

## **DEPENDENCE OF THE BASIC DYNAMIC PARAMETERS ON THE DISTURBING FORCE OF THE IMPACT IMPULSE TYPE WITH DIFFERENT FORMS**

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**Abstract.** *In the modern civil engineering design, especially in the field of machine foundations, the dynamic actions in the form of the impulse load that is short-term, high intensity loads are very frequent. When an acting force, during a short time interval remains constant, the impulse is of a rectangular form, as opposed to the trapezoid, triangular, parabolic and sinusoid form, with defined time of increase and decrease of the load. This paper considers the influence of the form of the impact impulse load on the dynamic response of the foundation structure, without changing the remaining parameters.*

**Key words:** *impulse load, form of the impulse, foundation, dynamic response.*

### 1. INTRODUCTION

In the field of civil engineering, machine engineering, civil and military industry design, the short-term high intensity dynamic loads are frequent. They occur mostly during the course of work of a machine and machine facilities or under the influence of impact and explosive actions such as falling of the weight onto the ground, seismic actions, explosive impact waves etc. In literature, the short term impulse loads of medium intensity – in exploitation, and the short term impulse loads of very high intensity – in machine failure, are separately treated.

The result of such actions on the structure depends on the interaction in the dynamic system, which according to the observed problem can be defined as the machine-foundation- bedding, impact body - structure - bedding, impact wave – structure – bedding.

This paper considers the short term impulse loads of medium intensity, which occur mostly during the work of the machines. Despite the short duration, such loads have certain regularity of change during the course of time, that is, certain form. The aim is to determine the influence of the impulse load form on the dynamic response of the

foundation structure, in order to substitute the more complex forms with simpler ones in practical design calculations.

## 2. IMPACT IMPULSE LOAD FORMS

The impulse load is a short-term high intensity load that does not change the direction of its action and has not more than one maximum, during the constant action  $\tau$ .

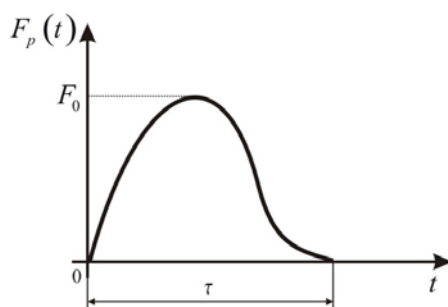


Fig. 1. Single impulse – graphic representation

As represented in the analytic form, the impulse load can be defined by the expression:

$$\begin{aligned} F_p(t) &= F_0 \cdot f(t) \text{ for } 0 \leq t \leq \tau \\ F_p(t) &= 0 \text{ for } t > \tau \end{aligned} \quad (1.1)$$

Where:  $\tau$  – is the duration of impulse  
 $f(t)$  – function characterizing the form of impulse  
 $F_0$  – maximum load during constant action

The value defined by the expression:

$$S = \int_0^{\tau} F_p(t) dt = F_0 \int_0^{\tau} f(t) dt \quad (1.2)$$

Represents the force impulse and according to the (Fig. 1) is numerically equal to the surface of the force – time diagram.

The regularity of change of the dynamic impulse load during time – form of the impulse, can be different, but in practice, the rectangular, triangular, trapezoid, parabolic and sinusoid form are most often encountered.

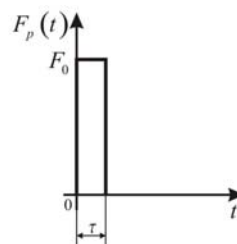


Fig. 2. Rectangular impulse forms

$$F_p(t) = \begin{cases} F_0 = \text{const} \rightarrow 0 \leq t \leq \tau \\ 0 \rightarrow t > \tau \end{cases} \quad (1.3)$$

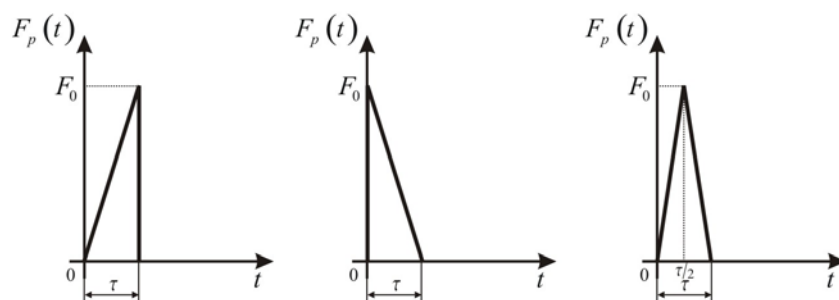


Fig. 3. Triangular impulse forms

$$F_p(t) = \begin{cases} F_0 \frac{t}{\tau} \rightarrow 0 \leq t \leq \tau \\ 0 \rightarrow t > \tau \end{cases}$$

$$F_p(t) = \begin{cases} F_0 \left(1 - \frac{t}{\tau}\right) \rightarrow 0 \leq t \leq \tau \\ 0 \rightarrow t > \tau \end{cases} \quad (1.4)$$

$$F_p(t) = \begin{cases} 2F_0 \frac{t}{\tau} \rightarrow 0 \leq t \leq \frac{\tau}{2} \\ 2F_0 \left(1 - \frac{t}{\tau}\right) \rightarrow \frac{\tau}{2} \leq t \leq \tau \\ 0 \rightarrow t > \tau \end{cases}$$

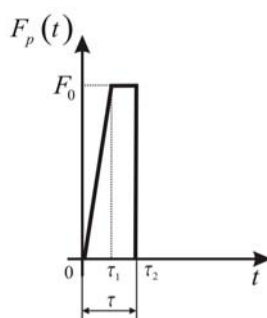


Fig. 4. Trapezoid impulse forms

$$F_p(t) = \begin{cases} F_0 \frac{t}{\tau_1} \rightarrow 0 \leq t \leq \tau_1 \\ F_0 = \text{const} \rightarrow \tau_1 \leq t \leq \tau_2 \\ 0 \rightarrow t > \tau_2 \end{cases} \quad (1.5)$$

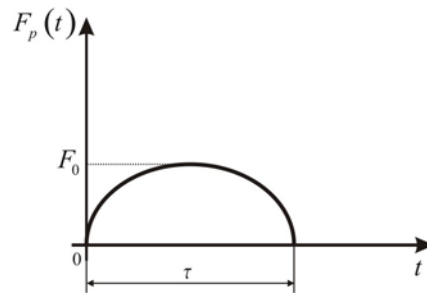


Fig. 5. Parabolic impulse forms

$$F_p(t) = \begin{cases} -F_0 \frac{4}{\tau^2} t^2 + F_0 \frac{4}{\tau} t \rightarrow 0 \leq t \leq \tau \\ 0 \rightarrow t > \tau \end{cases} \quad (1.6)$$

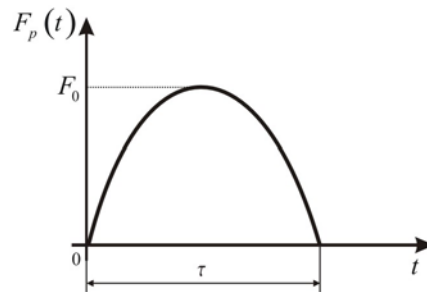


Fig. 6. Sinusoid impulse forms

$$F_p(t) = \begin{cases} F_0 \sin\left(\frac{\pi}{\tau} t\right) \rightarrow 0 \leq t \leq \tau \\ 0 \rightarrow t > \tau \end{cases} \quad (1.7)$$

The impulse duration time ( $\tau$ ) ranges from thousandth parts of a second to several seconds and more and in certain cases can be determined by calculation or experimental methods [6], [12]. In the absence of the required data for determination of ( $\tau$ ) value, the value  $\tau = 0.001 \text{sec}$  can be adopted with sufficient certainty, especially when it comes to the machine impulse loads.

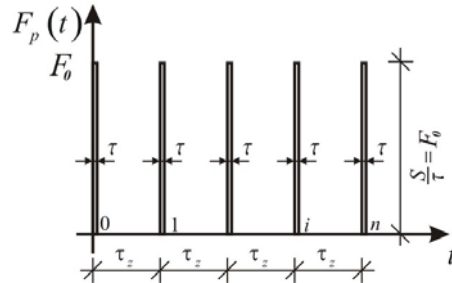


Fig. 7. Rectangular form impulse series

In the course of special technological processes, the short-term impulse load can be periodically repeated in certain time intervals ( $\tau_z$ ), resulting in the final perturbation force as a series of impulses.

### 3. THE OBSERVED DYNAMIC SYSTEM

The foundations of the forging hammers have been chosen for the analysis in this paper, since those are the machines with typical impact impulsive high intensity load. The dynamic models, that represent the real dynamic system, comprise the masses of the foundation and the machine, the bedding that supports the foundation (natural bedding – soil) and the slabs installed as the vibration dampers (artificial layers).

In accordance to the applied manner of vibration damping and the type of soil on which the foundation rests, the stiffness of the bedding ( $k$ ) and damping ( $c$ ) are defined. The dynamic system is observed with one or two degrees of freedom, as it is shown with corresponding models in (Fig. 8), depending on whether the foundation block lies directly on the soil or in a casing with vibration damping layers.

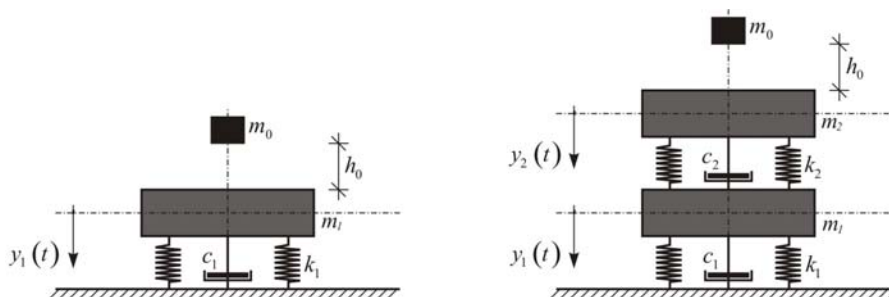


Fig. 8. Dynamic models for the system: a) with one degree of freedom, b) with two degrees of freedom ( $m_0$  – mass of the falling load,  $h_0$  – height of drop)

The adopted dynamic models need to be appended with the corresponding movement equations, whose solution will finally yield the response of the foundation structure to an impact impulse load.

## 4. MATHEMATICAL MODEL OF THE ADOPTED DYNAMIC SYSTEM

For the solution of problem, the movement equations known from the structure dynamics are used, derived on the basis of the D'Alembert principle for the event of forced damped oscillations and given in a general case for the system with  $m$  degrees of freedom in matrix form:

$$[m] \{\ddot{y}\} + [c] \{\dot{y}\} + [k] \{y\} = F_0 \{f(t)\} \quad (1.8)$$

Where:

$[m]$  – mass matrix

$\{\ddot{y}\}$  – column vector whose elements are the accelerated mass of the  $m_i$  oscillating system

$[c]$  – system damping matrix,

$\{\dot{y}\}$  – column vector whose elements are the speeds of the masses  $m_i$  of the oscillating system

$[k]$  – stiffness matrix

$\{y\}$  – column vector whose elements are the movement of the masses of  $m_i$  oscillating system.

$\{f(t)\}$  – column vector whose functions are the functions that define the impulse load regularity of change

$F_0$  – maximum amplitude of the impulse load

For the system with one degree of freedom, in respect to the markings of the Figure 8, and I accordance with the said system of equations (1.8), the expression will be:

$$m_1 \ddot{y}_1 + c_1 \dot{y}_1 + k_1 y_1 = F_0 f(t) \quad (1.9)$$

In the case of the dynamic system with two degrees of freedom, the expression will be:

$$\begin{aligned} [m] &= \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, & \{\ddot{y}\} &= \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} \\ [c] &= \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}, & \{\dot{y}\} &= \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} \\ [k] &= \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, & \{y\} &= \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} \\ F_0 \{f(t)\} &= F_0 \begin{Bmatrix} 0 \\ f(t) \end{Bmatrix} \end{aligned} \quad (1.10)$$

As it can be seen, the movement equations are non-homogenous differential equations of the second order with constant coefficients. The initial conditions that relate to the mass movement and the speeds of mass movement  $m_1$  and  $m_2$  are appended to the movement equations. The initial conditions for the system with one oscillating mass ( $m_1$ ), on which the impact load is acting directly, are:

$$\begin{aligned} y_1(0) &= 0 \\ \dot{y}_1(0) &= v_0 = \frac{S}{m_1} \end{aligned} \quad (1.11)$$

In the system with two oscillating masses, the impact impulse load acts directly on the mass ( $m_2$ ), thus:

$$\begin{aligned} y_1(0) &= 0 \quad \dot{y}_1(0) = 0 \\ y_2(0) &= 0 \quad \ddot{y}_2(0) = v_0 = \frac{S}{m_2} \end{aligned} \quad (1.12)$$

In the equations (1.11) and (1.12),  $v_0$  is the initial speed with which the mass to which the force impulse  $S$  is imparted starts to oscillate.

In solving the differential equation of movement, the modern software has been used – program package MATHEMATICA<sup>®</sup>.

## 5. ANALYSIS OF OBTAINED RESULTS

Dynamic responses of the structure which were used for the comparative analysis have been obtained from the equations (1.8), in accordance with equations (1.9) and (1.10), for the various displayed forms of impact impulse loads. The influence of the form of the impulse load on the size of the amplitudes of movement ( $A$ ), number of oscillations ( $f$  and  $\lambda$ ) and the damping parameters (logarithmic decrement  $\delta$  and relative damping  $\xi$ ) has been observed, without altering the remaining parameters in the dynamic system.

Simultaneously, the comparison of the results obtained for the simplest rectangular form of the impulse from [9], where the movement equations have been solved with numerical procedure (Runge-Kutta-Merson method), has been done.

### 5.1 System with one degree of freedom

This part of the analysis takes into consideration the data for the forging hammer VC 500, where the dynamic system is presented by the model (Fig. 8a). The size of the impulse, calculated according to Lipinski [6] is  $S = 4.45kNsec$  and the duration of the impulse is  $\tau = 0.001sec$ .

### 5.2 System with two degrees of freedom

The forging hammer MPM 1000 belongs to the high impact energy machines. It is supported by the foundation, consisting of the foundation block in the protective reinforced concrete casing with vibration damping insulation. The corresponding dynamic system is presented by the model (Figure 8b).

The size of the impulse, calculated according to the data for the said hammer, according to the instructions of Lipinski [6], amounts to  $S = 13.56kNsec$ . Duration of the impulse is equal to the previous case and is  $\tau = 0.001sec$ .

Table 1. Relative departures of the observed parameters for the different forms of impulses in respect to the 1. rectangular form – approximate solution (PFP), 2. rectangular form – correct solution (PFT)

Observed parameters	$A_{max}$ (mm)	$f = n / t$ (Hz)	$\lambda = 2\pi f$ ( $sec^{-1}$ )	$\delta$	$\xi$ %
Form of impulse					
<b>Rectangular form – approximate solution (PFP)</b>	<b>0.331</b>	<b>18.00</b>	<b>113.00</b>	<b>0.528</b>	<b>8.40</b>
<b>Rectangular form – exact solution (PFT)</b>	<b>0.348</b>	<b>18.33</b>	<b>115.17</b>	<b>0.485</b>	<b>7.72</b>
Relative departure from PFP %	+5.14	+1.83	+1.92	-8.14	-8.09
<b>Trapezoidal</b>	<b>0.349</b>	<b>18.33</b>	<b>114.98</b>	<b>0.491</b>	<b>7.81</b>
Relative departure from PFP%	+5.44	+1.83	+1.75	-7.01	-7.02
Relative departure from PFT%	+0.29	0	-0.16	+1.24	+1.17
<b>Parabolic</b>	<b>0.348</b>	<b>18.00</b>	<b>113.10</b>	<b>0.486</b>	<b>7.73</b>
Relative departure from PFP%	+5.14	0	+0.009	-7.95	-7.97
Relative departure from PFT%	0	-1.80	-1.79	+0.21	+0.13
<b>Sinusoid</b>	<b>0.344</b>	<b>18.33</b>	<b>115.19</b>	<b>0.459</b>	<b>7.31</b>
Relative departure from PFP%	+3.93	+1.83	+1.92	-13.06	-12.97
Relative departure from PFT%	-1.15	0	0	-5.36	-5.31

Table 2. Relative departures of the observed parameters for the different forms of impulses in respect to the 1. rectangular form – approximate solution (PFP), 2. rectangular form – correct solution (PFT)

<b>Foundation block</b>					
Observed parameters	$A_{max}$ (mm)	$f = n / t$ (Hz)	$\lambda = 2\pi f$ ( $sec^{-1}$ )	$\delta$	$\xi$ %
Form of impulse					
<b>Rectangular form – approximate solution (PFP)</b>	<b>2.43</b>	<b>5.12</b>	<b>32.17</b>	<b>0.403</b>	<b>6.40</b>
<b>Rectangular form – exact solution (PFT)</b>	<b>2.40</b>	<b>5.00</b>	<b>31.42</b>	<b>0.368</b>	<b>5.90</b>
Relative departure from PFP %	-0.81	-2.34	-2.34	-8.68	-7.81
<b>Trapezoidal</b>	<b>2.31</b>	<b>5.5</b>	<b>34.56</b>	<b>0.378</b>	<b>6.02</b>
Relative departure from PFP%	-4.94	+7.42	+7.43	-6.20	-5.94
Relative departure from PFT%	-3.75	+10.0	+10.0	+2.72	+2.03
<b>Parabolic</b>	<b>2.40</b>	<b>5.50</b>	<b>34.56</b>	<b>0.373</b>	<b>5.90</b>
Relative departure from PFP%	-0.81	+7.42	+7.42	-7.44	-7.81
Relative departure from PFT%	0	+10.0	+10.0	+1.24	0
<b>Sinusoid</b>	<b>2.40</b>	<b>5.50</b>	<b>34.56</b>	<b>0.392</b>	<b>6.20</b>
Relative departure from PFP%	-0.81	+7.42	+7.42	-2.72	-3.22
Relative departure from PFT%	0	+10.0	+10.0	+6.52	+5.08
<b>Protective casing</b>					
Observed parameters	$A_{max}$ (mm)	$f = n / t$ (Hz)	$\lambda = 2\pi f$ ( $sec^{-1}$ )	$\delta$	$\xi$ %
Forms of impulses					
<b>Rectangular form – approximate solution (PFP)</b>	<b>0.127</b>	<b>5.12</b>	<b>32.17</b>	<b>0.360</b>	<b>5.70</b>
<b>Rectangular form – exact solution (PFT)</b>	<b>0.127</b>	<b>5.00</b>	<b>31.42</b>	<b>0.387</b>	<b>6.20</b>
Relative departure from PFP %	0	-2.34	-2.34	+7.5	+8.77
<b>Trapezoidal</b>	<b>0.127</b>	<b>5.5</b>	<b>34.56</b>	<b>0.389</b>	<b>6.21</b>
Relative departure from PFP%	0	+7.42	+7.43	+8.06	+8.95
Relative departure from PFT%	0	+10.0	+10.0	+0.52	+0.16
<b>Parabolic</b>	<b>0.130</b>	<b>5.50</b>	<b>34.56</b>	<b>0.412</b>	<b>6.60</b>
Relative departure from PFP%	+2.36	+7.42	+7.42	+14.44	-15.79
Relative departure from PFT%	+2.36	+10.00	+10.00	+6.45	+6.45
<b>Sinusoid</b>	<b>0.127</b>	<b>5.50</b>	<b>34.56</b>	<b>0.427</b>	<b>6.80</b>
Relative departure from PFP%	0	+7.42	+7.42	+18.61	+19.29
Relative departure from PFT%	0	+10.00	+10.00	+10.33	+9.67



#### 4. CONCLUSION

The impact impulse load defined by the expression (1.1) is characterized by three significant parameters: duration of impulse action ( $\tau$ ), form of the impulse represented by the function  $f(t)$  and the intensity of the impulse  $F_0 = S / \tau$ . In this paper only the impulse form influence on the dynamic response of the foundation structure has been analysed, since it can be simple, more or less complex or completely unknown.

The comparative analysis, performed on the basic parameters of the dynamic response, shows that the relative departures for the considered forms, in respect to the simplest rectangular one, range within limits that can be neglected in practical designing. There is no essential difference either, between the results of the individual forms. It is also obvious, in comparison of the approximate and correct solution for the rectangular form, that the favourable numerical procedures for the solution of the problem can be used in their entirety, as shown in the literature [9].

The results of the analysis conducted in this paper coincide with the conclusions stated in the literature [6] and [12] that the form of the impulse has a second rate influence on the dynamic response of the system, in respect to the intensity and duration of the impulse.

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### ZAVISNOST OSNOVNIH DINAMIČKIH PARAMETARA OD POREMEĆAJNE SILE TIPA UDARNOG IMPULSA RAZLIČITE FORME

**Verka Prolović, Zoran Bonić, Srđan Živković**

*U savremenom građevinskom konstrukterstvu, posebno u oblasti temelja mašina, česti su dinamički uticaji u vidu impulsnog opterećenja odnosno kratkotrajnog opterećenja visokog inteziteta. Kada delujuća sila tokom kratkog vremenskog intervala ostaje konstantna, impuls je pravougaone forme, za razliku od trapezne, trougaone, parabolične i sinusoidne, sa definisanim vremenom rasta i opadanja opterećenja. U radu je razmatran uticaj forme udarnog opterećenja na ponašanje dinamičkog sistema temeljna konstrukcija-podloga ne menjajući ostale parametre.*