. They have the associated planes, through the points $\mathbf{B_2}$ and $\mathbf{E_2}$ in the space θ^2, β_2 and ε_2, parallel to the vanishing plane $\mathbf{M_2}$. The space axis $\mathbf{o_2}$, penetrates these planes in the points $\overline{\mathbf{B}_2}$ and $\overline{\mathbf{E}_2}$. The points $\overline{\mathbf{B}_1}$, $\overline{\mathbf{E}_1}$ and $\overline{\mathbf{B}_2}$, $\overline{\mathbf{E}_2}$ are mutually associated.

For this constructive procedure, the vanishing planes are in the such a position, in second projection, so the planes parallel to them, where the circumferences of the cylinders and cones are intersected, are seen as straight lines.

Through the point $\mathbf{B_1}$ and axis $\mathbf{o_1}$, a rotating cylinder $\psi_1^{\mathbf{B}}$, is set, which is intersected by the planes parallel to the vanishing plane, $\mathbf{N_1}$, across the circumferences $\Sigma\mathbf{t_1}^{\mathbf{B}}$, whose radius is $\mathbf{r_1}^{\mathbf{B}}$, which are viewed in their true size in both projections. To the cylinder $\psi_1^{\mathbf{B}}$ the rotating cone $\psi_2^{\mathbf{B}}$ in the space θ^2, whose apex is $\mathbf{W_2}$, axis $\mathbf{o_2}$ and runs through the associated point $\mathbf{B_2}$. The radius of the circumference of the rotating cone $\psi_2^{\mathbf{B}}$, in the plane β_2, is $\mathbf{r_2}^{\mathbf{B}}$. The planes parallel to the vanishing plane $\mathbf{M_2}$, of the space θ^2, intersect the cone $\psi_2^{\mathbf{B}}$ across the circumferences $\Sigma\mathbf{t_2}^{\mathbf{B}}$. Two planes, form the set $\Sigma\mathbf{t_2}^{\mathbf{B}}$, which intersect the cone $\psi_2^{\mathbf{B}}$ across the circumferences of the radius $\mathbf{r_1}^{\mathbf{B}}$ are directrix planes $\mathbf{K_2}^{\mathbf{B}}$ and $\mathbf{Q_2}^{\mathbf{B}}$, of the space θ^2. The directrix planes $\mathbf{K_1}^{\mathbf{B}}$ and $\mathbf{Q_1}^{\mathbf{B}}$ in the space θ^1, are associated to them.

The penetration of the axes $\mathbf{o_1}$ and $\mathbf{o_2}$, of the spaces θ^1 and θ^2, through the directrix planes $\mathbf{K_1}^{\mathbf{B}}$, $\mathbf{Q_1}^{\mathbf{B}}$ i $\mathbf{K_2}^{\mathbf{B}}$, $\mathbf{Q_2}^{\mathbf{B}}$, are the points $\mathbf{Y_1}^{\mathbf{B}}$ and $\mathbf{Z_1}^{\mathbf{B}}$ in the space θ^1 and $\mathbf{Y_2}^{\mathbf{B}}$ and $\mathbf{Z_2}^{\mathbf{B}}$ in the space θ^2. From the relationship $\lambda=(\mathbf{W_2X_2}^{\infty}\mathbf{Y_2Z_2})=(\mathbf{W_1}^{\infty}\mathbf{X_1Y_1Z_1})$ the points $\mathbf{Y_1}^{\mathbf{B}}$ and $\mathbf{Z_1}^{\mathbf{B}}$ can be determined in the space θ^1 on the axis $\mathbf{o_1}$, and by this the positions of the directrix planes $\mathbf{K_1}^{\mathbf{B}}$ and $\mathbf{Q_1}^{\mathbf{B}}$.

If in the space θ^2, through the point $\mathbf{B_2}$ and axis $\mathbf{o_2}$, a rotating cylinder $\xi_2^{\mathbf{B}}$, of radius $\mathbf{r_2}^{\mathbf{B}}$, is set, it has a rotating cone $\xi_1^{\mathbf{B}}$, associated through the pinnacle $\mathbf{X_1}$, axis $\mathbf{o_1}$ and point $\mathbf{B_1}$, of the space θ^1. As in the previously described procedure, the planes parallel to the vanishing plane $\mathbf{N_1}$ will intersect the cone $\xi_1^{\mathbf{B}}$ across the circumferences $\Sigma\mathbf{t_1}^{\mathbf{B}}$, and only two planes from this set will intersect the cone across the circumferences of the radius $\mathbf{r_2}^{\mathbf{B}}$. In this way the same positions of the directrix planes $\mathbf{K_1}^{\mathbf{B}}$ and $\mathbf{Q_1}^{\mathbf{B}}$ in the space θ^1, are obtained, as well as with the previously described procedure, and by this it is possible to better determine the directrix planes $\mathbf{K_2}^{\mathbf{B}}$ and $\mathbf{Q_2}^{\mathbf{B}}$ in the space θ^2, by the cross ratio $\lambda=(\mathbf{X_1}^{\infty}\mathbf{W_1Y_1Z_1})=(\mathbf{X_2W_2}^{\infty}\mathbf{Y_2Z_2})$, which is a proof of biunivocality of the collinear mapping.

If the previously described procedure for determination of the directrix planes is applied to the point $\mathbf{E_1}$ in the space θ^1 and for the point $\mathbf{E_2}$ in the space θ^2, the same positions of the directrix planes are obtained, $\mathbf{K_1}^{\mathbf{B}}=\mathbf{K_1}^{\mathbf{E}}$, $\mathbf{Q_1}^{\mathbf{B}}=\mathbf{Q_1}^{\mathbf{E}}$ and $\mathbf{K_2}^{\mathbf{B}}=\mathbf{K_2}^{\mathbf{E}}$, $\mathbf{Q_2}^{\mathbf{B}}=\mathbf{Q_2}^{\mathbf{E}}$, as well as for the points $\mathbf{B_1}$ and $\mathbf{B_2}$. At that, the penetration points of the axes, $\mathbf{o_1}$ and $\mathbf{o_2}$, through these directrix planes coincide $\mathbf{Y_1}^{\mathbf{B}} = \mathbf{Y_1}^{\mathbf{E}}$, $\mathbf{Z_1}^{\mathbf{B}} = \mathbf{Z_1}^{\mathbf{E}}$, as well as $\mathbf{Y_2}^{\mathbf{B}} = \mathbf{Y_2}^{\mathbf{E}}$, $\mathbf{Z_2}^{\mathbf{B}} = \mathbf{Z_2}^{\mathbf{E}}$.

On the basis of the previous assertions, it can be concluded that SC spaces set in the special form have two associated pairs of directrix planes (identical fields of points), where the first associated pair is set in advance by the identical quadrangles. The distance of the second pair are equal to the distance of the first pair from the corresponding vanishing plane. This implies that the position of the second associated pair of the directrix planes is known when the position of the vanishing planes is determined.

This ensures that there are also foci in these space, which agrees with the conclusion in the doctoral thesis of A. Jovanovic. Coinciding of one associated pair of directrix planes, the SC spaces can be brought into the perspective position from the general position.

2.4. Determination of foci in SC space in the general case

Foci are the apexes of the associated identical bundles of straight lines, whose all the associated rays are at the same angle in respect to the centers of the space. The foci are situated on the axes of space, which are only two associated straight lines which are normal to the corresponding vanishing planes. The distance of the foci from the centers is equal to he distance of the directrix planes from the vanishing planes in the second space and vice versa.

One pair of foci is $\mathbf{F_1}$ and $\mathbf{O_1}$ in the space θ^1, and the pair associated to them in the space θ^2 is $\mathbf{F_2}$ and $\mathbf{O_2}$. Foci $\mathbf{F_1}$ and $\mathbf{O_1}$, are symmetrical in respect to the center $\mathbf{X_1}$ of the space θ^1, and they are determined by the distance of the directrix planes $\mathbf{K_2}$ and $\mathbf{Q_2}$ from the vanishing plane $\mathbf{M_2}$ in the space θ^2, so that $\mathbf{F_1X_1}=\mathbf{O_1X_1}=\mathbf{Y_2W_2}=\mathbf{Z_2W_2}$. Foci $\mathbf{F_2}$ and $\mathbf{O_2}$, are symmetrical in respect to the center $\mathbf{W_2}$ of the space θ^2, and they are determined by the distance of the directrix planes $\mathbf{K_1}$ and $\mathbf{Q_1}$ from the vanishing plane $\mathbf{N_1}$ in the space θ^1, so that $\mathbf{F_2W_2}=\mathbf{O_2W_2}=\mathbf{Y_1X_1}=\mathbf{Z_1X_1}$.

Considering that the foci are the apexes of the associated identical bundles of straight lines, all the associated rays form the same angles to the corresponding vanishing planes, and by their determination, the mapping in the general space is considerably simpler.

By coinciding one of the associated pair of foci, the SC spaces can be brought from the general into perspective position.

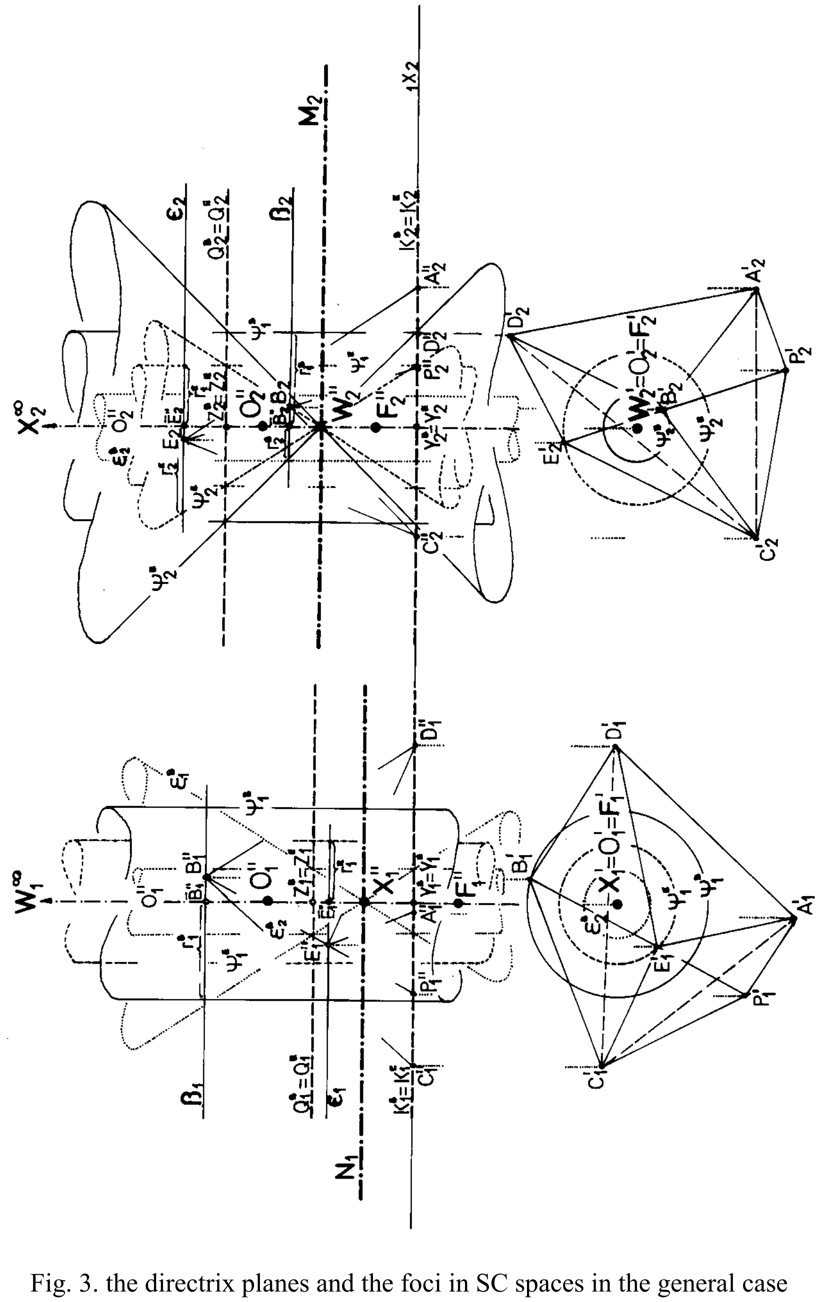

Fig. 3. the directrix planes and the foci in SC spaces in the general case

3. Conclusion

In the paper, the constructive methods were used to determine the characteristic parameters of to SC spaces in the general case: vanishing planes, axes of space, foci (apexes of the associated identical bundles of straight lines) and directrix planes (associated identical fields of points. SC spaces in the generalial form are set with five pairs of biunivocally associated points, so that the quadrangle I the first space, obtained from three basic and the penetration point of the remaining two through the plane of the first three, is identical or similar to the associated quadrangle obtained in the same way in the second space.

The vanishing planes are associated to the infinitely distant planes of space, so by them, the fictitious elements can be brought into finiteness. The axes of the space are only two associated straight lines which are normal to the corresponding vanishing planes, so the points in them can be mapped by the cross ration in the associated sequences of points.

In the SC spaces in the general form, there are the foci (apexes of the associated bundles of straight lines) and directrix planes (associated identical fields of points), which has been proven in this paper. It enables association of two SC spaces and determination of other projective forms, which will be a subject of further research.

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ODREDJIVANJE KARAKTERISTIČNIH PARAMETARA U SPECIJALNIM KOLINEARNIM PROSTORIMA U OPŠTEM SLUČAJU

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Specijalni kolinearni prostori u opštem položaju zadati su sa pet parova jednoznačno (biunivoko) pridruženih tačaka, čiji je položaj specijalno podešen. I u ovim prostorima potrebno je da se odrede karakteristični parametri: nedogledne ravni, ose prostora, žiže prostora, (temena pridruženih identičnih svežnjeva pravih) i direktrisne ravni, (pridružena identična polja tačaka). žiže i direktrisne ravni u specijalnim kolinearnim prostorima u opštem položaju postoje, {to je na konstruktivan način dokazano u ovom radu, u paru Monžovih projekcija.