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OPTIMAL DESIGN OF TIMBER BEAM

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Abstract. *In modern timber structures a large number of beams are loaded with bending. Contemporary engineering practice has shown that the effect of tangential stress and deflection of beams are not controlled during the design of this kind of beams. This could produce negative consequences in structure behavior during the exploitation. Using optimal design procedure in this article, the member is designed for bending in two planes. The following criteria must be observed: normal bending stress, shear stress and maximal deflection.*

Key words: *timber structures, bending, optimal design.*

1. INTRODUCTION

By applying the optimal design of long span beams in structure systems made of monolithic timber [MT] and glued laminated timber [GLT], the quantity of material used per m^2 of the structure base can be reduced, that is, the price of the supporting structure can be reduced.

The portion of timber used up for these beams is relatively high in respect to the total usage of timber, and this calls for the significant optimization, that is, reduction of the cross section surface area to the minimum.

The minimum dimensions of cross section surface area of the beams loaded with bending will be determined from the conditions of total employment of permissible normal and tangential stresses as well as of permissible deflection.

2. THEORETICAL IDENTIFICATION OF THE PROBLEM

Beams in timber structures are very often loaded either with bending, with or without a normal force, or with unsymmetrical bending. In order to determine the optimal dimensions of the cross section surface area, in terms of material consumption, the case of a simple beam static system support, of an arbitrary cross section, loaded with normal bending will be analyzed.

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The analysis does not take into account a normal force impact. In design of the beams loaded with normal bending, in a general case, there are 3 criteria for determination of the cross section area dimensions:

– Usage of true normal bending stresses - σ_m

$$\sigma_m = \sigma_{md} \Rightarrow \frac{M_y}{W_y} = \sigma_{md} \text{, that is, section modulus: } W_y = \frac{M_y}{\sigma_{md}} \ [\mathrm{cm}^3] \tag{1.1}$$

– usage of true shear stress - $\tau_{m\|}$

$$\tau_{m\|} = \tau_{m\|d} \Rightarrow \frac{T_z S_y^{ods}}{I_y b_{(z)}} = \tau_{m\|d} \text{, or } A_T = \frac{I_y b_{(z)}}{S_y^{ods}} \text{, that is surface area: } A_T = \frac{T_z}{\tau_{m\|d}} \ [\mathrm{cm}^2] \tag{1.2}$$

– usage of true maximal deflection - f_{stv}

$$f_{stv} = f_{dop} \Rightarrow \int_l \frac{M_y \overline{M}_y}{E_\| I_y} dx = \Psi \frac{M_y}{E_\| I_y} = \frac{l}{m} \text{, that is moment of inertia: } I_y = \Psi \frac{M_y m}{E_\| l} \ [\mathrm{cm}^4] \tag{1.3}$$

In the expression (1.1) the section modulus increases with the square degree of height, the surface area A_T in the expression (1.2) increases linearly, and the stiffness $E_\| I_y$, in the expression (1.3) with the third degree of height.

On the basis of the expression (1.1) and (1.3) it may be concluded that with the increase of the cross section height at the expense of its width, more cost-effective cross sections are obtained. However, one must consider that such increase of height and the decrease of width gives way for the decrease of lateral and torsional stiffness of the beam, and that it can disrupt the lateral-torsional stability of the beam..

These three conditions are almost never simultaneously satisfied in practice, but one of them is always a design value, that is, meeting of this condition means that the other two conditions are also satisfied, that is why the following relations are indicative for control:

$$\sigma_m \le \sigma_{md} \tag{1.1.1}$$

$$\tau_{m\|} \le \tau_{m\|d} \tag{1.1.2}$$

$$f_{stv} \le f_{dop} \tag{1.1.3}$$

However, in the analysis and the practical design, it is possible to have a condition that two conditions are simultaneously satisfied, for example:

$$\sigma_m \le \sigma_{md} \text{ and } \tau_{m\|} \le \tau_{m\|d} \text{, or} \tag{1.2.1}$$

$$\sigma_m \le \sigma_{md} \text{ and } f_{stv} \le f_{dop} \text{, or} \tag{1.2.2}$$

$$f_{stv} \le f_{dop} \text{ and } \tau_{m\|} \le \tau_{m\|d} \tag{1.2.3}$$

In theory, it is possible to have all three conditions simultaneously satisfied, but, in practice, due to the limitations of the elastic-mechanic components, such case is of no practical importance.

In this case an analysis of the elastic simply supported beam has been done, the beam having the span *l*, and rectangular cross section *b/h*, loaded with uniformly distributed load *q* (Fig. 1.), where the maximal influence is required design value:

$$A_T = \frac{2}{3}bh \tag{2.1}$$

$$\Psi = \frac{5l^2}{48} \tag{2.2}$$

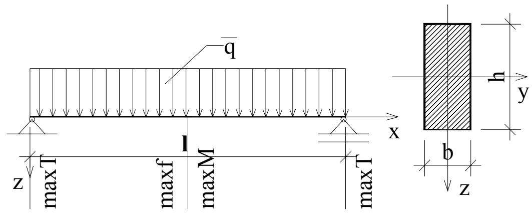


Fig. 1. Static system and the load

By substituting the values (2.1), (2.2) in the expressions (1.1), (1.2) and (1.3) and mathematical transformation of the values obtained in this manner, the criteria for design are obtained, as parameter values:

1. $\frac{\overline{q}}{b} = f(\sigma_{md}, \frac{h}{l})$

$$\left[\frac{\overline{q}}{b}\right] = \alpha\sigma_{md}\left(\frac{h}{l}\right)^2 \tag{3.1}$$

2. $\frac{\overline{q}}{b} = f(\tau_{m\|d}, \frac{h}{l})$

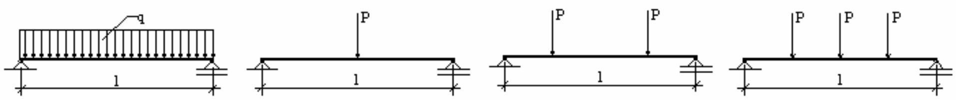
$$\left[\frac{\overline{q}}{b}\right] = \beta\tau_{\|md}\left(\frac{h}{l}\right) \tag{3.2}$$

3. $\frac{\overline{q}}{b} = f(f_{dop}, \frac{h}{l})$

$$\left[\frac{\overline{q}}{b}\right] = \gamma\frac{E_{\|}}{m}\left(\frac{h}{l}\right)^3 \tag{3.3}$$

Where:

$\overline{q}$ - is the substituted calculation load depending on the given load and is determined according to the Figure 2.



$\overline{q} = q$ $\qquad \overline{q} = \frac{P}{l}$ $\qquad \overline{q} = \frac{2P}{l}$ $\qquad \overline{q} = \frac{3P}{l}$

Fig. 2. Load constellations

Expressions (3.1), (3.2) and (3.3) are valid for the simple beam static system support of rectangular cross section. These expressions can be applied for the arbitrary cross section, provided that instead of the values defining the cross section (*b* and *h*) the corresponding dimensions of the arbitrary cross section are taken. So, for the circular cross section, $b = r$ and $h = r$, where r – radius of the cross section of the beam.

The parameter equations for the simple beam static system with circular cross section are as follows:

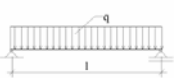
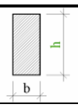
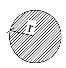
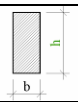
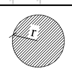
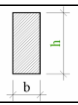
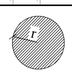
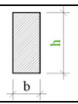
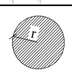
$$\left[\frac{\bar{q}}{r}\right] = \alpha\sigma_{md}\left(\frac{r}{l}\right)^2 \tag{4.1}$$

$$\left[\frac{\bar{q}}{r}\right] = \beta\tau_{\|md}\left(\frac{r}{l}\right) \tag{4.2}$$

$$\left[\frac{\bar{q}}{r}\right] = \gamma\frac{E_{\|}}{m}\left(\frac{r}{l}\right)^3 \tag{4.3}$$

In the expressions (3.1), (3.2), (3.3) and (4.1), (4.2), (4.3) they are:
– α, β and γ - constants depending on the load and the cross section of the support beam. Their values are determined from the equations (1.1), (1.2) and (1.3) or from the Table 1.

Table 1. Coefficient values α, β and γ

Static system	Cross section	α	β	γ
		$\frac{4}{3}$	$\frac{4}{3}$	$\frac{384}{60}$
		2π	$\frac{3\pi}{2}$	$\frac{96\pi}{5}$
		$\frac{2}{3}$	$\frac{4}{3}$	4
		π	$\frac{3\pi}{2}$	12π
		1,0	$\frac{4}{3}$	$\frac{108}{23}$
		$\frac{3\pi}{2}$	$\frac{3\pi}{2}$	$\frac{324\pi}{23}$
		1,0	$\frac{4}{3}$	$\frac{96}{19}$
		$\frac{3\pi}{2}$	$\frac{3\pi}{2}$	$\frac{288\pi}{19}$

2. Analysis for the Glued Laminated Timber Beams

For the elastic simply supported beam of rectangular cross section, loaded with the uniformly distributed load, according to the derived expressions (3.1), (3.2) and (3.3), and depending on the argument l / h the function graph $\overline{q} / b$ is given (Fig. 3), for glued laminated timber, 1st class conifers (JUS standard).

Basic permissible stresses for GLT, 1ST class conifers, are:

$\sigma_{md} = 1400\, N/cm^2 = 14000\, kN/m^2$ - permissible normal bending stress

$\tau_{m\|d} = 120\, N/cm^2 = 1200\, kN/m^2$ - permissible shear stress

$E_{\|} = 1100\, N/cm^2 = 11000000\, kN/m^2$ - elasticity modulus || fibers

$f_{dop} = \dfrac{l}{m} = \dfrac{l}{200}$ - permissible deflection

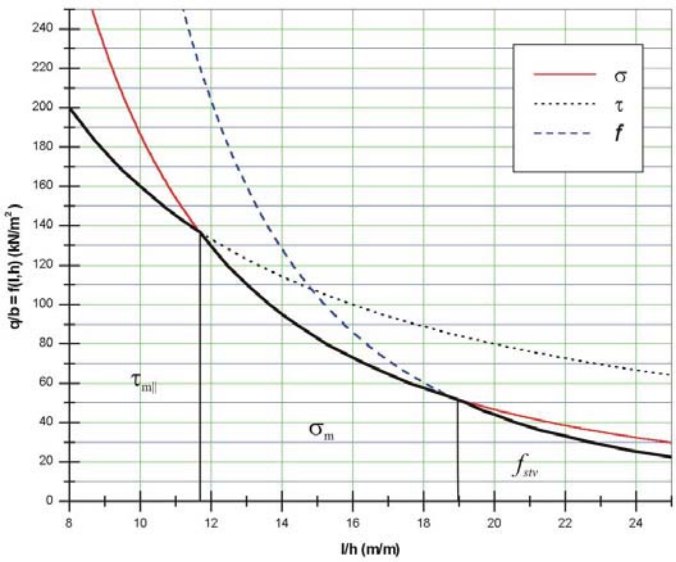


Fig. 3. Analysis of GLT timber bending

3. Examples for Usage of the Diagrams

Example 1. Determine the bearing capacity of the simple beam static system support, 4,20m in span, cross section dimensions, b/h = 8/24cm.

– Span height ratio $\dfrac{l}{h} = \dfrac{420}{24} = 17{,}5$

– For this value, the diagram indicated the ratio $\dfrac{q}{b} = 61\, kN/m$

– So the desired value is: $q = 61 \cdot 0{,}08 = 4{,}88\, kN/m^2$

Example 2. Determine the height of the cross section of the beam b = 10cm in width, of the simple beam static system support, 4,5m in span, if it is loaded with uniformly distributed load q = 6,5kN/m.

– Ratio $\frac{q}{b} = \frac{6,5}{0,10} = 65\,kN/m^2$

– For this value, the diagram indicated the ratio $\frac{l}{h} = 17$

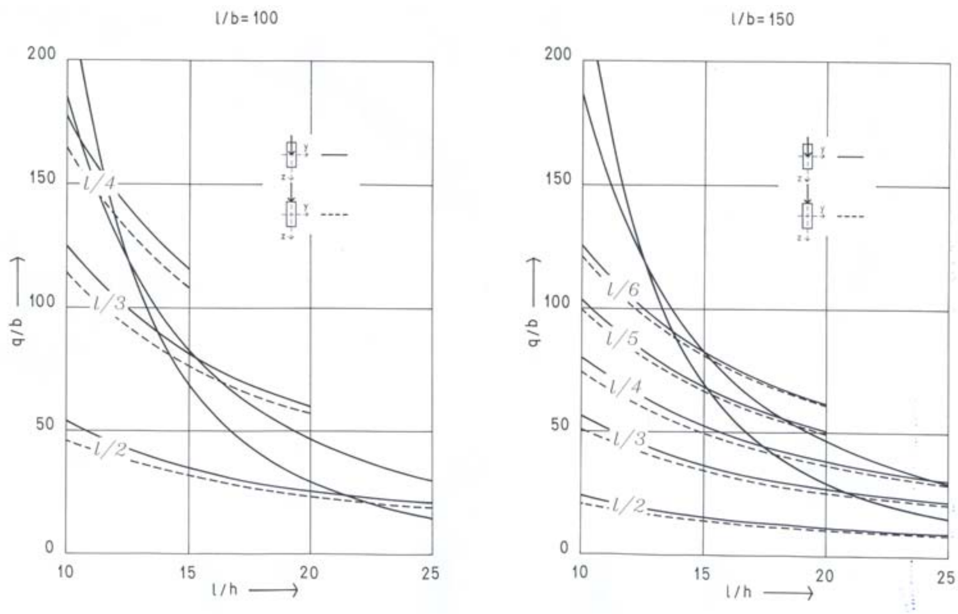
– So the desired height is $h = \frac{l}{17} = 0,265m$

If in the spatial model of the elastic simply supported beam, the following limit conditions are assumed:

– the beams have the fork support at the ends, the beam supports prevent both lateral deflection and twist,
– the beam can have free lateral-torsional buckling in the field

Therefore, the expression (3.1) is: $\left[\frac{\bar{q}}{b}\right] = \alpha\sigma_{md}\left(\frac{h}{l}\right)^2 \chi$

where χ - is a coefficient of lateral-torsion buckling for the elastic field of buckling. This is valid for the simple beam which has lateral support in the upper zone at a distance l,l/2, l/3, l/4, l/5, l/6, l/7, l/8, l/9 and l/10.
where l – is a span of the simple beam.



_____Chear centre loading
------- Top flange loading

f_{dop}= l/300

Fig. 4. Analysis of the lateral supported beam and efect of the level of application of a central point load

The program FEM : STAN 3D, for the simple beam of rectangular cross section, loaded with the uniformly distributed load, calculates the values of χ according to the bifurcation theory .

The values for $[\overline{q}\,/\,b]$ are significantly reduced and depend on the parameter *l*/*b*.

This influence of the lateral-torsional buckling is presented in the Figure 4.

4. CONCLUSION

From the ratio diagrams $\frac{\overline{q}}{b}$ and $\frac{l}{h}$ the following may be concluded:

1. If the ratio is $\frac{l}{h} \le \frac{\alpha}{\beta}\frac{\sigma_{md}}{\tau_{m\|d}}$ - *short beams* – then the tangential stress is the required design value;
2. If the ratio is $\frac{\alpha}{\beta}\frac{\sigma_{md}}{\tau_{m\|d}} < \frac{l}{h} < \frac{\gamma}{\alpha}\frac{E}{\sigma_{md}}\frac{1}{m}$ – *medium length beams* – the normal stress is the required design value;
3. If the ratio is $\frac{l}{h} \ge \frac{\gamma}{\alpha}\frac{E}{\sigma_{md}}\frac{1}{m}$ – *long beams* – the beam deflection is the required design value.

The equality sign in these expressions indicates the intersection points of curved lines.

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OPTIMALNO DIMENZIONISANJE NOSAČA OD DRVETA

Dragoslav Stojić, Srđan Živković

U savremenim drvenim konstrukcijama veliki broj nosača je opterećen na pravo savijanje. Dosadašnja inženjerska praksa je pokazala da se, veoma često, prilikom dimenzionisanja ovakvih nosača u praksi ne kontrolišu uticaji tangencijalnih napona i ugiba nosača, što može izazvati negativne posledice na ponašanje konstrukcije za vreme njene eksploatacije.

Koristeći proceduru optimizacije, ovaj rad razmatra štap pod takvim opterećenjem. Pri tome je korišćen uslov iskorišćenja dopuštenih normalnih napona savijanja, smičućih napona i maksimalnog ugiba.