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THE PLANE SECTION OF THE SURFACE OF REVOLUTION

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Abstract. *In this paper a procedure for determining a plane section of the surface of revolution was described. The surface of revolution is given by an axis of the revolution and a meridian which is coplanar with the axis. The axis of revolution is parallel to the z axis of the coordinate system. The intersecting plane is given by its three points; these are the points where the plane intersects axes x, y and z. For the determination of a plane section of the surface of revolution we can use horizontal planes which intersect the surface of revolution on parallel circles and each auxiliary plane intersects the intersecting plane on the horizontal line. Two points of the intersecting space curve are given as intersecting points between the horizontal line and the circle and new points of the intersecting curve have been determined by using several auxiliary planes.*

Key words: *the Surface of Revolution, the Plane Section, Computer Geometry.*

1. THE PLANE IN A SPECIAL POSITION

The surface of revolution is given by a vertical axis of the revolution and a meridian (Fig. 1), the axis and meridian being in *Oxz* plane. The axis *o* contains the point $0_1(x_1, y_1, 0)$, and the meridian is described with the equation $z = f(x)$. Plane α intersects the surface, and the plane is given with $\alpha(\alpha_x, \alpha_y, \infty)$, where α_x is the intersecting point between the plane and *x* axis.

In the second orthogonal projection (the plane *Oxz*), the vertical axis *o* is shown in real length, and in the first orthogonal projection (the plane *Oxy*) the axis is shown as a point (in the projecting position). In the first orthogonal projection the plane α is in the projecting position (we can see it as a line) and it is coincident with the line α_1 (α_1 is an intersecting line between the plane α and plane *Oxy*). By using horizontal auxiliary planes we can find the intersecting points between the surfaces of revolution and plane α. The horizontal plane β at a height z_A intersects surfaces of revolution on the circle with radius

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r_A. In the first orthogonal projection the intersecting curve is shown as a line and the second projection of the curve points can be determined by using the horizontal circle. With a sufficiently large number of auxiliary planes it will be possible to find an intersecting curve as a set of intersecting points for all auxiliary planes.

Each meridian point $A''(x_A, z_A)$, with the rotation about o axis by the circle with radius $r_A = x_A - x_1$ can be transformed into the axis-symmetrical point $A_1''(x_{A_1}, z_A)$ with coordinate $x_{A_1} = 2x_1 - x_A$.

The intersecting line α_1 between the plane α and Oxy is given with the following equation

$$y = a_{\alpha_1} x + b_{\alpha_1};\ a_{\alpha_1} = -\frac{\alpha_y}{\alpha_x};\ b_{\alpha_1} = \alpha_y$$

The points P_1, P_2 from the intersecting curve, which are at the same height z_A as point A, can be found at first in the first orthogonal projection as the intersecting points of the circle (radius r_A) and the line α_1. By substituting the equation of line α_1 into circles with the equation

$$(x - x_1)^2 + (y - y_1)^2 = r_A^2$$

one can find the next quadratic equation

$$ex^2 + fx + g = 0$$

where coefficients e, f and g have values described with

$$e = 1 + a_{\alpha_1}^2;\ f = 2[a_{\alpha_1}(b_{\alpha_1} - y_1) - x_1];\ g = x_1^2 + (b_{\alpha_1} - y_1)^2 - r_A^2$$

By solving the quadratic equation we can find solutions for x coordinates

$$x_{P_{1,2}} = \frac{-f \pm \sqrt{f^2 - 4eg}}{2e}$$

which can be real and different, real and equal or complex. The sign of discriminant $D = f^2 - 4eg$ defines which of these three variants will be accomplished. If $D \geq 0$ then we have two intersecting points; one point case is when $D = 0$ and if $D < 0$then there are not real intersecting points.

Now, y coordinates are defined with

$$y_{P_{1,2}} = a_{\alpha_1} x_{P_{1,2}} + b_{\alpha_1}$$

The second projection of intersecting points are given with coordinates x and z.

$$P_1''(x_{P_1}, z_A);\ P_2''(x_{P_2}, z_A)$$

2. THE PLANE IN A GENERAL POSITION

If an intersecting plane is in a general position then we have a general case (the plane is given with three points: α_x, α_y, α_z). The procedure for the determination of the intersecting curve is therefore a little complex. Again, we can use horizontal auxiliary planes (Fig. 2), which intersect the plane α on horizontal lines, and the surfaces of revolution on circles.

First, we can find the intersecting point $S(x_1, y_1, z_S)$ between the axis o and the given plane. In the first orthogonal projection the axis o and point S are coincident, and because the point S is on the plane α, we can find the second projection S'' by using lines h' and h''. The line h' is given by equation

$$y = a_{\alpha_1} x + b_1$$

where $$b_1 = y_1 - a_{\alpha_1} x_1 .$$

The line h' intersects x axis in the point $2'_h(x_H, 0)$ where x_H is $x_H = -\frac{b_1}{a_{\alpha_1}}$

The line α_2 of the plane α in the system Oxz is given with the equation

$$z = p_{\alpha_2} x + r_{\alpha_2} \qquad p_{\alpha_2} = -\frac{\alpha_z}{\alpha_x}, \quad r_{\alpha_2} = \alpha_z$$

Now, the z coordinate of the point S is

$$z_S = p_{\alpha_2} x_H + r_{\alpha_2}$$

The line g_1 from the plane α contains point S and in the first projection g_1 is orthogonal to the line h' (here we can determine a coefficient $m_{g'_1}$ of the line (1)), and in the same projection the line contains the point $O'_1(x_1, y_1)$, i.e.

$$y = m_{g'_1} x + n_{g'_1} \tag{1}$$

$$m_{g'_1} = -1/a_{\alpha_1}$$

$$n_{g'_1} = y_1 - m_{g'_1} x_1$$

The point Q' is an intersecting point between the line α_1 (it is an intersecting line between the plane α and the horizontal plane Oxy) and the line g''_1, where point Q'' has coordinates:

$$x_Q = \frac{n_{g'_1} - b_{\alpha_1}}{a_{\alpha_1} - m_{g'_1}}$$

$$y_Q = a_{\alpha_1} x_Q + b_{\alpha_1}$$

The line g''_1 contains the second projection of the points S and Q and it is given with $z = m_{g''_1} x + n_{g''_1}$

where $$\begin{aligned} m_{g_1''} &= \frac{z_S}{x_1 - x_Q} \\ n_{g_1''} &= -m_{g_1''} x_Q \end{aligned}.$$

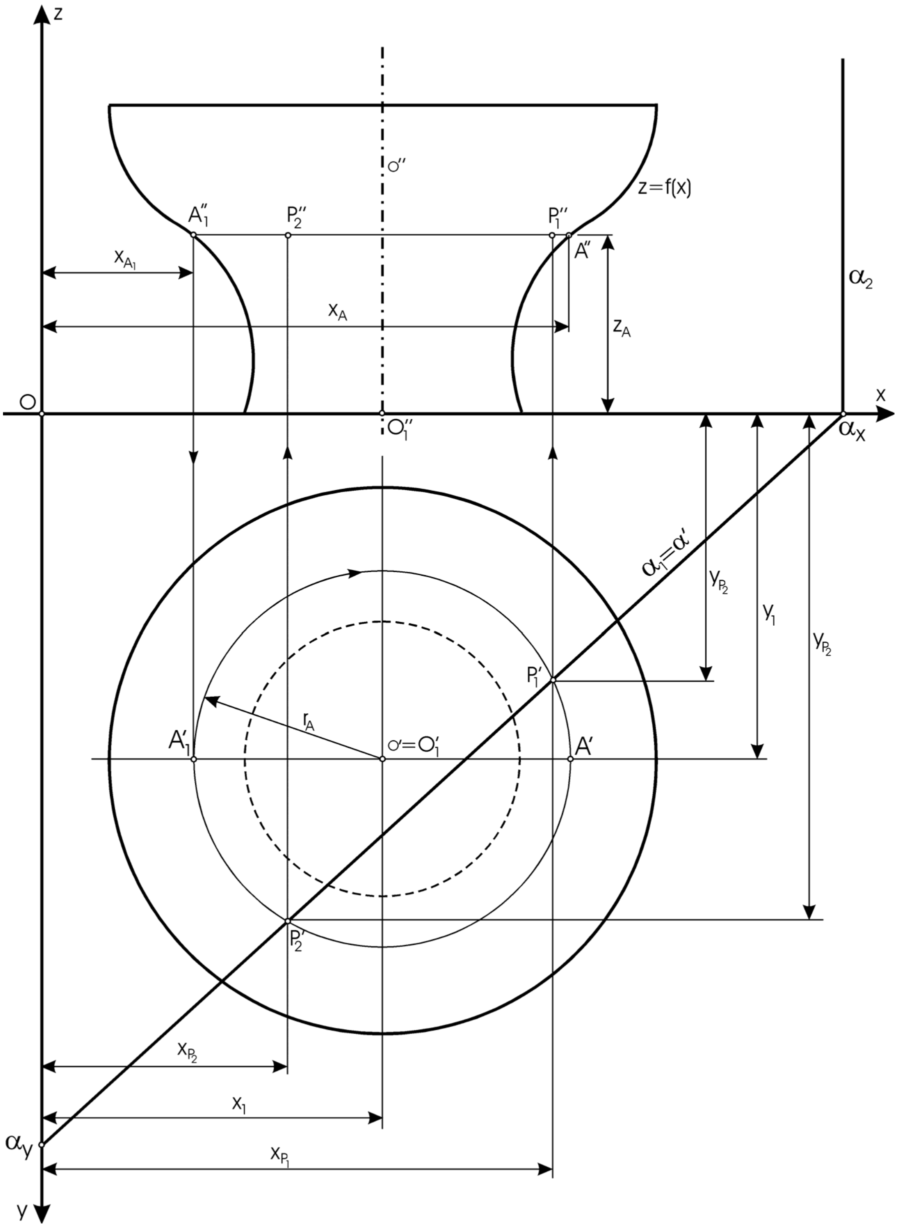

Fig. 1. The plane section of the surfaces of revolution; a special case

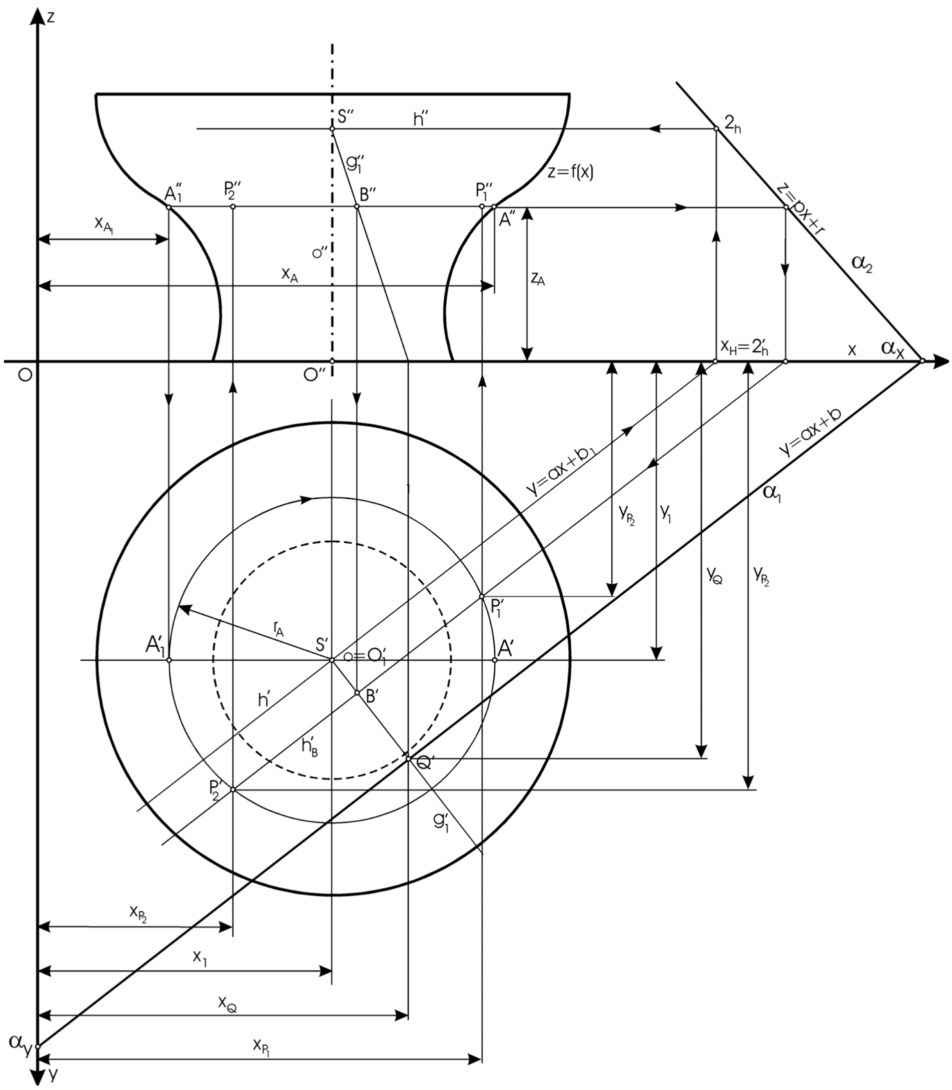

Fig. 2. The plane section of the surfaces of the revolution; a general case

Now we can determine the intersecting curve. The horizontal line at a height z_A intersects the line $g_1^{"}$ in the point $B^{"}(x_B, z_B)$

$$x_B = \frac{z_A - n_{g_1^{"}}}{m_{g_1^{"}}}$$

The point $B^{'}(x_B, y_B)$ belongs to the line $g_1^{'}$

$$y_B = m_{g_1^{'}} x_B + n_{g_1^{'}}$$

Through the point B' a line $h_B^{'}$ is drawn with the equation

$$y = a_{\alpha_1} x + s$$

$$s = y_B - a_{\alpha_1} x_B$$

By substituting the equation of the line $h_B^{'}$ into the equation of the parallel of the point *A* one can find the following quadratic equation

$$e_1 x^2 + f_1 x + g_1 = 0$$

where

$$e_1 = 1 + a_{\alpha_1}^2$$

$$f_1 = 2[a_{\alpha_1}(s - y_1) - x_1]$$

$$g_1 = x_1^2 + (s - y_1)^2 - r_A^2$$

The intersecting points for the circle containing the point *A* are given by equations

$$x_{P_{1,2}} = \frac{-f_1 \pm \sqrt{f_1^2 - 4e_1 g_1}}{2e_1} \qquad y_{P_{1,2}} = a_{\alpha_1} x_{P_{1,2}} + s$$

$$P_1^{''}(x_{P_1}, z_A)\,;\; P_2^{''}(x_{P_2}, z_A)$$

3. Results

In figures 3 and 4 the plane section of the surfaces of revolution were shown, with the plane in a special position (the meridian is given with 620 points).

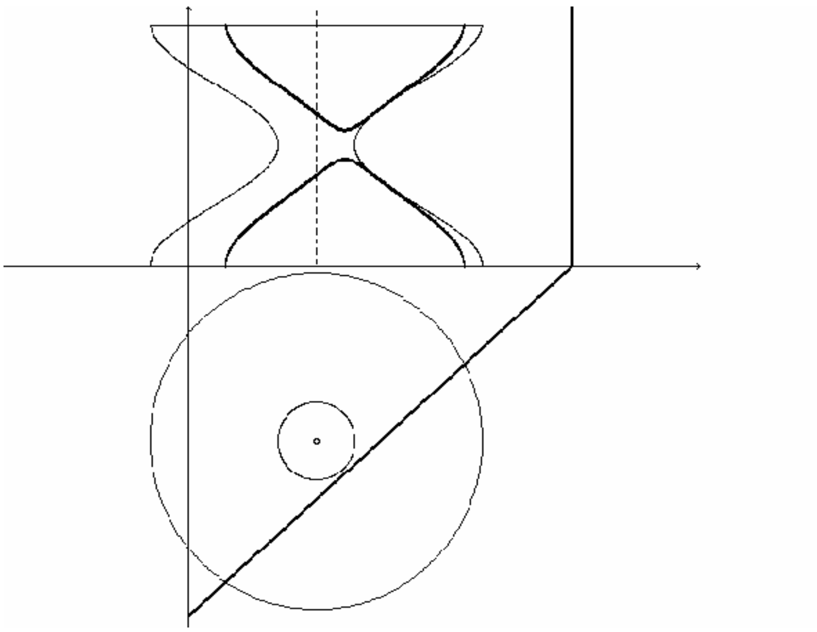

Fig. 3. The plane section; the intersecting plane is in a special position

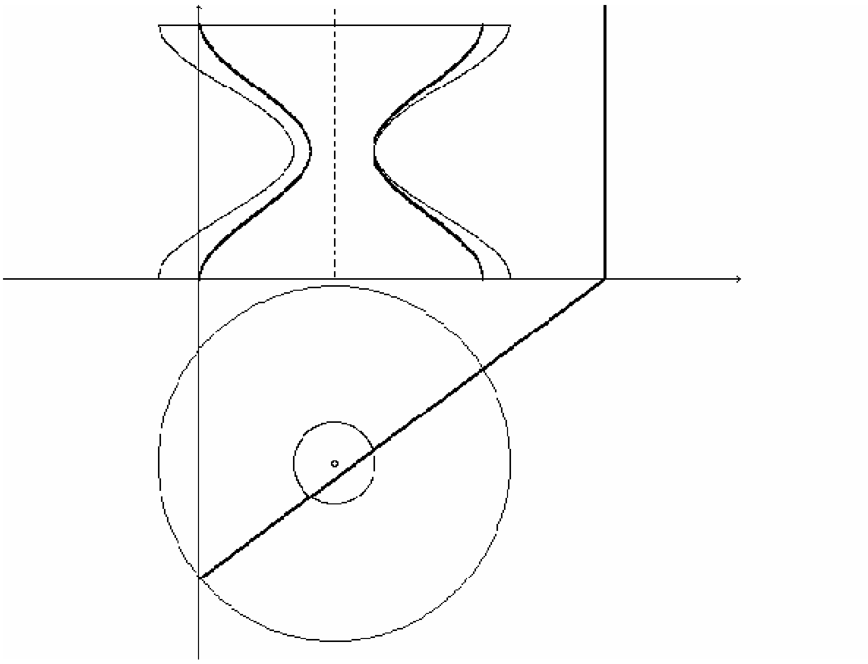

Fig. 4. The plane section; the intersecting plane is in a special position and the intersecting plane intersects the smallest circle

4. CONCLUSION

In this paper the mathematical models for determining a plane section of the surfaces of revolution were shown, in cases when the intersecting plane is in a special or a general position. For determining the intersecting curve auxiliary planes are used and these planes intersect the surfaces of revolution on parallels. Based on the well-known descriptive geometry facts, procedures were formed which enabled us to solve this problem by using a computer.

In the future research a procedure for 3D presentation of these results will be possible, i.e. in the oblique, axonometric or perspective projection.

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RAVAN PRESEK ROTACIONE POVRŠI

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U ovom radu je opisana procedura za određivanje ravnog preseka rotacione površi pomoću računara. Površ je zadata sa meridijanom i osom površi koji se nalaze u frontalnici, a osa površi je paralelna sa z osom koordinatnog sistema. Ravan α koja seče površ je zadata sa svoja tri osna traga. Za određivanje ravnog preseka rotacione površi korišćene su horizontalne pomoćne ravni koje rotacionu površ seku po paralelama, a svaka pomoćna ravan seče ravan α po horizontali. U preseku ove horizontale sa paralelom povši za istu pomoćnu ravan dobijaju se dve tačke prostorne presečne krive, dok se cela kriva dobija kao skup parova tačaka za sve pomoćne ravni.