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CONTRIBUTION TO THE DESIGN OF BOLTED ANGLE CONNECTION IN THE STEEL STRUCTURES

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Abstract. *The paper presents the equations for the determination of the neutral axis position, when it is on the face, that is, on the bearing plate, and for the highest normal tensile stress in the fastener, as well as for the determination of highest and lowest normal compressive stress occurring on the face, that is bearing plate when the neutral axis is outside the face, that is, the bearing plate. Which of these three possible cases will occur, depends solely on the force field occurring in the given cross-section for the dimensioning of the joint. The connection is effected by the face or bearing plate, depending on the task being solved and the bolt fasteners. The expressions for the calculation of the said connections are derived for the general case when all three forces intersecting, M, T and N, in the cross section given for their dimensioning, are other than zero or any one of them is other than zero. The paper also presents a tabular presentation of the results for all of the three possible cases of the force fields, that is, the stress fields most frequently occurring in the civil engineering – building practice at these joints.*

Key words: *Steel structure, angled connection, rigid connection, face plate, bearing plate, fastener.*

1. INTRODUCTION

In the existing professional and university literature in the field of steel structures, there is a number of examples the calculation procedures for the dimensioning of two angled members, through the permissible stress field and stress or deformation field limit state methods. At such connections, the most common case is, that apart from the transversal force, in the given cross section for the connection dimensioning, a bending moment occurs. Such case has been very clearly explained in the literature, when a connection is effected via the face plate, while it is not the case with the bearing plate connection with the concrete foundation. Also, the case when, apart from the transversal force and bending moment in the cross section at the point of angle connection the normal

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force occurs, which is a very frequent case, has not been extensively treated in the scientific and professional literature yet. This is true both for the connections effected by the face plate and for those realized by the bearing plate, so this type of connection, with all three cases of force field that may occur in the given cross section for the dimensioning of the connection, has been the subject of this paper.

The derived equations are the solution of the problem in respect of the elastic behavior of the connection. The equations have been obtained on the basis of the formulated mathematical models for all three possible force field statuses, that is, stresses in the observed given angle connection, by using the conditional equilibrium equations for the relieved face, that is, bearing plate.

The solution of the derived equations, with the required numerical exactness, may be obtained by utilization of any of the modern computer software programs.

2. Fundamental Postulates

It is adopted that the basic material, both for the members and the fasteners which immediately realize the connection, is in the load phase for the entire time in the elastic behavior zone.

Out of this fundamental postulate, the linear distribution of forces is adopted; that is of normal tensile strain in the fasteners as well as the normal compressive stress on the face, that is, bearing plate.

3. Problem Solution

On the basis of the adopted postulates, the problem of dimensioning of angle connections effected with the face, that is bearing plate and the bolted fasteners, is reduced to the determination of the maximum tensile force in the fastener, that is, to the calculation of the maximal normal tensile stress in the fasteners, Figure 1 case (b), that is the highest and the lowest normal compressive stress on the face plate, Figure 1 case (c). Which of these three mentioned cases of stress fields on the face, that is, bearing plate and in the fasteners will occur, depends primarily on the internal forces field M, T, and N, occurring in the intersection of the connecting point of the face, that is, bearing plate.

In the general case, the bending moment, normal and transversal force occur in the cross section 1-1 (Figure 1) which is important for the dimensioning of the connection. By using the conditional equilibrium equations for the relieved face, that is, bearing plate, the problem is reduced to solving two non-linear equations with three and two variables, depending on the force field occurring in the given cross section for dimensioning of the connection. The following values and the corresponding designations for the problem solution have been introduced for the arrangement of the fasteners according to Figure 1:

A. For the known values

M, T and N – are the intersecting forces in the cross section given (1-1) for dimensioning of the connection,

– d – is the distance of the center of gravity of the transversal section of the console, bolt or column from the edge of the face, that is bearing plate which is pressed due to the bending moment M,

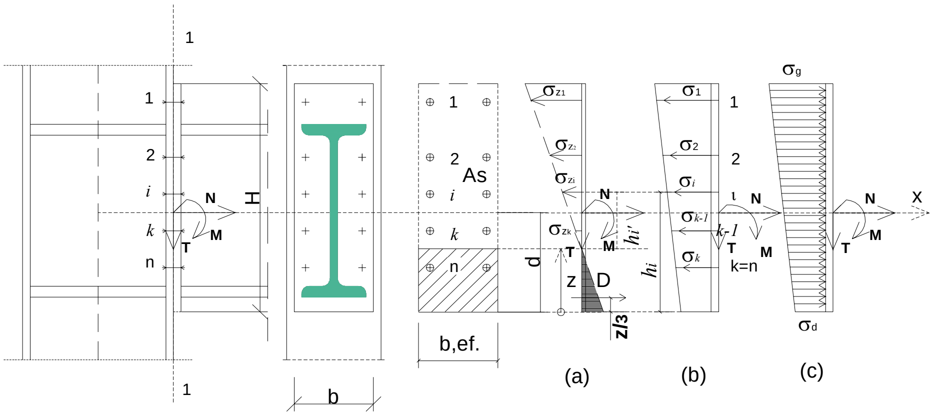

Fig. 1. Angled connection realized with the face plate and bolts.

– h_i – is the distance of i–th row of fasteners from the edge of the face, that is, bearing plate which is pressed due to the bending moment M,

– h_i' – is the distance of i–th row of tensioned fasteners from the neutral axis on the face, that is, bearing plate,

– $b_{ef.}$ – is the effective width of the face, that is, bearing plate for the observed angled connection.

– A_s – is the surface area of the tested cross section, that is, core of the transversal section of the adopted fasteners on the face, that is, bearing plate,

– m – is the adopted number of columns of fasteners on the face, that is, bearing plate,

– $m_{ef.}$ – is the effective number of columns of fasteners on the face, that is, plat bearing e,

– $e = E_{steel}/E_{support}$ – relation of the elasticity modulus of the basic material of the bolts, that is anchors and the basic material of the support on which the normal compressive stress is transmitted through the bearing plate.

– n – is the adopted number of columns of fasteners on the face, that is, bearing plate.

B. For variables

– k – is the number of rows of tensioned fasteners on the face, that is, bearing plate,

– z – is the distance of the neutral axis from the edge of the face, that is, bearing plate which is pressed due to the bending momentum M,

– σ_{z1} – is the maximum normal tensile stress in the fastener,

– σ_1 – is the normal tensile stress in the 1^{st} row of fasteners on the face, that is, bearing plate,

– σ_n – is the normal tensile stress in n-th row of fasteners on the face, that is, bearing plate,

– σ_g – is the normal compressive stress on the upper edge of the face, that is, bearing plate and

– σ_d – is the normal compressive stress on lower edge of the face, that is, bearing plate.

3.1 First Case – Figure 1.a

From the first condition of equilibrium and Fig. 1.a that is, from the condition:

$$\sum X = 0 \Rightarrow \ N + D - m_{ef.} \cdot \sum_{i=1}^{k} Z_i = 0 \,, \tag{1}$$

where: $$D = \frac{b_{ef.}}{2 \cdot e} \cdot \frac{\sigma_{z1}}{(h_1 - z)} \cdot z^2 \text{ and } Z_i = \frac{Z_1}{h_1'} \cdot h_i' = \frac{\sigma_{z1} \cdot A_s}{(h_1 - z)} \cdot (h_i - z), \quad (2)$$

$m_{ef.} = m = 2$ – if two columns of fasteners on the face, that is, bearing plate are adopted, and $m_{ef.} = 2 \cdot 1.0 + 2 \cdot 0.8 = 3.6$ – if four columns of fasteners on the face plate are adopted, because the bearing capacity of the outer columns of fasteners is 20% less because of the increased deformability of the part of the face plate.

By the substation of the said values given under (2) in the equation (1) and its processing, the first equation for the solution of the problem is obtained:

$$\begin{aligned} & N + \frac{1}{2} \cdot \frac{b_{ef.}}{e} \cdot \sigma_{z1} \cdot \frac{z^2}{(h_1 - z)} - \frac{m_{ef.} \cdot A_s}{(h_1 - z)} \cdot \sigma_{z1} \cdot \sum_{i=1}^{k} (h_1 - z) = 0 \Rightarrow, \\ & b_{ef} \cdot \sigma_{z1} \cdot z^2 - 2 \cdot N \cdot e \cdot z + 2 \cdot m_{ef} \cdot e \cdot A_s \cdot k \cdot \sigma_{z1} \cdot z - \\ & - 2 \cdot m_{ef} \cdot e \cdot A_s \cdot \sigma_{z1} \cdot \sum_{i=1}^{k} h_i + 2 \cdot N \cdot e \cdot h_1 = 0 \end{aligned} \quad (3)$$

From the second condition of equilibrium and Fig. 1.a that is from the condition:

$$\sum M = 0 \Rightarrow \quad N \cdot \left(d - \frac{z}{3} \right) + M - m_{ef} \cdot \sum_{i=1}^{k} Z_i \cdot \left(h_i - \frac{z}{3} \right) = 0. \quad (4)$$

By the substitution of expression for Z_i given under (2) in the equation (4) and its processing, the second equation for the solution of the problem is obtained, as follows:

$$N \cdot \left(d - \frac{z}{3} \right) + M - \frac{m_{ef}}{3} \cdot \frac{A_s \cdot \sigma_{z1}}{(h_1 - z)} \cdot \sum_{i=1}^{k} (3 \cdot h_i^2 - 4 \cdot h_i \cdot z + z^2) = 0. \quad (5)$$

The variables in equations (3) and (5) are k, z and σ_{z1} and their obtained values are real only if they are positive.

3.2 Second Case – Figure 1.b

From the first condition of equilibrium, and Fig. 1.b that is, from the condition:

$$\sum X = 0 \Rightarrow N - m_{ef.} \cdot A_s \cdot \sum_{i=1}^{n} \sigma_i = 0, \quad (a)$$

where: $$\sigma_i = \sigma_1 - \frac{(\sigma_1 - \sigma_n)}{(h_1 - h_n)} \cdot (h_1 - h_i) \quad (b)$$

By the substitution of the expression for σ_i given under (b) in the equation (a) the first equation of the solution of the problem is obtained, as follows:

$$N - m_{ef.} \cdot A_s \sum_{i=1}^{n} \left[\sigma_1 - \frac{(\sigma_1 - \sigma_n)}{(h_1 - h_n)} \cdot (h_1 - h_i) = 0 \right] \quad (6)$$

From the second condition of equilibrium, and Fig.1.b that is, from the condition:

$$\sum M = 0 \Rightarrow \quad Nd + M - m_{ef.} \cdot A_s \sum_{i=1}^{n} \left[\sigma_1 - \frac{(\sigma_1 - \sigma_n)}{(h_1 - h_n)} \cdot (h_1 - h_i) \right] h_i = 0 \tag{7}$$

The variables in the equations (6) and (7) are σ_1 and σ_n and their obtained values are real only if they are positive, that is if all the fasteners on the plate are tensioned.

3.3 Third Case – Figure 1.c

From the first condition of equilibrium, and Fig. 1.c that is, from the condition:

$$\sum X = 0 \Rightarrow \quad N + \left(\frac{\sigma_g + \sigma_d}{2} \right) \cdot b_{ef} \cdot H = 0 \tag{8}$$

From the first condition of equilibrium, and Fig. 1.c that is, from the condition:

$$\begin{aligned} &\sum M = 0 \Rightarrow \quad N \cdot d + M + \sigma_g \cdot \frac{b_{ef.} \cdot H}{2} \cdot \frac{2}{3} \cdot H + \sigma_d \cdot \frac{b_{ef.} \cdot H}{2} \cdot \frac{1}{3} \cdot H = 0 \Rightarrow \\ &N \cdot d + M + \frac{b_{ef.} \cdot H^2}{6} \cdot (2 \cdot \sigma_g + \sigma_d) = 0 \end{aligned} \tag{9}$$

The variables in the equations (8) and (9) are σ_g and σ_d and their obtained values are real only if they are positive, that is, if both normal stresses are the compressive stresses.

By solving the equation system (3) and (5) or (6) and (7) or (8) and (9), which depends on the force field in the given cross section of the connection, eventually the solution of the postulated problem is reached.

The solution of the equations (3) and (5), that is the solution of the postulated problem for that case of the force field and the occurrence of the state of stress in the fasteners and the face, that is, bearing plate, may be obtained by the iteration method, assigning a value from 1 to n to the k variable, where n – is the number of rows of fasteners on the face that is, bearing plate. After assigning values to the k, the problem is reduced to solution of two non-linear equations with two variables by z_u and σ_{z1}. The obtained solution is correct if for the assumed k the value for the position of the neutral axis (z) between the k-th and k+1-th row of the fasteners on the face that is, bearing plate is obtained.

Non-linear equations (3) to (9) are most frequently solved by using some of the ready-to-use software numeric programs dealing with mathematics. The transversal force T, in the given cross section for the dimensioning of the angled connection, is evenly distributed to all the fastener on the face that is, bearing plate, and it is conveyed from the butt plate through the steel pin to the foundation basis, which is most often made of concrete.

In the end, it is necessary to carry out the control of the parallel stress in the most loaded fastener on the face, that is, bearing plate.

Table 1. Numerous examples for all three possible cases of force and stress fields in the connection of members with the face plates or the connection of the bearing plate with the anchors to the concrete support.

H=60.0cm; $b_{ef.}$=25.0cm; d=30.0cm; $m_{ef.}$=2; n=5; A_s=3.53cm^2; h_1=50.0cm; h_2=40.0cm; h_3=30.0cm; h_4=20.0cm and h_5=10.0cm										
No.	N /kN/	M /kNcm/	k	e	z /cm/	σ_{z1} /kN/cm^2/	σ_1 /kN/cm^2/	σ_n /kN/cm^2/	σ_g /kN/cm^2/	σ_d /kN/cm^2/
Force field belongs to⇒					Case from Fig. 1.a		Case from Fig. 1.b		Case from Fig. 1.c	
1	300	5000	5	1	3.83	18.71	–	–	–	–
2	0	5000	5	1	7.90	7.41	–	–	–	–
3	-300	5000	2	1	40.37	0.14	–	–	–	–
4	300	5000	5	7	8.47	19.71	–	–	–	–
5	0	5000	4	7	16.91	8.93	–	–	–	–
6	-300	5000	1	7	41.98	0.78	–	–	–	–
7	500	0	5	–	–	–	14.16	14.16	–	–
8	500	50	5	–	–	–	14.31	14.02	–	–
9	500	-50	5	–	–	–	14.02	14.31	–	–
10	-1200	0	0	–	–	–	–	–	0.80	0.80
11	-1200	500	0	–	–	–	–	–	0.77	0.83
12	-1200	-500	0	–	–	–	–	–	0.83	0.77

4. Conclusion

The paper gives a contribution to the calculation of the bolted angle connections in steel structures. All three possible cases of force fields, and therefore the stress field, which may occur in the fasteners both on the face, that is, bearing plate, when in the given cross section of the connection all three M, T, and N forces intersect, and are other than zero, or if any one of them is equal to zero, have been processed.

The derived non-linear multi-parameter equations may be used in calculation of the angle connection of column-bolt, column-console, transversal-longitudinal girder, column bearing plate as well in calculation of the connection of the bearing plate-foundation basis.

The paper gives a considerable contribution to the correction and completing of our scientific, professional and university-teaching literature in the field of steel structures, for dimensioning of the angle connections, according to the permissible stress fields and deformations, that is, dimensioning of these connections for the elastic area of the stress fields and deformations in them, for the entire time of the load action on the structure, during the existence of the structure.

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PRILOG PRORAČUNU VEZA POD UGLOM U ZAKOVANOJ IZRADI OSTVARENE ČEONOM ILI LEŽIŠNOM PLOČOM

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U radu su izvedene jednačine za određivanje položaja neutralne ose i vrednosti normalnog napona zatezanja u najopterećenijem spojnom sredstvu za slučaj kada se neutralna osa nalazi na čeonoj, odnosno ležišnoj ploči. Takođe, izvedeni su i izrazi za određivanje najvećeg i najmanjeg normalnog napona zatezanja u spojnim sredstvima, odnosno najvećeg i najmanjeg normalnog napona pritiska koji se javljaju na čeonoj, odnosno ležišnoj ploči kada se neutralna osa nalazi van čeone tj. ležišne ploče. Koji će se od ova tri moguća slučaja javiti za rešenje problema veze isključivo zavisi od polje sila koje se javlja u merodavnom preseku za dimenzionisanje veze. Veza se izvodi pomoću čeone ili ležišne ploče, zavisno od postavljenog zadatka koji se rešava, i spojnih sredstava u zakovanoj izradi. Izrazi za proračun pomenutih veza izvedeni su za opšti slučaj tj. za slučaj kada su sve tri sile u preseku M, T i N, u merodavnom preseku za njihovo dimenzionisanje , različite od nule ili bilo koja od njih različita od nule.