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# SYSTEM IDENTIFICATION APPROACH APPLICATION FOR EVALUATION OF SYSTEM PROPERTIES DEGRADATION

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**Abstract**. *In respect to the subspace identification method as one of the possible variants of inverse dynamic analyses, behavior of real structural systems with real load and real noise contaminated input/output data was investigated in this work. A useful and non-destructive dynamic parameters evaluation tool - vibration monitoring of the structure - is proposed. The report of original investigation on real models excited by an impulse load in laboratory is also presented. A special contribution is a software program for experiment monitoring and for determination of relevant mechanical characteristics as well as of the location of a possible structural damage.*

## 1. INTRODUCTION

Erosion, friction, fatigue, internal damages and cracks cause gradual degradation of structural performances during a long term service: the stiffness of the system is weakening, whereas the damping of the system is strengthening. The accumulation of this degradation could cause the system fail to continue performing a safe and satisfactory service as required and guaranteed in the design period. When the structural damage is small or it is in the interior of the system, it cannot be visually detected. A useful non-destructive evaluation tool is the vibration monitoring. It is based on the fact that the occurrence of damage or loss of integrity in a structural system leads to the changes in the dynamic properties of the structure.

In recent years, various analytical and experimental techniques have been proposed and developed to deal with the issue of damage or fault detection in structural systems. One of them is a simple and effective method called the *subspace identification method for identification of state space models* introduced by Ho and Kalman (1966). The main

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idea of this work was to test and prove this method and detect the damage position in the structure.

## 2. Introduction to Subspace Identification Method

### 2.1. Structural analysis and FEM model

The dynamic behaviour of complex structures is often modelled by a system of second order linear ordinary differential equation (e.g. Bathe 1990, Zienkiewicz and Taylor 1998), i.e.,

$$\mathbf{M}\ddot{\mathbf{w}}(t) + \mathbf{D}\dot{\mathbf{w}}(t) + \mathbf{S}\mathbf{w}(t) = \mathbf{f}(t) \quad (1)$$

where **M**, **D** i **S** are **mass**, **damping** and **stiffness matrices** of the structure respectively and **w**($t$) and **f**($t$) are the **displacement vector** and the **force vector**. Let $n$ be the number of degrees of freedom of the system. Then, $\mathbf{M},\mathbf{S},\mathbf{D} \in \mathrm{R}^{\mathrm{nxn}}; \mathbf{w}(t), \mathbf{f}(t) \in \mathrm{R}^{\mathrm{nx1}}$. This model is well known from classical structural analysis for finite degree-of-freedom systems or from FE discretization procedure for continuous systems.

Having in mind various influences on the structure during its course of service, it is understandable that the dynamic properties of the system are changing. There can be significant differences between initial values $\mathbf{D_0}$ and $\mathbf{S_0}$ compared to current values **D** and **S**. An evaluation of the current values can be a very difficult task. If the traditional methods known from damage mechanics and other relevant fields are used it is necessary to model the evolution of a system property, obtain system property parameters, trace the history of motion and loading, carry out complicated analysis and computation under prescribed initial and boundary conditions, and finally derive the degraded property and responses of the system of interest.

Another way, which will be considered in this work, is to use an "inverse" method, i.e. extract information about system properties from experimental input/output data; hence it does not require such a costly foregoing procedure. This way is based on the following procedures.

($i$) Given is the structural system. It is possible to arrange some actuators at some locations in the structure which will produce excitation of the system. Let $m$ be the number of actuators. ***The force vector*** **f**($t$) can be replaced by an *input* **Gu**($t$), i.e.,

$$\mathbf{f}(t) = \mathbf{G}\mathbf{u}(t), \quad (2)$$

where $\mathbf{u}(t) \in \mathrm{R}^{\mathrm{mx1}}$ is an *m-**input force vector*** ; and $\mathbf{G} \in \mathrm{R}^{\mathrm{nxm}}$ represents ***the input location influence matrix***.

($ii$) At the same time, $l$ sensors arranged at some locations in the system measure the response of the structure under foregoing excitation and the *output* can be expressed as:

$$\mathbf{y}(t) = \mathbf{C}_d\mathbf{w}(t) + \mathbf{C}_v\dot{\mathbf{w}}(t) + \mathbf{C}_a\ddot{\mathbf{w}}(t) + \overline{\mathbf{D}}'\mathbf{u}(t) \quad (3)$$

where $\mathbf{y}(t) \in \mathrm{R}^{\mathrm{lx1}}$ is an *l-**sensor output vector***; $\mathbf{C}_d \in \mathrm{R}^{\mathrm{lxn}}$, $\mathbf{C}_v \in \mathrm{R}^{\mathrm{lxn}}$ and $\mathbf{C}_a \in \mathrm{R}^{\mathrm{lxn}}$ represent ***output displacement***, ***velocity*** and ***acceleration location influence matrices***, respectively, and $\overline{\mathbf{D}}' \in \mathrm{R}^{\mathrm{lxm}}$ is the ***direct transmission matrix*** corresponding to direct input/output feedtrough. Note that there is no transmission matrix $\overline{\mathbf{D}}'$ if the position is measured. If the measured data are displacements (resp. velocities, accelerations), they will be referred to as *displacement* (resp. *velocity*, *acceleration*) *sensing*. In this three cases, two of the matrices $\mathbf{C}_d$, $\mathbf{C}_v$ and $\mathbf{C}_a$ vanish, respectively.

(*iii*) With foregoing data it is possible to find **M**, **S**, **D** so that the dynamic system modeled by Eq. (1) exactly supplies the output data (3) measured with input data (2).

*The subspace identification method for identification of state space models* as one of the latest among the several approaches for the mass, stiffness and damping identification is proposed here. It can be said that this method has a property of a black-box system because the full information about the stiffness and damping matrices are not available. This comes from the fact that the structural model equations identified either by the modal analysis based on FFT or by the subspace identification method for state space models or by other known identification techniques, are not really the second order dynamic differential equations (1). They are only a form of Eq. (1) under an unknown coordinate transformation. Generally, it is difficult to transform, in complete and unique sense, the former into the Eq. (1).

The main goal of this method is to evaluate three structural property matrices **M**, **S** and **D**. The well known principle of mass conservation can be applied here; so, the mass matrix of the system is constant given by its initial value $\mathbf{M_0}$.With information about the construction of the stiffness matrix it is possible to detect the location of the damaged or faulty elements, if any, in the structural system.

### 2.2. Subspace identification method for state-space models

Linear and quasi-linear ordinary differential equations of any given order with input/output, including the second order differential equations (1) with (2)-(3), may be equivalently expressed in a form of state space model. The equations below are known as state space model with multi-inputs and multi-outputs.

$$\dot{\mathbf{x}}(t) = \overline{\mathbf{A}}\mathbf{x}(t) + \overline{\mathbf{B}}\mathbf{u}(t), \tag{4}$$

$$\mathbf{y}(t) = \overline{\mathbf{C}}\mathbf{x}(t) + \overline{\mathbf{D}}\mathbf{u}(t) \tag{5}$$

where $\mathbf{x}(t) \in \mathrm{R}^{\mathrm{Nx1}}$ is ***state vector***, $\mathbf{u}(t) \in \mathrm{R}^{\mathrm{mx1}}$ is ***input vector***, $\mathbf{y}(t) \in \mathrm{R}^{\mathrm{lx1}}$is ***output vector***, $\overline{\mathbf{A}} \in \mathrm{R}^{\mathrm{NxN}}$is ***system matrix***, $\overline{\mathbf{B}} \in \mathrm{R}^{\mathrm{Nxm}}$ is ***control matrix***, $\overline{\mathbf{C}} \in \mathrm{R}^{\mathrm{lxN}}$ is ***observer matrix*** and $\overline{\mathbf{D}} \in \mathrm{R}^{\mathrm{lxm}}$ is ***direct transmission matrix***.

In the theory of the system identification and realisation the only available information is the input, i.e., system excitation, and output, i.e., the system response on the given excitation; hence initial behavior of the system is totally unknown. Mathematically, the main problem is to find such a state-space model (Eqs. (4)-(5) ) of a minimal dimension for the given experimental input/output data, so that input and output are satisfied. There are several methods and techniques in the system identification theory which attempt to solve this problem. Ho and Kalman subspace identification method is one of them.

Let us consider the structural system with the impulse input excitation and let the $\mathbf{y}(t)$ = $\mathbf{y}(i\Delta t)$ be the measured response of the structure on the impulse input $\begin{cases} \mathbf{u}(0) = 1 \\ \mathbf{u}(i\Delta t) = 0, i \geq 1 \end{cases}$.

Response is measured in equidistant time steps $\Delta t$ that have to be "very" small. Our job is to calculate system matrices $(\overline{\mathbf{A}}, \overline{\mathbf{B}}, \overline{\mathbf{C}}, \overline{\mathbf{D}})$ from Eqs (4) – (5) for given $\mathbf{y}(t_i)$, $\Delta t$ and $\mathbf{u}(t_i)$. The next step will be the evaluation of the dynamic properties **S** and **D** from the system matrices using a special algorithm.

Thus the system realisation problem may be reformulated as follows: for the given impulse response functions of the system, i. e., a set of Markov parameters,

$$\{\mathbf{Y}(s)\} = \begin{cases} \mathbf{D}, & s = 0 \\ \overline{\mathbf{C}}\mathbf{A}^{s-1}\mathbf{B}, & s > 0 \end{cases} \tag{6}$$

find a triplet $\{\overline{\mathbf{A}}, \overline{\mathbf{B}}, \overline{\mathbf{C}}\}$, called realisation of a state space model (4) – (5).

A standard algorithm based on a subspace identification method, called *Eigen- system Realisation Algorithm* (ERA) is accepted and a widely used method for solving the latter problem. One of the main steps in ERA algorithm is calculating of the **Hankel** matrix. For the derivation of Hankel matrix and for more details see the reference [1].

### 2.3. Iteration algorithm for the case of incomplete output data

The number of sensors which is required depends on the total number of DOF of the system under consideration (see reference [1]). That means that $l$ (number of sensors) has to be equal to $n$ (number of DOF) for uniquely evaluating the stiffness and damping matrices. For the system which has a very large $n$ it is difficult to put so many sensors on the structure. Moreover, due to the fact that the singular value decomposition of the Hankel matrix involves the dependence on the system eigenvalues, an increased number of eigenvalues generates eigenvalues with higher values. That causes the choice of shorter time steps $\Delta t$. One of possibilities is to make the whole large set of output data collecting data from several measurements with different pattern of sensor's and actuator's arrangements.

Usually, for the system with a large number of DOF only a set of incomplete data may be available. If $l < n$, we can use an iteration algorithm to provide a set of lacking data using following procedure.

We can arrange $l$ sensors at $l$ locations in structural system. The missing $n$-$l$ number of real sensors can be replaced with the fictive sensors which are properly arranged. If we know the state vector (response) $\mathbf{x}(t)$ for input $\mathbf{u}(t)$ we can calculate the missing output data. The initial values for stiffness and mass matrices $\mathbf{S}_0$, $\mathbf{M}_0$ are calculated using structural analysis for discretized system or from discretization methods for continuous systems. The evaluation of initial damping matrix is a difficult task. Hence $\mathbf{D}_0 = 0$ can be used as initial value and the current values of **D** as well as **S** can be calculated through the algorithm.

Using these initial values, system matrices $\overline{\mathbf{A}}, \overline{\mathbf{B}}, \widetilde{\mathbf{C}}$ can be calculated using Eqs. (7)-(9) given bellow. By solving the differential equation (10) we can get the state vector. By using the state vector, the output of the missing data, and afterwards the complete data, can be calculated. Using ERA it is possible to derive system matrices $\overline{\mathbf{A}}', \overline{\mathbf{B}}', \overline{\mathbf{C}}'$ with respect to an unknown coordinate system and from them to evaluate dynamic system properties: stiffness, damping matrices $\mathbf{S}_1$ and $\mathbf{D}_1$, and input location influence matrix $\mathbf{G}_1$

using the proposals from ref.[1]. In the next step these new values $\mathbf{S}_1$, $\mathbf{D}_1$, $\mathbf{G}_1$ will have the position of the initial values $\mathbf{S}_0$, $\mathbf{D}_0$, $\mathbf{G}_0$.

$$\overline{\mathbf{A}}_\alpha = \begin{bmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{M}_0^{-1}\mathbf{S}_\alpha & -\mathbf{M}_0^{-1}\mathbf{D}_\alpha \end{bmatrix}, \quad (7) \qquad \overline{\mathbf{B}}_\alpha = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_0^{-1}\mathbf{G}_\alpha \end{bmatrix}, \quad (8)$$

$$\widetilde{\mathbf{C}}_\alpha = [\mathbf{0} \quad \mathbf{I}_{n-l}]([\hat{\mathbf{C}}_d \quad \hat{\mathbf{C}}_v] - \hat{\mathbf{C}}_a \mathbf{M}_0^{-1}[\mathbf{S}_\alpha \quad \mathbf{D}_\alpha]), \tag{9}$$

$$\dot{\mathbf{x}}_\alpha(t) = \overline{\mathbf{A}}_\alpha \mathbf{x}_\alpha(t) + \overline{\mathbf{B}}_\alpha \mathbf{u}(t), \quad (10) \qquad \widetilde{\mathbf{y}}_\alpha(t) = \widetilde{\mathbf{C}}_\alpha \mathbf{x}_\alpha(t) + \widetilde{\mathbf{D}}_\alpha \mathbf{u}(t), \quad (11)$$

$$\hat{\mathbf{y}}_\alpha(t) = \begin{bmatrix} \mathbf{y}(t) \\ \widetilde{\mathbf{y}}_\alpha(t) \end{bmatrix}, \tag{12}$$

$$(\overline{\mathbf{A}}'_{\alpha+1}, \overline{\mathbf{B}}'_{\alpha+1}, \overline{\mathbf{C}}'_{\alpha+1}) = \mathbf{ERA}(\hat{\mathbf{y}}_\alpha(t)), \tag{13}$$

$$(\mathbf{S}_{\alpha+1}, \mathbf{D}_{\alpha+1}, \mathbf{G}_{\alpha+1}) = \Phi(\overline{\mathbf{A}}'_{\alpha+1}, \overline{\mathbf{B}}'_{\alpha+1}, \overline{\mathbf{C}}'_{\alpha+1}). \tag{14}$$

The procedure stops when the algorithm provides satisfactory values for matrices **S, D** and **G**. In the previous equations $\overline{\mathbf{C}} \in R^{lxn}$ is the observer matrix for the measured data, $\widetilde{\mathbf{C}} \in R^{(n-l)xn}$ - observer matrix for the missing data, $\overline{\mathbf{D}} \in R^{lxn}$ - direct transmission matrix for the measured data, $\widetilde{\mathbf{D}} \in R^{(n-l)xn}$ - direct transmission matrix for the missing data, **0** is zero matrix of size $l$x$(n-l)$, $\mathbf{y}(t) \in R^{lx1}$ - measured output, $\widetilde{\mathbf{y}}(t) \in R^{(n-l)x1}$ - the output of a missing data, $\hat{\mathbf{y}}(t) \in R^{nx1}$ - complete output.

## 2.4. Location estimation of damaged and faulty elements

If the data for the stiffness matrix **S** of a structural system at some stage are available, it is possible to estimate whether or not the stiffness of this system has been considerably weakened, and inside which element or part of the system this phenomenon of stiffness weakening occurs.

We can consider the initial system state without damage and the current state where the damage has already occurred. It means that the stiffness and strength of the system are changed in comparison with the initial values. But, regarding only the stiffness matrix we cannot conclude where the damage occurred. So, some procedure is needed to detect the right damage place. The aggregate stiffness may be computed through the assembling procedure known from FEM, i.e.,

$$\mathbf{S} = \sum_e \mathbf{S}^{(e)}$$

So, it is possible to do the "inverse" procedure, see [1], i.e., to find the stiffness of each element if the data for the aggregate stiffness matrix are available. Comparing the initial and current values of stiffness (e.g., axial, flexural, torsional rigidities) we can deduce the location of the damage or the faulty element.

## 3. Experimental Equipment

### 3.1. Physical properties of the tested structural systems

In the scope of testing and proving the theoretical and numerical part of this work some experiments have been done. Two steel bars (IPB 100 profile) with different way of supporting were tested in laboratory. The length of the bars was 4 m. First of them was hanging on the rubber ropes which were placed at a distance of 0.5m from both edges. The second bar was simply supported at the both ends on steel rollers and clamped with a special mechanism and rubber rolls which allowed deflections, but vertical and horizontal displacement at the ends were constrained.

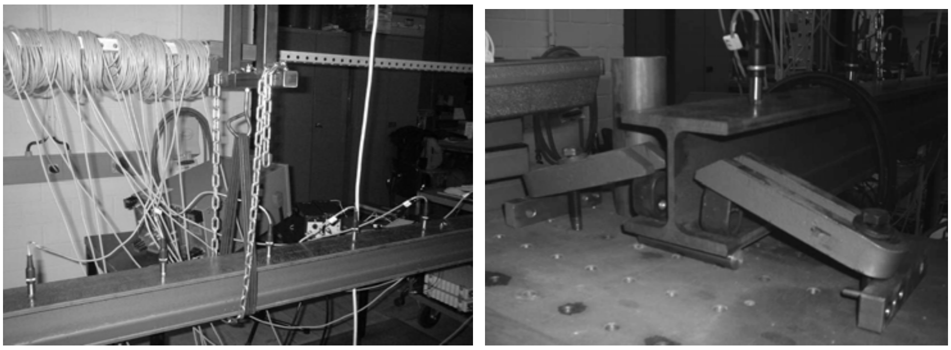


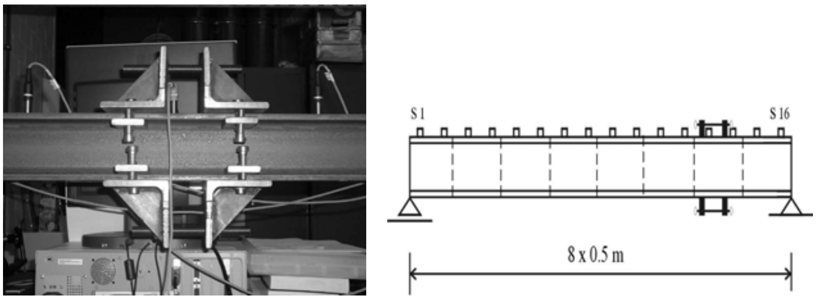
Fig. 1. Supports– a) Bar I, b) Bar II

### 3.2 Equipment

The structure was instrumented using the piezoelectric accelerometers. 16 sensors were screwed in the beam equidistantly on the upper side. Locations were chosen so that for both bars sensors were not at the supports; indeed this would be pointless due to the boundary conditions. The task was to measure the response of the structure at the sensor locations due to the impulse excitation which was applied at different positions of the beam (middle, near the supports of the first bar and at the end). The sensors were the special devices which were connected to a SPIDER 8. It had two devices and both of them had 8 inputs. The instrument was transferring the measured data in mV/V to a PC where all data were converted into acceleration sense. The software which was used for measurement monitoring and data converting to the desired form and graphs was **Catman$^{R}$ 31** (Hottinger Baldwin Messtechnik GmbH). The type of the accelerometers was B12/500 (produced by HBM) with characteristic frequency 500 Hz, nominal range 1000 m/s$^2$ and precision class ±2%. The used sensors were new with calibration certificate of the producer and only a small test was carried out in a laboratory. Spider 8/SR 55 was produced by the same company and had the characteristic of 9600samples/sec per each channel at the same time exactly in the same phase. Each channel has an analogue to digital converter (A/D). The precision class of the Spider was 0.1% over a full range. Before the measuring instruments were tested. A digital FFT analyser was used for testing eigenfrequencies of the bars which were expected during the measuring.

### 3.3 Simulation of the damage

In both experiments the bars were tested without and with simulated damage. A steel clamp (Fig.3.a) was placed but not tightened at some position of the beam. The response of the beam with this additional mass was measured. Then the clamp was tightened and the response was measured again. In the second case one part of the beam was clamped and it was expected that it would be enough to show that some damage occurred in that part. The difference between responses of the beams in both cases can be seen trough diagrams in section 4.2. Experimental results.




a) b)

Fig. 3. a) Clamp (simulation of the damage), b) position of the damage

## 4. EXPERIMENTAL PROCEDURE

### 4.1. Load details

Impulse load was applied by using hammer in three different positions: middle of the beam, near the $14^{th}$ sensor, and at the end, near sensor 16,. The responses of the structure in each of these cases were measured separately. It was checked that the duration of acting force was really "short" - less then 0.2 ms. This assured that acting force could be considered as an impulse load; indeed, the sample frequency was 4800Hz, i.e., the time step was $\approx$0.2ms. In each case the duration of the structure response measuring was 3 sec. The whole set of data was supplying information from 16 locations of the beam. Data were stored in such a way that they could be used as input data for the written program in Scilab as well as for a finite element program.

### 4.2. Experimental results

Although a large amount of data was collected, only few characteristic cases have been selected for presentation in this section. In the diagrams (1) to (4), the whole sets of data of one measurement for one characteristic sensor are shown where the influence of damping can be clearly seen. Instrument was not set to trigger, i.e., measurement does not start when some impact is applied, because we wanted to record the initial noise as well as the noise after finishing of the beam vibration. It can be seen that some negative influences of surrounding have occurred. First three diagrams represent the response of the first bar which was hanging on the rubber ropes. Because of the elastic supports the influence of damping is not too strong as in the case of second bar where the supports were at the edges and stiffer. Here, (Diagram 4) damping is acting very strongly and very soon after the vibration starts it can be seen that the beam is vibrating with its first eigenfrequency because all higher frequencies are smothered. Due to these soft supports in Bar I, it can be said that the rigid body motion plays an important role. This leads to the conclusion that Bar II is more acceptable structural system for this purpose.

In Diagrams (5) and (6) data were collected from different measurements and, because of that, they were rejected from the moment when the response of structure occurred and the time axis was chosen to start from zero at that point. The sample frequency in each measurement was the same, so, the rejection was allowed. These diagrams represent the response of the structure at the same place of the bar but in three different cases: first, called **free,** is without additional mass of clamp which will later simulate damage; second, named **undamaged** is case where a clamp was placed on the structure but it was only an additional mass to a new structure, i.e., it was strongly tightened and the wave could pass trough the beam without delay; and third, **damaged,** where clamp was not tightened strongly; hence the structure was damaged in the clamped part, compared to the foregoing case two. We can conclude also that the influence of the additional mass causes a change in the shape of the diagram; the response is not as smooth as in the *free* case (Diagrams 1–3). Comparing the results it can be concluded that this kind of simulation of damage can be applied because it evidently causes the expected differences in output data.

In Diagrams 5 and 6 the delay of the response of the system can be seen in an *undamaged* and *damaged* case in comparison with the *free*. The greater mass in the first two cases produces greater inertia and the answer of the structure is delayed. The stiffness of the beam is the same as in the case *free* but the mass is greater; that means, the frequency of oscillation of the beam is smaller and period of oscillation is greater. The amplitudes cannot be compared because in each test random intensity of the impulse force was applied. The amplitudes of acceleration are related to the load and hence different in each measurement. But the shape of the diagrams can be compared.

According to the Nyquist sampling theorem, the sampling rate must be greater than twice the signal frequency of interest and then no information of interest would be lost. Although the sample frequency was 4800 Hz for all measurements all eigenfrequencies up to 2400 Hz have to be present in the signal. Having in mind the values of theoretical eigenfrequencies shown in the (Tab.1.), first 7 eigenfrequencies have to be present in the signal. This was proved using Digital FFT Analyser (see Tab.2.) If we analyse the signal from first 8 sensors in the case without an additional mass (*free*) and the load in the middle of the beam, we obtain the results for the first bar as shown in Tab. 3. For the sensors 9-16 it was expected to get the similar results because the load was applied in the middle of the bar.

Tab. 1. Theoretical eigenvalues

| Profile IPB 100; l = 4m; Theoretical values | | | | | | | |
|---|---|---|---|---|---|---|---|
| Bar | Eigenfrequencies (Hz) | | | | | | |
| I | 46.35558 | 128.7655 | 252.3804 | 417.2002 | 623.225 | 870.4548 | 1158.889 |
| II | 20.60248 | 82.40992 | 185.4223 | 329.6397 | 515.062 | 741.6892 | 1009.522 |

Tab. 2. Eigenfrequencies of Bar II

| Profile IPB 100; l = 4m; Digital FFT Analyser (sample freq. 500 Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Bar | Eigenfrequencies (Hz) | | | | | | |
| II | 21.25 | 50 | 80 | 170 | 313 | 430 | - |

Tab. 3. Eigenfrequencies of Bar I

| Profile IPB 100; Bar I; l = 4m; Digital FFT Analyser (sample freq. 500 Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Sen.1 | Sen.2 | Sen.3 | Sen.4 | Sen.5 | Sen.6 | Sen.7 | Sen.8 |
| Eigenfrequences (Hz) | | | | | | | |
| 47.23 | 47.83 | 47.23 | - | 48.44 | 47.84 | 47.84 | 47.84 |
| 243.42 | 242.82 | 243.42 | 243.42 | 243.42 | 243.42 | 242.81 | 242.82 |
| 548.00 | 546.79 | 548.00 | 549.82 | 546.79 | 546.79 | 552.24 | 548.00 |
| 921.00 | 928.88 | 928.88 | 922.83 | 923.43 | 919.19 | 923.43 | 922.00 |
| 1301.25 | 1303.68 | 1301.25 | 1305.49 | 1301.25 | 1123.25 | 1301.25 | 1301.0 |

From these we can conclude that only the first eigenfrequency is similar to the theoretical. The difference in higher frequencies is caused by the influence of the soft supports behaviour and because of that the Bar II was tested too.

### 4.3. Presentation of the measurement results

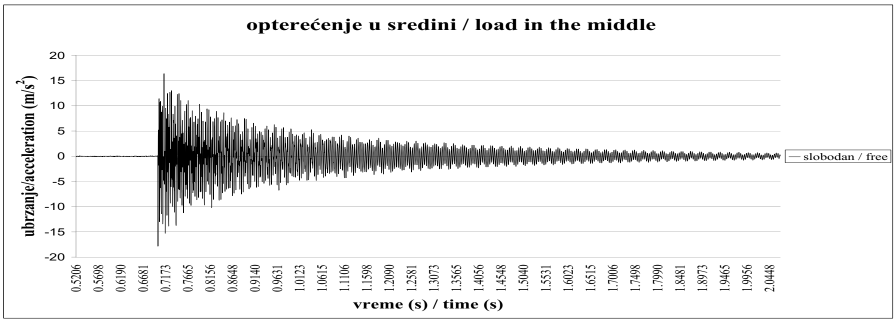



Diagram 1. Response of the Bar I- free case

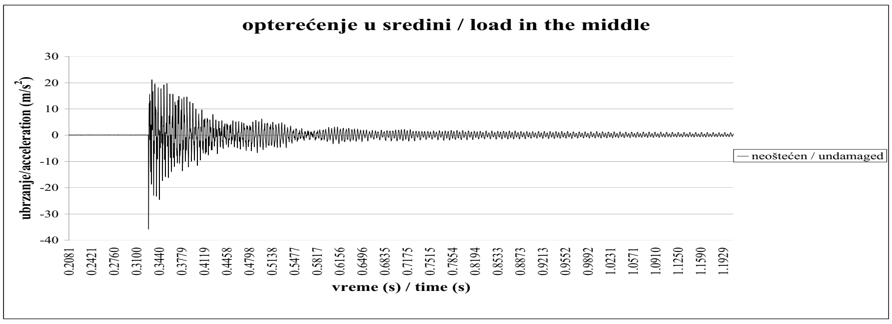



Diagram 2. Response of the Bar I – undamaged case

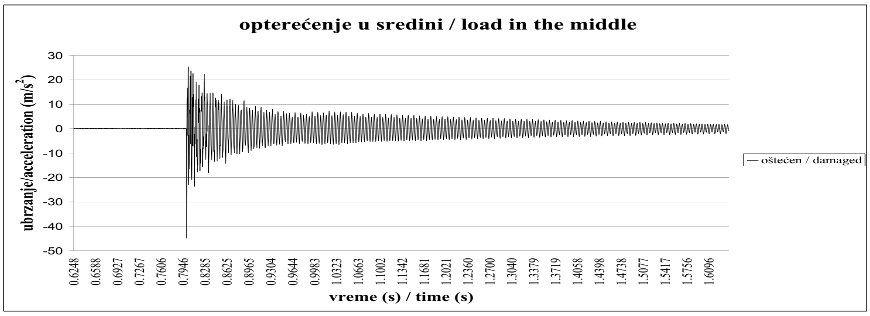



Diagram 3. Response of the Bar I - damaged case

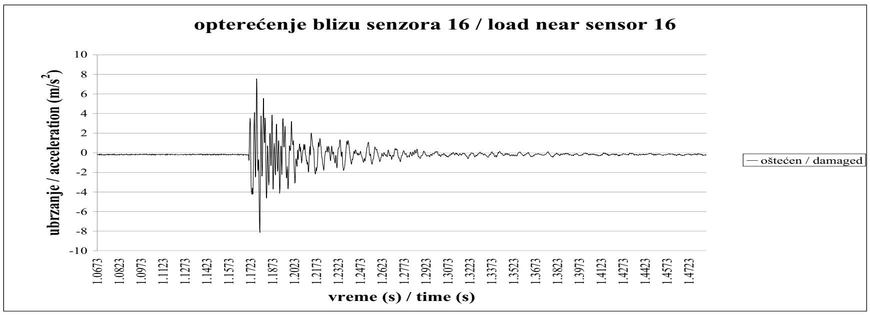



Diagram 4. Response of the Bar II

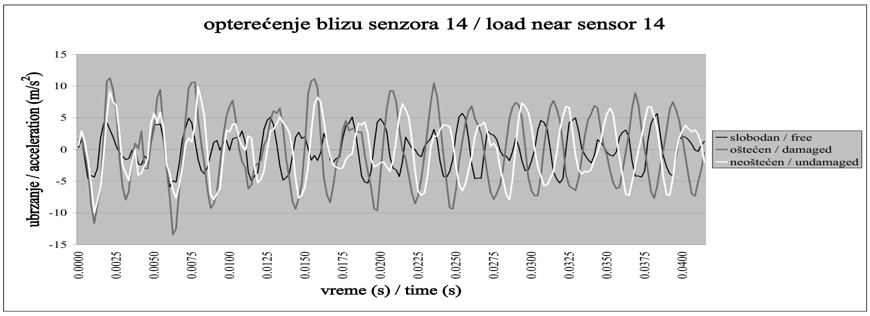



Diagram 5. Three cases measured by sensor 11- Bar I

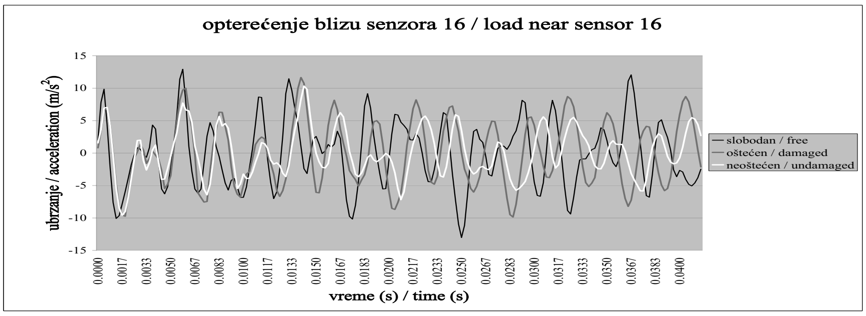



Diagram 6. Three cases measured by sensor 11- Bar I

For more measurement results see [2].

### 4.4. Subspace identification overview

Subspace identification method is one of the latest linear system identification algorithms. Ho and Kalman proposed it in 1966 but it started to develop by the middle of 1980s when Juang and Pappa proposed ERA. Very soon it has become accepted and widely used. Some packages are available by Internet, e.g. SMI package (T.U. Delft). Ready-to-use routines are incorporated in mathematical programs like Scilab. In system identification we firstly collect measurements of input/output behaviour of the system and afterwards compute a model that explains the measured data. Collected input/output measurements are used for evaluating the Markov parameters that are the entries of the Hankel matrix. The Hankel matrix plays a major role because the state space model can be obtained from a factorisation of a Hankel matrix via its singular value decomposition (SVD). It is well known that the rank of the Hankel matrix determines the order of the system. With perfect noise free data, the minimum order realisation can be easily obtained by keeping only the non-zero Hankel singular values. But, with real or noise – contaminated data, the Hankel matrix tends to be a full rank, thus making the problem of determining of a minimum – order state space model. In this case, we have to be sure that decision of how many highest singular values are significant is well done. The model order can for instance be found by looking the maximal distance between two successive singular values. The remaining singular values will be neglected. While this can happen with low noise simulated data, it rarely happens with real data. Reduced – order model obtained by retaining only "significant" singular values tends to be poor in accuracy. This represents one of the most unsatisfactory discrepancies between what is expected in theory and what is actually observed in practice. Real systems are also non – linear and infinite dimensional. It is possible to keep all Hankel singular values and to determine a high – dimensional state space model and after that in post processing procedure reduce the dimension of the identified model. But, this also leads to some loss of accuracy. The main idea is to find the right order directly in the first place. B. Peeters et al. recommend to "play" with choosing different orders that give different state space model and corresponding model parameters. They can be compared and discussed and after that choice of the "right" order can be made.

Another problem is that in practice we will never have the exact Markov parameters, but we will have measured data which are disturbed by noise. Hence the identified Markov parameters, calculated directly or indirectly from input/output data, will have an error. Used as the entries of the Hankel matrix they will cause in it a constant error which cannot be reduced. Lim, Phan and Longman (see reference [3]) recommend evaluating of the Hankel matrix directly from the input output data. The intent is to avoid the intermediate steps so that Hankel matrix identified in this way does not suffer from having an error of Markov parameter identification.

Also, in practice, the measurements that are available will not necessarily be impulse load responses, but general input/output data. Since these data will in general contain noise, an exact realisation of data by minimal order state space model will be cumbersome because it is always accompanied by the mentioned differences. The process noise caused by the disturbances and the modelling inaccuracies and measurement noise created due to the sensor inaccuracy are both immeasurable values and usually assumed to be zero mean, white and stationary.

The recorded signals need cutting off, i. e., reduction and filtration with low pass filter with some cut off frequency and have to be resampled. These operations reduce the number of data points and make the identification more accurate in the considered frequency range.

All sensors have a certain spatial distribution over the structure, leading to the signals of different quality. Some sensors may be located at nodal points of a mode shape and others may be located at points close to the fixed boundaries. In "good" signals, all modes of interest have to be present.

But, all these require a significant investigation, discussion and analysis, and request that for subspace application each intermediate step has to be well observed and controlled. Hence, system identification cannot be performed fully automatically. A lot of work and analysis has to be done first.

## 5. Numerical Procedure

As it was mentioned before, the necessary number of sensors has to be equal to the total number of DOF. Hence the beams were considered as a finite DOF discretized structure with 8 constitutive elements (i.e., 9 nodes) and in each node 2 DOF were allowed: lateral deformation and bending angle. Although in two nodes at supports lateral deformation is restrained the total number of DOF was considered to be 16. So, the number of sensors (16) was equal to the number of DOF (16). Hence the theoretical result of the stiffness and damping matrices **S** and **D** had to be unique. But, to prove the case when we have the set of incomplete data (section 2.4.) the idea was to ignore measured results from some of sensors which could be considered as fictive and hence their data have to be calculated trough algorithm (7)-(14). Further, the results from algorithm could be compared with measured values for the same sensors. This comparison could give the information about validity of the algorithm.

The program was written in Scilab, a free MatLab$^R$ clone. Scilab is a powerful interactive open source programming environment that greatly facilitates the task of numerical computations and data analysis and it has been designed for engineering and scientific applications. Scilab is a user-friendly environment such as MatLab$^R$. Numerous numerical operations, plots, etc. are programmed and ready to be used. A state space realization is also one of the powerful algorithms that are implemented in this software package (routine imrep2ss).

The program has been written to make the full investigation of the system identification problem (for complete code see the reference [2]). All routines are commented. The program, essentially, consists of three parts:

(*i*) Input data
(*ii*) Subroutines
(*iii*) Evaluation.

In the first part of the program the user is defining the characteristics of the structural system under consideration, and which are necessary for the further evaluation. In evaluation part the Scilab incorporated routines for the system realization are also recommended.

It has been seen that the numerical procedure, though ready to be used, stuck at the output of the subspace method. The main reason for this behaviour is the fact that this method is still not developed for use of automatic execution. Indeed, for each single set of data, the right subset of data to be used in the subspace method has to be selected by hand. For this purpose the subset data have to be analyzed by FFT in order to see if the eigenvalues are comparable to eigenvalues of a similar beam. Also, the sample time step plays a prominent role for the quality of subspace identification.

The routines developed so far may be used with any other identification method or for subspace methods that are less sensible on the data window or the used time step. Whenever such a method and/or routine are available the program may be of valuable use.

## 6. CONCLUSION

Based on the theoretical proposals from reference [1] and through the experimental and numerical work, this paper proves that subspace identification algorithm is not yet a method where real system parameters can be automatically identified.

Further investigation and development in this field will be the topic of the future work, especially in overcoming the noticed inadequacies and also should show if the linear system description is sufficient for the detection of damage; alternately, more sophisticated methods should be used to take into account the nonlinear effects. The noise and other disturbing influences that may substantially change the input data should be particularly considered.

## REFERENCES

1. Xiao H., Bruhns O. T., Waller H. and Meyers A.: An input/output-based procedure for fully evaluating and monitoring dynamic properties of structural systems via subspace identification method, Journal of Sound and Vibration 264/4, 2001. pp. 601-623.
2. Trajković M., *Investigation on dynamic properties of linear systems/ Ispitivanje dinamičkih karaktersitika linearnih sistema,* Edition Academia, Andrejević Endowment, Belgrade 2004.
3. Lim Ryoung K., Phan Minh Q. and Longman Richard W: State-Space System Identification with Identified Hankel Matrix, Department of Mechanical and Aerospace Engineering, Technical Report No.3045, Princeton University, Princeton, NJ., 1998.
4. Peeters B., Wahab M. Abdel, De Roeck G., De Visscher J., De Wilde W. P., Ndambi J.-M. and Vantomme J: Evaluation of structural damage by dynamic system identification., Proceedings of ISMA 21, Leuven, Belgium, 1996.pp. 1349-1361.
5. De Schutter B.: Minimal state-space realization in linear system theory: An overview, Journal of Computational and Applied Mathematics, Special Issue on Numerical Analysis in the 20th Century- Vol. I: Approximation Theory, vol. 121, no.1-2, 2000. pp. 331-354.
6. Doebling S. W., Hemez F. M., Peterson L. D. and Ferhat C.: Improved damage location accuracy using strain energy-based mode selection criteria., American Institute of Aeronautics and Astronautics Journal 35, 1997. pp. 693-699.
7. Peterson L. D. and Alvin K. F.: Time and frequency domain procedure for identification of structural dynamic models, Journal of Sound and Vibration 201(1), 1997. pp. 137-144.

# PRIMENA METODE IDENTIFIKACIJE SISTEMA ZA PROCENU DEGRADACIJE SISTEMSKIH PARAMETARA

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*U radu je prikazano istraživanje ponašanja realnih sistema sa realnim opterećenjem i realnim podacima koji u sebi nose i uticaj šumova uz korišćenje metode identifikacije podprostora kao jedne od mogućih varijanti inverzne dinamičke analize. Preporučuje se i monitoring vibracija konstrukcije kao ne-destruktivno sredstvo procene dinamičkih parametara. Dat je i prikaz originalnog istraživanja na realnim modelima opterećenim impulsnim opterećenjem u laboratorijskim uslovima. Poseban doprinos je softver za određivanje relevantnih mehaničkih karakteristika kao i položaja eventualnog oštećenja konstrukcije. Rezultati eksperimentalnog dela rada su korišćeni kao ulazni podaci kako za slučaj kompletnog skupa podataka (broj senzora je jednak broju stepeni slobode) tako i za slučaj nekompletnih podataka (za sisteme sa velikim brojem stepeni slobode) gde se podaci koji nedostaju računaju korišćenjem iteracionog alogirtma.*