FACTA UNIVERSITATIS
Series: **Architecture and Civil Engineering** Vol. 3, N° 1, 2004, pp. 109 - 120

PROPOSITION OF A MODEL FOR THE LIMIT STATE OF RUPTURE OF TIMBER STRUCTURES

UDC 624.011.1:624.042 (045) = 20

Dragoslav Stojić

Faculty of Civil Engineering and Architecture, University of Niš, Serbia and Montenegro

Abstract. *This paper deals with a general state of European norms for timber structures, according to "Eurocode 5: Design of timber structures, part 1.1, General rules and rules for buildings", together with the following European norms EN. The concept of design of timber structures, according to limit states and Eurocode 5 (EC5). draft is briefly shown. In some areas a comparative analysis of EC5/JUS/DIN norm is given. The paper contains the proposition of the possible model for limit state of rupture of the timber structures during bending and eccentric stress.*

Key words: *Limit state of rupture, Timber Structures, Eurocode 5.*

1. INTRODUCTION

In the recent years, in Europe in particular, a grate effort is made to harmonize the existing national standards with the European EN norms and ISO. A number of Yugoslav standards for timber and timber structures is already adapted to the international standards, but there is still a large part of the job to be done, to amend the old and create the new standards. The EU member states and EFTA are bound to give up their national standards after they adopt the uniform and common European standards. Thus, in spite of the resistance of some groups in scientific and professional circles, the well known DIN, BS, SIA and other standards will vanish for ever.

The European Committee for Standardization (CEN) is dealing, on the basis of the corresponding mandates of the Commission of the European Community CEC, with the creation of the standards in the European area in order to improve the transfer of goods and services, by removing the obstacles presented by the variety of technical regulations. Building this common concept for creation of the unified European civil engineering market, the CEN has, in the framework of the technical committee TC 250, made the standards for design of structures in civil engineering (Structural Eurocode) and the accompanying standards for building materials, industrial production of certain building

Received December 12, 2004

materials and testing of the material properties. That is how CEN/TC 250/SC5 PT-1 for the design of timber structures according to the report EUR 9887 has made "Eurocode 5" – common regulations for the timber structures since 1987. The draft of the Eurocode was prepared by the working group W18 – CIB (*Conseil International du Bâtiment pour la Recherche, I', Etude et la Documentation*)in cooperation with CEI (*Confédération Européen des Industries du Bois*) and the working group of RILEM (*Réunion Internationale des Laboratoires d' Essais et de Recherches sur les Matériaux et les Constructions*) and ISO (*International Standardisation Organisation*). The report CIB-REPORT of 1983 "*CIB Structural Timber Design Code*" was a working version of Eurocode 5. The CEN issued a new version of Eurocode 5[1] titled Design of timber structures, part 1.1 " General regulations and building design regulations (shortened EC 5), on 20th November 1992 for a temporary use as an European pre-standard ENV 1995-1-1:1993 in order to develop it into the European standard. Remaining parts of the Eurocode 5 are:

Part 1, 2: Additional regulations for fire protection;

Part 2: Bridges;

The members of the CEN are the national institutes for standardization of the EU countries and EFTA. The seat of the CEN is in Bruxelles. The CEN has *Technical committees - TC*, *Subcommittees - SC* and *Project Teams – PT* for different areas. The subcommittee SC 5 works On the Eurocode 5, and for the corresponding parts of the Eurocode 5, there are the project teams PT1, PT2,..., PT10.

According to the way they are processed, the European Standards can be:

pr EN - Draft of European Standards,

EN - European standard,

pr ENV - Draft of European pre-standard and,

ENV - European pre-standard.

Along with issuing the EN simultaneously the corresponding national standards are being withdrawn.

In transitional (trial) period of the application of the ENV, all the necessary data for design which are not present in the Eurocode are comprised in the *National Application Document* of EC5. Each country member has to issue the NAD, so the parallel trial application of EC5 and national standards is rendered possible. The NAD contains all the required values for application of EC5, and all the regulations and values of constants of materials which differ from those of EC5 must be specially marked. The NAD in the field of timber structures in certain countries is still being prepared. The NAD enables to overcome the differences of the EC5 and the existing national standards. The NAD comprises some additional regulations, and some modifications aimed at simplification of some articles of the EC5

As the EC5 is based on the concept of limit states, and the other national standards in Europe on the concept of permissible stress, it is necessary to use and test the EC5 along with the national document in practice, in multiple projects, so that the differences in results could be identified, and the EC5 possibly corrected.

[1] Work group is expanded to include A.Ceccotti (Italy), J. Kuipers (The Netherlands) and P. de Sousa (Portugal)

2. The Fundamentals of Timber Structures Design

2.1 General

The fundamentals of timber structures design which are in the EC 5 are the same as in the structures of some other materials (Concrete EC2 and steel EC3)

The basic concept design of timber structures is the limit states concept:

-Ultimate Limit States-ULS

-Serviceability Limit States-SLS

Design situations are classified as: persistent, transient and accidental situations

The limit states concept significantly differs from the concept of permissible stress, which is a still valid concept in many countries. It comprises a solution of two problems, and those are:

- design values of actions, that is a design value of an internal force or moment
- Corresponding design resistance
- Calculation of action upon the timber structures given in the EC1 is the same as the of the structures of other materials.

2.2 Characteristic values of actions

The design value Fd of an action is expressed in general terms as:

$$F_d = \gamma_F F_K \tag{1}$$

where:

F - symbol for the actions: G - permanent, Q - variable and A - accidental.

F_k - characteristic values of actions: Symbol for: Gk -permanent -, Qk- variable i Ak-accidental.

γ_F - partial safety factors for the action.

2.3. Combinations of actions

For each case, design values Ed for the effects of actions shall be determined from combination rules involving design values of actions as identified by Table 1

Table. 1. Design values of actions for use in the combination of actions

Design Situation	Permanent actions G_d	Variable actions		Accidental actions
		One Q_d	All other	A_d
Persistent and Transient	$\gamma_G G_k$	$\gamma_Q Q_k$	$\psi_0 \gamma_Q Q_k$	–
Accidental	$\gamma_{GA} G_k$	$\psi_1 Q_k$	$\psi_2 Q_k$	$\gamma_A A_k$ (if A_d not specified directly)

The design values of table 1 shall be combined using the following rules

- Persistent and Transient design situations (fundamental combination)

$$\sum \gamma_{Gj} G_{kj} + \gamma_{Q,1} Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \tag{2}$$

Accidental design situations (if not specified differently elsewhere)

$$\sum \gamma_{GA,j} G_{k,j} + A_d + \psi_{1,1} Q_{k,1} + \sum_{i>1} \psi_{2,i} Q_{k,i} \quad (3)$$

where the symbols are defined as follows:
$G_{k,j}$ – characteristic values of permanent actions
$Q_{k,1}$ – characteristic values of one of the variable actions
$Q_{k,1}$ – characteristic values of the other variable actions
A_d – design values (specified values) of the accidental actions
$\gamma_{G,j}$ – partial safety factors for permanent actions
$\gamma_{GA,j}$ – as γG,j but for accidental design situations
$\gamma_{Q,j}$ – partial safety factors for variable actions
ψ_0, ψ_1, ψ_2 – factors are defined s follows:

- combination value: $\psi_0\ Q_k$
- frequent value: $\psi_1\ Q_k$
- quasi-permanent value: $\psi_2\ Q_k$

2.4 Material properties

The specific values and design values of the material properties must be determined as material characteristics.

The design value X_d of a material property is defined as:

$$X_d = K_{mod}\ X_k / \gamma_m \quad (4)$$

where:
K_{mod} – modification factor, taking into account the effect on the strength parameters of the duration of the load and the moisture content in the structure[2].

The design resistance *Rd* is defined as:

$$R_d = R_d(X_d, a_d) \quad (5)$$

where a_d is the geometrical data.

Characteristic values X_k in general corresponds to a fractile in the assumed statistical distribution of the particular property of the material, specified by the relevant standards and tested under specified conditions: (20°C/60%/ 300s.)

Characteristic strength and stiffness values (E i G) and densities shall be derived according to the method given in EN 384.

The proposed strength classes and the relative values for various properties are given in EN 338 (*Structural timber, Strength classes*).

γ_m – partial safety factor for the property of material

Table 2 presents the comparative values of the existing standards JUS.UC9.200, DIN 1052, DIN 4074 and EN 338.

[2] Load-duration classes data are specified in EC 5 (Table 3.1.6).

Table. 2. Comparative analysis of classes JUS/ EN/ DIN

Strength Classes EN 338	Classes according to JUS.UC9.200 (DIN 1052)	Classes according to DIN 4074
C18 or C22	III	S7
C24	II	S10
C30	I	S13
C35 or C40	–	MS17

2.5. Serviceability limit states

Serviceability limit states correspond to the states beyond which specified service criteria are no longer met. This state comprises:

A. Deformations or deflections which affect the appearance or effective use of structure (including the malfunction of machines or services) or cause damage to finishes or nonstructural elements.

B. Vibration which causes discomfort to people, damage to the building or which limits its functional effectiveness.

The deformation of a structure which results from the effects of actions and from moisture shall remain within appropriate limits, having regard to the possibility of damage to surfacing materials, ceilings, partitions and finishes, and to the functional needs as well as any requirements of appearance.

2.6 Ultimate limit states

ULTIMATE LIMIT STATES comprise the following (Verification conditions):

A. When considering a limit state of state equilibrium or of gross displacement or deformations of the structure, it shall be verified that

$$E_{d,\ dst} \leq E_{d,\ stb} \quad (6)$$

where:

E – effect of action

d – design

dst – destabilizing

stb – stabilizing

B. When considering a limit state of rupture or excessive deformation of a section, member or connection it shall be verified that:

$$S_d \leq R_d \quad (7)$$

where:

S – force or moment

R – resistance

S_d and R_d – the design value -of an force or moment (S_d) and design resistance (R_d)

C. When considering a limit state of transformation of the structure into a mechanism, it shall be verified that a mechanism does not occur unless actions exceed their design values-associating all structural properties with the respective design values.

D. When considering a limit state of stability induced by the second-order effects it shall be verified that instability does not occur unless actions exceed their design values –

associating all structural properties with the respective design values. In addition, sections shall be verified according to EC5: 2.3.2.1P(2)

3. PROPOSITION OF THE LIMIT STATE OF RUPTURE MODEL

In calculation of the rectangular cross-sections loaded on bending, which are the most frequent ones, it is adopted that the neutral axis divides the height of the cross-section in half. Observing the realistic behavior of the timber member of such cross-section loaded only by the bending moment, one may conclude that the distribution of stress and strain corresponds to these suppositions only at very low values of normal bending stress (Fig. 1a).

In the phase of elastic deformations, due to the various modules of elasticity in the compressed and tensioned zone of cross-section, the neutral axis is dislocated towards the tensioned edge, where a bilinear distribution of stress occurs (Fig. 1b). For the calculation of the geometrical values of the rectangular section the idealized I –section, rectangular section and reduced modulus E, the double modulus theory is used.

As the load increases, that is the normal stress, and since the limit strength of timber stressed to compression (σ_{c*}) is by the absolute value lower than the limit strength to tension (σ_{t*}) up to three times, the fibers of the cross section exposed to compression will be sooner brought to the elastic stress limit than the fibers stressed to tensioning. The moment when the tensioned fibers are still in the phase of elastic deformations, and when in the most loaded compressed fiber the stress σ_{c*} is achieved, can be defined as the beginning or threshold of the pressure flow. (Fig. 1c) the moment when the most loaded tensioned fiber reaches the tensile strength of timber σ_{t*}, the cross section is not capable to accept the increase of the external load, so the cross-section fails (ruptures) (Fig. 1d).

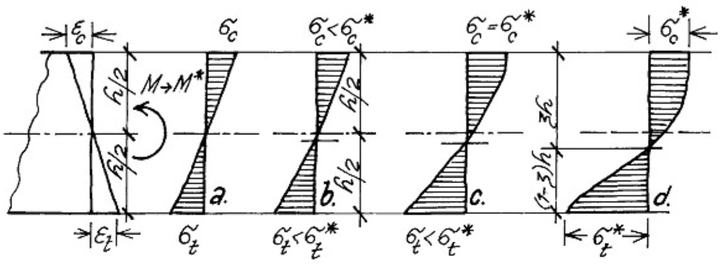

Fig. 1. Stress and strain distribution

A part of the diagram which characterizes the compressive stress is rounded, and the neutral axis is moved towards the tensioned edge of the rectangular cross-section. In the moment of failure, the distribution of dilatation in the cross section remains approximately linear.

If instead of the real diagrams σ – in the calculation an idealized diagram of stress and dilations is introduced, especially in the case of compression, the behavior of timber can be represented by Prantl-Royce material (ideally elasto-plastic material – without reinforcement), and in the case of tensioning by Hook's (brittle-elastic- material) the non-linear distribution of stresses can be approximated by the Mirko Ros model (Fig. 2.).

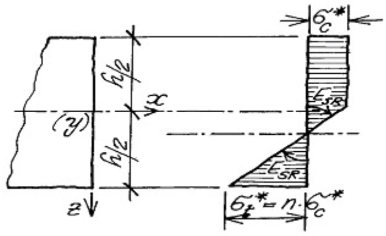

Fig. 2. Mirko Ros model

3.1. Bending members

3.1.1. The position of the neutral axis at the moment of failure

The position of the neutral line will be defined by the coordinate zh (measured from the compressed edge), and it can be obtained from the interior forces equilibrium conditions. (Fc = Ft):

$$\sigma_c^* \cdot \varsigma \cdot h - \frac{1}{2} \cdot \sigma_c^* \cdot \frac{\sigma_c^*}{E} = \frac{1}{2} \cdot \sigma_t^* \cdot (1-\varsigma) \cdot h \tag{8}$$

If the compressive stress is expressed by the tensile stress

$$\sigma_t^* = n \cdot \sigma_c^* \tag{9}$$

and (on the basis of Figure 3.) a substitution is introduced

$$\frac{\sigma_t^*}{E} = (1-\varsigma) \cdot h \tag{10}$$

the position of the neutral axsis is determined by the expression:

$$\varsigma = \frac{1+n^2}{(1+n)^2} \tag{11}$$

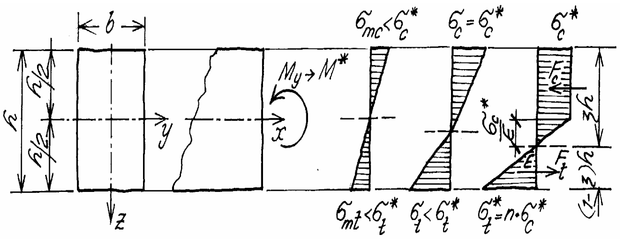

Fig. 3. The position of the neutral axsis

3.1.2. Failure momentum expressed by the internal forces

The failure momentum (or bearing capacity momentum) can be obtained as the sum of momentum of interior compressive and tensile forces in respect to the neutral axis of cross-section.

$$M^* = F_{c1} \cdot z_1 + F_{c2}\ z_2 + F_t \cdot z_3 \tag{12}$$

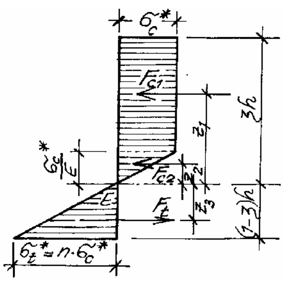

Fig. 4. The interior forces

If from the similarity of triangles (Fig. 4) the following value is determined

$$\frac{\sigma_c^*}{E} = \frac{1}{n} \cdot (1-\varsigma) \cdot h \tag{13}$$

the internal forces expressed via compressive strength can be obtained by the integration of the stress diagram, and the corresponding arms (distances from the neutral line):

$$\begin{aligned} F_{c1} &= \frac{\sigma_c^*}{n} b \cdot h \cdot (n \cdot \varsigma + \varsigma - 1); \\ z_1 &= \frac{1}{2 \cdot n} \cdot h \cdot (n \cdot \varsigma - \varsigma + 1) \end{aligned} \tag{14}$$

If the expression for the position of the neutral axis, defined only in function of the relation of tensile and compressive strength – n, is substituted in the said equations, a final expression for the momentum of failure or momentum of bearing capacity of the rectangular timber cross-section is obtained:

$$M^* = \sigma_c^* \cdot W_y \cdot \frac{3 \cdot n^4 + 8 \cdot n^3 + 6 \cdot n^2 - 1}{(1+n)^4} \tag{15}$$

where:

$$W_y = \frac{b \cdot h^2}{6} \tag{16}$$

is the section modulus for the main central axis around which the bending is performed.

3.2 Bending members with axial compression*)

If, apart from the bending momentum, the member is exposed to the centric compressive force, such that it is lower than the force of compression which effects crushing of the timber on the entire surface of cross-section,

$$N = A_N \cdot \sigma_c^* \leq N^* = A \cdot \sigma_c^* \tag{17}$$

and the problem of the stability is not considered, the procedure of determination of the bearing capacity of the cross section is as follows.:

3.2.1Position of the neutral axis for the case of BENDING MEMBERS with AXIAL COMPRESSION

If such force is applied into the centroid of the cross-section, it will cause a constant normal compressive stress in the carrier, along the entire height of the cross-section (Fig. 5)

$$\sigma_c = \frac{N}{A} \tag{18}$$

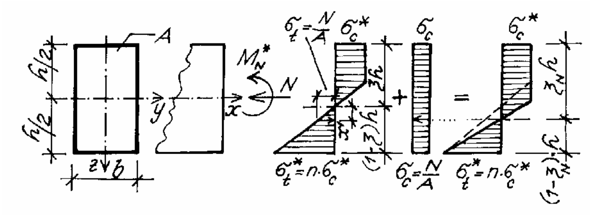

Fig. 5. Stress distribution and position of the neutral axis

It is can be easily concluded that the neutral line, defined for the case of occurrence only of the bending moment, will move towards the tensioned zone of the cross-section, exactly into a fiber where prior to the application of the centric force was the following stress: $\sigma_t = -\sigma_c = N/A$.

Then:

$$\varsigma_N \cdot h = \varsigma \cdot h + x' \tag{19}$$

The value of movement of the neutral line can be obtained from the similarity of triangles

$$x' = \frac{\sigma_c}{\sigma_c^*} \cdot \frac{h}{n} \cdot (1 - \varsigma) \tag{20}$$

If this substitution is done

$$\varsigma = \frac{1 + n^2}{(1 + n)^2} \tag{21}$$

then:

$$x' = \frac{\sigma_c}{\sigma_c^*} \cdot \frac{2h}{(1 + n)^2} \tag{22}$$

From the previous expressions, it results that the position of the neutral axis of the rectangular cross-section of timber loaded with the bending momentum with the normal force is:

$$\varsigma_n = \frac{1 + n^2 + 2 \cdot \frac{\sigma_c}{\sigma_c^*}}{(1+n)^2} = \varsigma + \frac{2}{(1+n)^2} \cdot \frac{\sigma_c}{\sigma_c^*} \tag{23}$$

3.2.2 Timber cross-section bearing capacity momentum at the moment of rupture in the case of action of bending momentum and normal force

The procedure for determination of the intensity of interior forces and their distances from the neutral axis of the totally plastified cross-section is analogous to the one for the case of action of bending momentum only, because the previous expressions can be used, and only instead of ς, one should take ς_N, that is:

$$\begin{aligned}
F_{c_1,N} &= \frac{\sigma_c^* \cdot b \cdot h}{n \cdot (1+n)^2} \cdot [n^3 - n + 2 \cdot (1+n) \cdot \frac{\sigma_c}{\sigma_c^*}] \\
z_{1,N} &= \frac{h}{2 \cdot n \cdot (1+n)^2} \cdot [n^3 + 3 \cdot n + 2 \cdot (1-n) \cdot \frac{\sigma_c}{\sigma_c^*}] \\
F_{c_2,N} &= \frac{\sigma_c^* \cdot b \cdot h}{n \cdot (1+n)^2} \cdot [n - \frac{\sigma_c}{\sigma_c^*}] \\
z_{2,N} &= \frac{4 \cdot h}{3 \cdot n \cdot (1+n)^2} \cdot [n - \frac{\sigma_c}{\sigma_c^*}] \\
F_t &= \frac{\sigma_c^* \cdot b \cdot h}{(1+n)^2} \cdot [n - \frac{\sigma_c}{\sigma_c^*}] \cdot n \\
z_{3,N} &= \frac{4 \cdot h}{3 \cdot (1+n)^2} \cdot [n - \frac{\sigma_c}{\sigma_c^*}]
\end{aligned} \tag{24}$$

The momentum of bearing capacity of the cross section cannot be described in respect to the neutral axis, because its position is not defined from the conditions of equality of interior forces. In the case of bending, the interior forces formed the composition whose momentum is invariable at any point of the cross-section. IN this case, the neutral axis of cross-section in the phase of failure is in the function of normal force (sum of all interior forces in the cross-section in the direction of the axis of the member must be equal to the given exterior normal force), so that the momentum of bearing capacity of the cross-section can be determined only in respect to the main central axis, in this case the gravity axis y-y, around which bending is performed.

The distance between the main central axis and the neutral axis of the cross-section at the moment of failure for the case when there is also loading with both with the momentum of bending and normal force, on the basis of Figure 6 will be:

$$x'' = \varsigma_N \cdot h - \frac{h}{2} = \frac{h}{2} \cdot (2 \cdot \varsigma_N - 1) = \frac{h}{2 \cdot n \cdot (1+n)^2} \cdot [4 \cdot \frac{\sigma_c}{\sigma_c^*} + (1+n)^2] \tag{25}$$

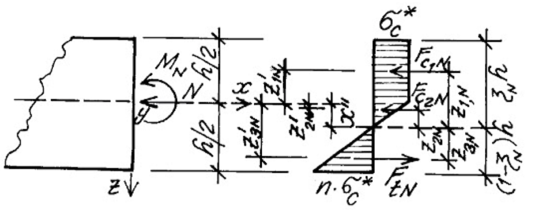

Fig. 6. The distance between the main central axis and the neutral axis of the cross-section at the moment of failure

Now the arms of the interior forces (distances from the main central axis of cross section) can be defined as:

$$z'_{1,N} = z_{1,N} - x''$$
$$z'_{2,N} = x'' - z_{2,N} \qquad (26)$$
$$z'_{3,N} = x'' - z_{3,N}$$

From the previous expressions, the following is obtained:

$$z'_{1,N} = \frac{h}{n \cdot (1+n)^2} \cdot [n - \frac{\sigma_c}{\sigma_c^*}]$$
$$z'_{2,N} = \frac{h}{6 \cdot n \cdot (1+n)^2} \cdot [4 \cdot (3 \cdot n + 2) \cdot \frac{\sigma_c}{\sigma_c^*} + 3 \cdot n^3 - 6 \cdot n - 5] \qquad (27)$$
$$z'_{3,N} = \frac{h}{6 \cdot (1+n)^2} \cdot [4 \cdot \frac{\sigma_c}{\sigma_c^*} + n \cdot (3 \cdot n^2 - 2 \cdot n + 3)]$$

So the momentum of failure is:

$$M^* = M^*_{1,N} - M^*_{2,N} + M^*_{3,N} = F_{c1,N} \cdot z'_{1,N} - F_{c2,N} \cdot z'_{2,N} + F_{t,N} \cdot z'_{3,N} \qquad (28)$$

The final expression for the momentum of failure of the rectangular timber cross-section, in the case when the momentum of bending and normal force act in the cross-section, is:

Knowing the value of the bearing capacity of the cross-section, and regarding the equality of the work of interior forces with the work of the exterior load, that is from the equality of this momentum with the momentum in the most loaded cross-section of the carrier, expressed through the exterior load, one can calculate the intensity of the failure load in the observed cross-section.

4. Conclusion

The proposed model for calculation of timber structures according to the limit state of rupture has only a research character for the time being. This model can be applied in more precise determination of bearing capacity and safety of timber structures.

Note: *Zoran Zoric, Prof. D. Stojic's assistant at the Faculty of Civil Engineering participated in this research.*

REFERENCES

1. Roš M.: Ergebnisse der Belastugnsversuche von Arth-Goldau an einem einstieligen Versuchsbinder der Kreisdirektion Luzern der SBB, Bericht, Nr 152 der EMPA, 1945.
2. EUROCODE 5: Design of Timber Structures, Part 1.1. 1993.
3. Borislav Zakić: Uvod u mehaniku drveta, FTN Novi Sad, IMS Beograd, Beograd, 1985.
4. Vlatko Brčić: Otpornost materijala, Beograd, 1978.

PREDLOG MODELA ZA GRANICNO STANJE LOMA DRVENIH KONSTRUKCIJA

Dragoslav Stojić

U radu je dat opšti pregled stanja evropskih standarda za drvene konstrukcije i to "EVROKOD 5: Proračun drvenih konstrukcija, deo 1.1, Opšta pravila i pravila za zgrade" sa pratećim evropskim standardima EN. Prikazan je ukratko koncept proračuna drvenih konstrukcija prema graničnim stanjima, prema nacrtu Evrokoda 5 (EC5). Data je u nekim oblastima i uporedna analiza standarda EC5 / JUS / DIN. Rad sadrži predlog mogućeg modela za granično stanje loma (limit state of reptere) drvenih konstrukcija pri savijanju i ekcentričnom naprezanju.