

## A CONSTRUCTIVE MODEL OF ORTHOGONAL AXONOMETRY

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**Abstract.** *A new method for object representation in arbitrary orthogonal axonometry is suggested. Constructions for a point drawing are shown, using two rotations - a rotation of the coordinate plane  $Oxy$  to the profile plane and a rotation of the profile plane to the projection plane. Using these simple constructions, an object can be projected on arbitrary plane. Plane figures (circles and polygons) can be pictured, too.*

**Key words:** *Axonometry, profile plane, projecting ray, rotation*

### 1. INTRODUCTION

A new method for representation of geometrical objects in orthogonal axonometry is suggested, and it possesses a lot of advantages. First of all, this method does not depend on any calculations (axonometry coordinates or ellipse axes), which are dealt with in some special projections like isometry and dimetry. The real sizes or scales have to be drawn on the picture here. The accumulation of calculation errors and drawing errors is avoided by this method because of the direct way for drawing.

The constructions of a point drawing are very clear and simple. A simple graphic way for drawing of a figure in arbitrary plane is given. There exists a possibility for a proper choice of the projection plane or of the projecting ray in order to achieve a better vizualization of some object details.

### 2. A CONSTRUCTIVE MODEL OF ORTHOGONAL AXONOMETRY

Let  $Oxyz$  be rectangular coordinates in the space. We denote by  $\omega$  the projection plane (see Fig. 1). Let  $O$  be projected orthogonally on  $\omega$  in  $O'$ , i.e. the projecting ray is  $s = OO'$ ,  $s \perp \omega$ . We denote the intersection point of  $\omega$  with the axes by  $X$ ,  $Y$  and  $Z$  respectively. The triangle  $XYZ$  is called the axonometric triangle.

Let  $ZO' \cap XY = M$ ,  $ZO' \perp XY$ . We denote also  $\gamma: = OMZ$ ,  $\mu: = XOY$ . Evidently that  $\gamma \perp \mu$ ,  $\gamma \perp \omega$ . If an arbitrary point  $K$  is given, then the orthogonal projection  $K'$  on  $\omega$  is the intersection point of the ray  $s$  through  $K$  with  $\omega$ . A way for finding  $K'$  will be demonstrated.

Denote by  $K_1$  the orthogonal projection of  $K$  on  $\mu$ ,  $K^0$  - the projection of  $K$  on  $\gamma$  and  $K_1^0$  is the projection of  $K^0$ . If we have the ray through  $K^0$ , it intersects  $ZM$  in a point, and we draw through this point a line, parallel to  $XY$  for finding  $K'$ .

It can be easily seen on the Fig. 1, that the point  $K'$  has a distance  $\xi_K = K_1K_1^0$  from the line  $ZM$ . Let us apply the following rotations:

1. we rotate  $\mu$  around  $OM$  to  $\gamma$
2. we rotate  $\gamma$  around  $ZM$  to  $\omega$ .

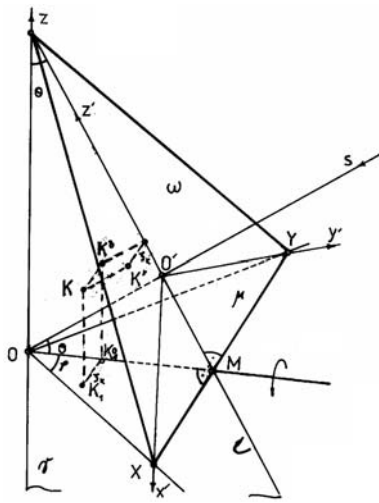


Fig. 1.

Let us suppose, that the axonometric triangle  $XYZ$  is given (to the right on Fig. 2). We find the orthocenter  $O'$  and the axonometric axes  $x' = O'X$ ,  $y' = O'Y$ ,  $z' = O'Z$ . Let us situate the rotated profile plane  $\gamma$  to the left on Fig. 2. The rotated situation of the triangle  $ZOM$  can be drawn: the point  $O^0$  is the intersection point of a half circle with diameter  $Z^0M^0$  and of the horizontal line through  $O'$ . Here  $O'O^0$  is the projection direction after the rotation.

We draw  $XY \perp z M^0$ ,  $XY \perp O^0M^0$  and  $M^0Y = MY$ ,  $M^0X = MX$ . Having the points  $X$  and  $Y$ , we draw  $O^0xy$ .

Let us draw the points  $L(3.5; 1; 0)$  and  $K(1; 2.5; 4)$ . We draw  $L_1$  and project it in  $L^0 = L_1$ . We have now the distance  $\xi_L = L_1L^0$ . We plot  $\xi_L$  on the horizontal line through  $L^0$  to the left of  $Z'M$  and find  $L'$ .

By the same way we draw  $K'$ : after  $K_1$  we project it in  $K_1^0$  and plot  $K_1^0K^0 = z_K = 4$ ; we have  $K_1K_1^0 = \xi_K$  and  $K'$  can be drawn.

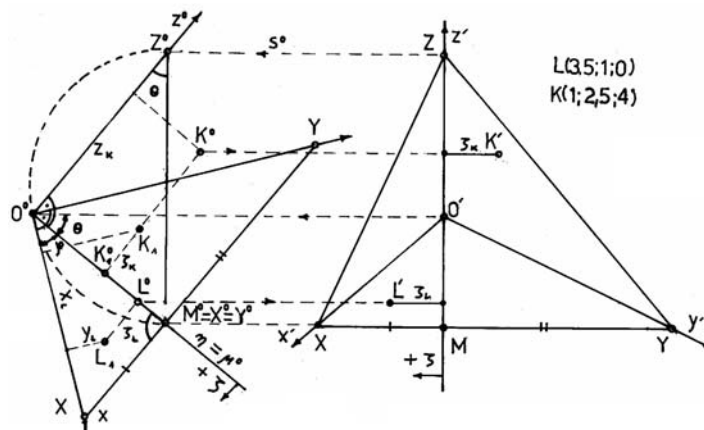


Fig. 2.

3. AN OBJECT DRAWING

On the Fig. 3 an object is given by its Monge projection. The projecting ray is defined by the angles  $\varphi = \angle(s_1, Ox)$ ,  $\beta = \angle(s_2, Oy)$ . We can find after a rotation the angle  $\theta = \angle(s, \mu)$ ; it is shown on the Fig. 3.

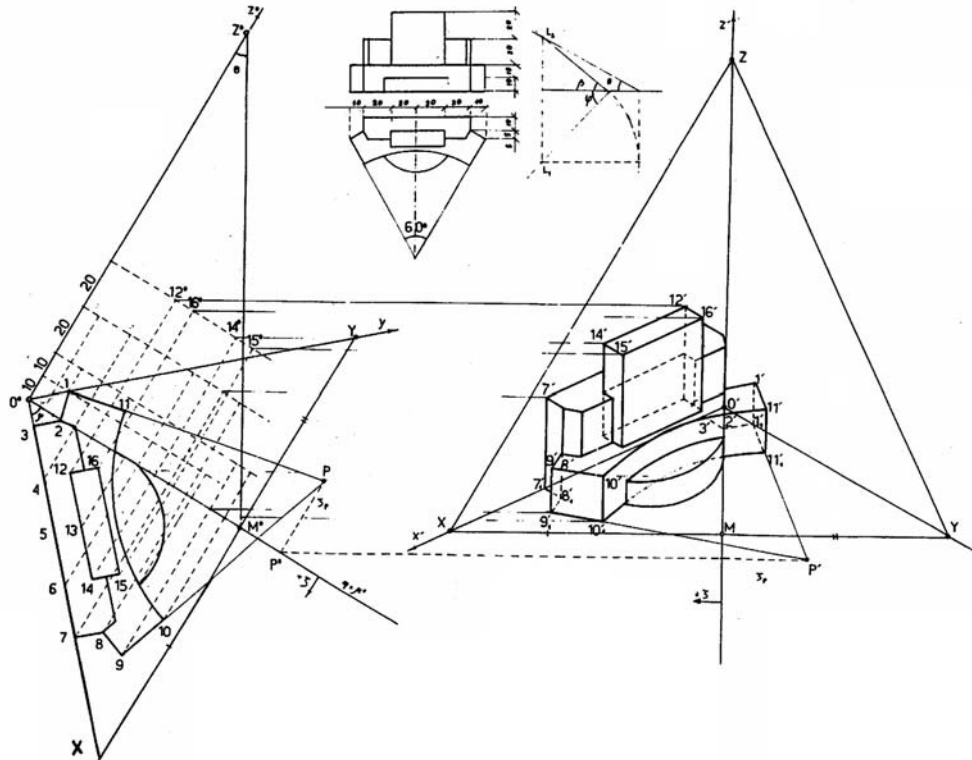


Fig. 3.

Having the projection ray, the projection plane  $\omega$  is determined and the triangle XYZ with accuracy due to them being parallel. First of all, we  $M^0Z^0=MZ$  is drawn. Let us find XM and YM.

The angles  $\theta = \angle(s, \mu) = \angle O'OM = \angle OZM$  are equal as angles with perpendicular rays (see Fig. 1). We plot the angle  $\theta$  with a vertex  $Z^0$ . The point  $O^0$  is derived and to the right is the projection  $O'$ .

Since  $O^0M^0 = s_1$ , then  $\angle(O^0x, O^0M^0) = \varphi$ , so we have the axes  $O^0x$  and  $O^0y \perp O^0x$ . The perpendicular line through  $M^0$  intersects them in X, Y. We plot  $XM = XM^0$ ,  $YM = YM^0$  to the right. The triangle XYZ is drawn.

Having  $O^0xy$ , we can draw the object plan which is given. The axonometry projection of each point is drawn in the way we have already mentioned. The projection of the given object is shown in the right side of the Fig. 3.

4. CIRCLES AND POLYGONS DRAWING

The axonometry triangle  $XYZ$  is given. We draw  $O^0MZ$  and  $O^0XY$  (see Fig. 4). Let us draw a circle in  $\mu$  with a centre  $P(4; 4; 0)$ ; a circle in  $v$  with a centre  $Q(4; 0; 5)$  and in  $\pi$  - with a centre  $R(0; 3.5; 3)$ .

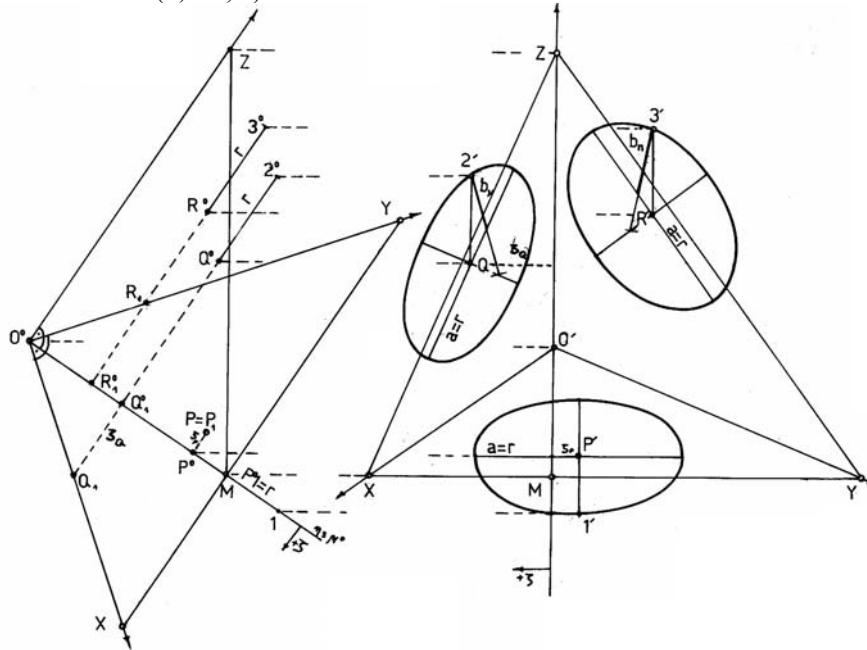


Fig. 4.

We draw the projection  $P'$ . The big axis of the ellipse is a projection of the diameter, which is parallel to the corresponding plane trace, and  $a = r$ . The little axis for the circle in  $\mu$  is a projection of the diameter, perpendicular to the first trace (it lies on a steep line). The radius is  $P^0I = r$  and we obtain  $I'$ .

For the circles in  $v$  and in  $\pi$  we use the end point of this radius, which is parallel to  $OZ$ :  $Q^02^0 = r$ ,  $R^03^0 = r$ . The points  $2'$  and  $3'$  are points on the corresponding ellipses.

On the Fig. 5 an circle in arbitrary plane  $\alpha$  is drawn. The big ellipse axis  $B'C'$  passes through  $S$ , parallel to the intersect line  $l_{\alpha\omega}$  among  $\alpha$  and  $\omega$  and  $B'C' = 2r$ .

We shall find a point  $A$  on the ellipse as a point of a horizontal line through the centre. For that purpose on the line  $h_1 \parallel X_\alpha Y_\alpha$  through  $S_1$  is plotted  $S_1A_1 = r$ ; the point  $A^0$  is on  $h^0$ ; the point  $A'$  is on  $h'$  to the right of Fig. 5. Now the construction for finding a little axis is applied.

If we need of the real size of a figure, lying in arbitrary plane, we can rotate this plane around the first trace in  $O^0xy$ . On the Fig. 6 an hexagon is given in the plane  $\alpha$  with a centre  $S$ , radius  $r$  and a point  $A$  on the horizontal line  $h_\alpha = AS$ ,  $A_1S_1 \parallel X_\alpha Y_\alpha$ . The plane  $\alpha$  is rotated around the first trace  $X_\alpha Y_\alpha$  to the  $\mu$  and the triangle  $ABC$  is drawn in its real size. Using the affinity, we find  $B_1$ . The third coordinate  $z_B$  is found and after that -  $B^0$  and  $B'$ . For the other vertexes a symmetry is used.

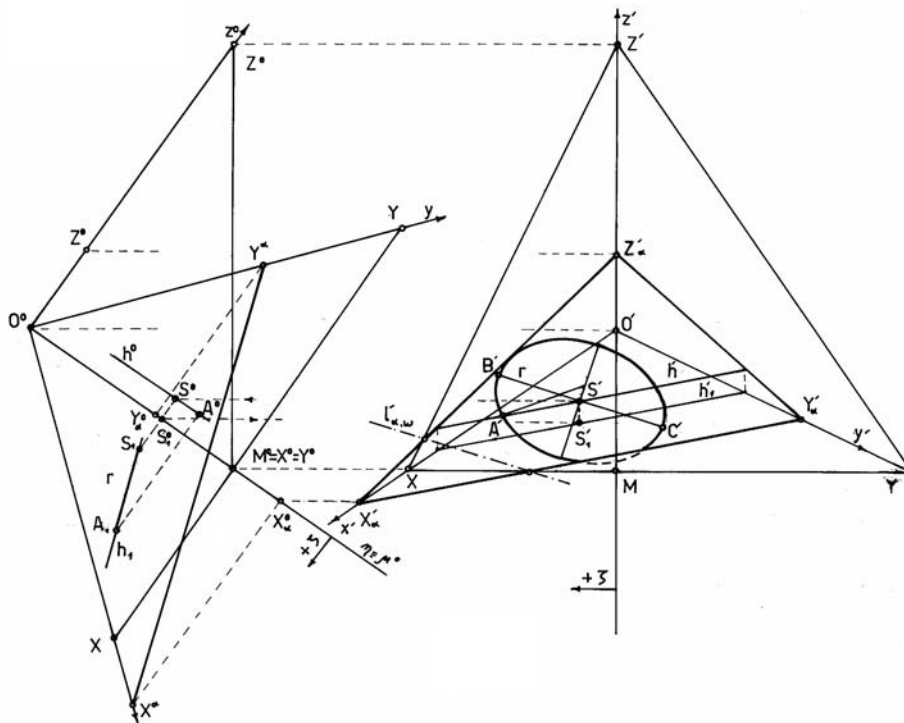


Fig. 5.

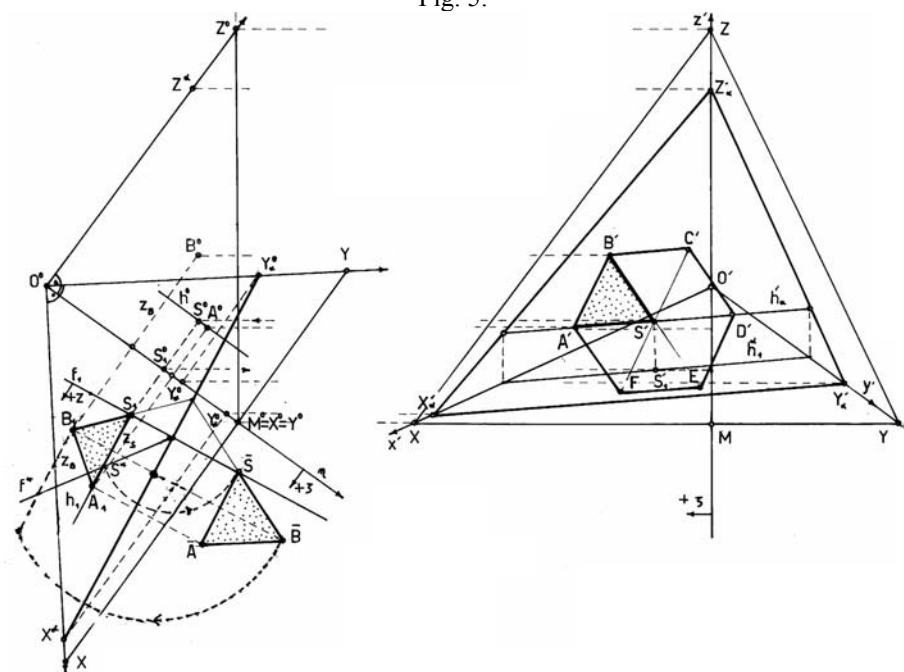


Fig. 6.



On the Fig. 8 above we have a look to the left and below (from V octant). Here  $M^0$  is over  $Z^0$ . We obtain  $O^0$  by  $\theta$  and use that again  $X > 0, Y > 0$ .

On the Fig. 8 below we have a look to the right and below (from VI octant). Here  $X < 0$  and  $Y > 0$ , which determines the directions of the axes.

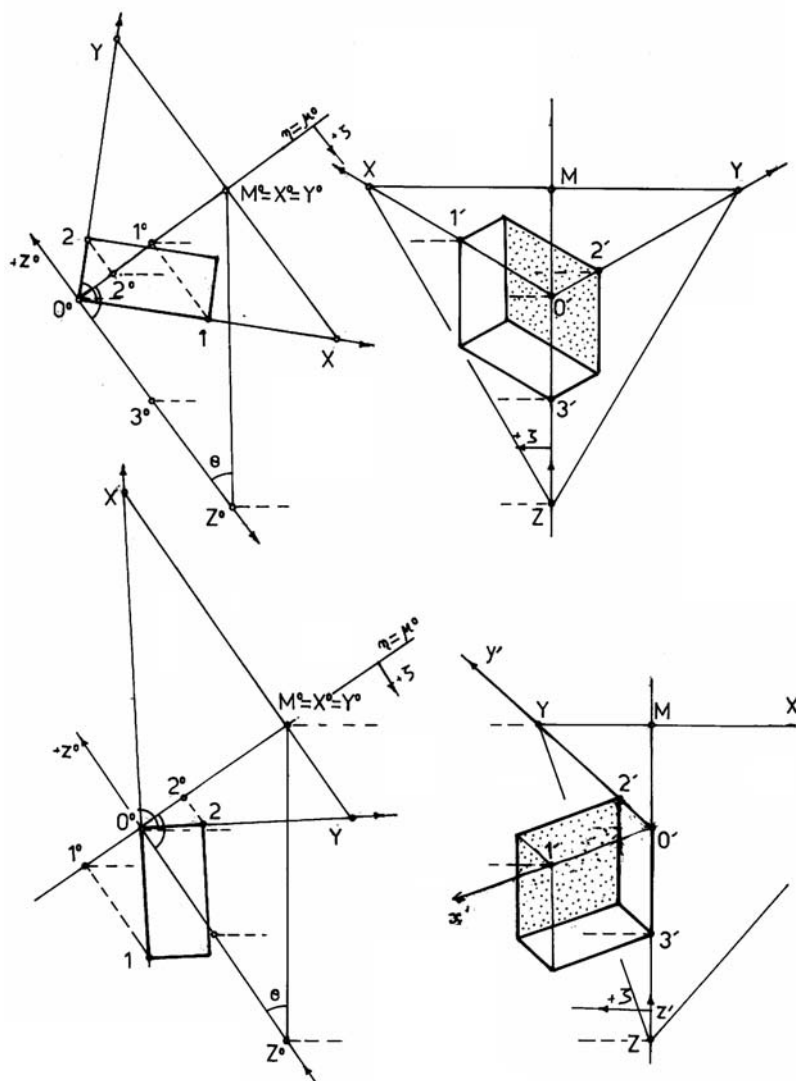


Fig. 8.

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**JEDAN MODEL KONSTRUKCIJE  
ORTOGONALNE AKSONOMETRIJE****Natasha Kirilova Dicheva, Jaklina Todorova Jivkova**

*Izlaže se novi metod predstavljanja objekta u ortogonalnoj aksonometriji. Pokazane su konstrukcije tačaka, primenom dve rotacije - rotacije koordinatne ravni  $Oxy$  do profilne ravni i rotacije profilne ravni do projekcijske ravni. Korišćenjem ove jednostavne konstrukcije objekat može biti projektovan na proizvoljnu ravan. Takođe se mogu crtati i ravne figure (kružnice i poligoni).*

Ključne reči: *nacrtna geometrija, konstrukcija, ortogonalna aksonometrija*