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IDENTICAL APPENDED SERIES OF POINTS AS INVARIANTS IN THE COLLOCAL GENERAL-COLINEAR FIELDS

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Abstract. *In order to bring the collocal collinear fields from the general into the perspective position, it is required to determine the identical appended series of points. Because of the properties depending on the projectivity that is given by the four appended points (straight lines) the appended identical series of the points and types are ranked among the invariants of general-collinear and perspectively-collinear fields. The procedure of determination of appended identical series of points is comprised of the following: in the set of ∞^1 of perspectively similar series in one field (whose center of perspective is a point on the vanishing line), find those that are identical to all the series in the set ∞^1 of perspective identical series of points in the other field (whose center of perspective is the point on the infinitely distant straight line). In the procedure, one begins from the appended similar methods obtained by the general method. The procedure is simplified by the introduction of the specially given similar series of points.*

Key words: *collocal general-collinear fields, appended identical series of points*

1. INTRODUCTION

The collinear relationship of two collocal fields is established by giving four pairs of uniformly appended points (straight lines). The mapping from one field into another in the general position of fields is very complex. By bringing the fields in the perspective position, the mapping is simplified. If the appended projective series of the first order points identical, then, by their coinciding, the double series is obtained, and the coincided carriers of the series became the axis of perspective (perspective position.

"If two similar series are parallel, that is perspectively appended so that their infinitely distant points D^{∞} and D_1^{∞} (that are mutually appended) coincide, then the center of perspective needs not be in the infinity. But if in this case the center of perspective is

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in the infinity, then such two projectively appended series are (identical)". - "Descriptive geometry" V. Nice

There is a question, how many appended identical series there are in two collocal, collinear fields in the general position, because with their coinciding the fields may be brought into the perspective position.

2. PROCEDURE OF DETERMINATION OF THE APPENDED IDENTICAL SERIES OF POINTS

The procedure of determination of the appended identical series of 1st order points is base on the following. Within one field, the perspectively identical series of points whose center of perspective on is on the infinitely distant straight line are chosen. The perspectively similar series of points in the other field have the point on the vanishing line of the first field as a center of perspective, which is appended to the point on the infinitely distant straight line. Using the methods of the projective geometry, in the set of ∞^1 perspective similar series of the other field, the ones identical with all the series in the set of ∞^1 perspective identical series of points in the first field, are identified.

In the procedure, one starts with the known postulates, and those are: projective series of 1st order points is determined with three collinear points; the segment on the straight line is the one that does not contain the fictitious point; three points on the straight line determine three segments, and by doing so the series of the 1st order points is determined; out of three segments, two always determine the third one (it is the sum of the first two segments); the series of the 1st order is determined with two segments that are determined with three points (one is the common point); the series of the points are similar, if the infinitely distant points are appended to them; all the appended segments are equal in the identical series of points.

3. GENERAL PROCEDURE

In the field $\overline{P}$ ∞^2 of the straight lines are the carriers of series which are completely determined with three finitely distant points. If ∞^2 of the randomly taken series are perspective, with perspective center in the infinitely distant point $\overline{V_\infty}$ on the fictitious straight line $_1\overline{n_\infty}$, then such series are mutually similar. The perspective rays form the bundle of straight lines ($\overline{V_\infty}$) in the field $\overline{P}$, which is one of the ∞^1 bundles that may be chosen in the field $\overline{P}$.

The set of ∞^2 straight lines (which are take n as the carriers of the perspective series), in the field $\overline{P}$, will map into the set ∞^2 of the straight lines (carriers of the perspective series), in the field P. The center of perspective of these series in the field P is the point V on the vanishing line $_1$n, which is appended to the point $\overline{V_\infty}$ on the fictitious straight line $\overline{_1n_\infty}$ in the field $\overline{P}$. The perspective rays form the pencil of the straight lines (V) in the field P. The set of ∞^2 straight lines in the field P, intersect the pencil V in the series which are general – projective, but not similar. To this dual relationship on the general – projective series, the simple relationship on the similar series is appended.

The random series of points $\overline{k'}(\overline{A'},\overline{B'},\overline{E_\infty})$, in the field $\overline{P}$, (Fig. 1), is chosen so that two points are finitely distant, and one is infinitely distant. If such series is repeated ∞^1 times, so that the infinitely distant points in all these series are appended, and the series are at the same time perspective, with the center of perspective in the point $\overline{V_\infty}$ on the fictitious straight line $_1\overline{n_\infty}$ (one of ∞^1 points on $_1\overline{n_\infty}$), then, they will be not only similar but identical - $\overline{k'}(\overline{A'},\overline{B'},\overline{E_\infty}) \equiv \overline{k''}(\overline{A''},\overline{B''},\overline{E_\infty})$, because the segments on them are equal $\overline{A'}\,\overline{B'} \equiv \overline{A''}\,\overline{B''}$, $\overline{B'}\,\overline{C'} \equiv \overline{B''}\,\overline{C''}$.

The perspective rays form the bundle of straight lines ($\overline{V_\infty}$), and the series of points which are parallel form the bundle of straight lines ($\overline{E_\infty}$).

The bundle of straight lines ($\overline{V_\infty}$) in the field $\overline{P}$, will map in the pencil of the straight lines (V) in the field P. The bundle of the straight lines ($\overline{E_\infty}$) in the field $\overline{P}$, is projectively appended to the pencil of the straight lines (E) in the field P. To the set of ∞^1 straight lines of the pencil (E), intersect the pencil (V) at the general-projective series. So, to the identical series in the field $\overline{P}$, the general-projective series in the field P are appended (Fig. 1).

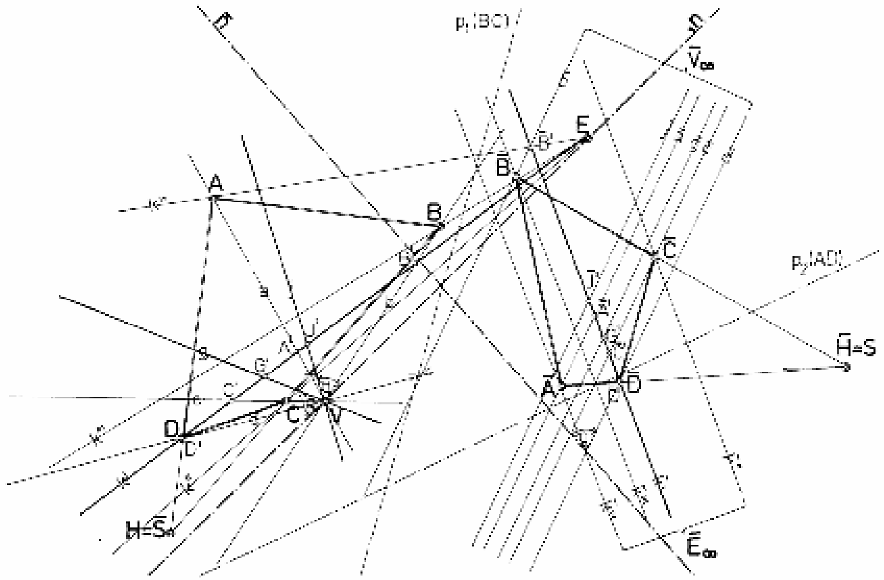


Fig. 1.

The similar series of first order points in the field $\overline{P}$, that will map into the similar series of 1st order points are identified. In order to have similar appended series, it is necessary that the infinitely distant point is appended to them, that is, the simple relationship appended.

There is an infinitely distant point in the field $\overline{P}$, that is mapped into the infinitely distant point of the field P. It is the point $\overline{N_\infty}$ (fictitious point of the vanishing line $\overline{n}$), which is appended to the point N_∞ (fictitious point of the vanishing line ${}_1n$). This implies that the only the straight lines parallel to the vanishing lines in both fields may me the carriers of the similar series of 1st order points.

When the series of points are considered, one may begin with taking on the same series:

1. two different segments,
2. two identical segments.

3.1. Different segments on the series of points ($\overline{k'}$)

The set of ∞^1 straight lines parallel to the vanishing line $\overline{n}$ in the field $\overline{P}$ (Fig. 2.), will intersect the bundle of the straight lines ($\overline{V_\infty}$) in the identical series of points. They have the appended set of ∞^1 straight lines parallel to the vanishing line ${}_1n$ in thefield P, intersecting the pencil of the straight lines (V) in the similar series of points.

In order to have the similar series of 1st order points in the field P identical with the appended series in the field $\overline{P}$, it is necessary to have the equal segments on them. If in the field $\overline{P}$ there is a series $\overline{k'}(\overline{A'},\overline{B'},\overline{C'},\overline{N_\infty})$, the identical series in the field P appended to it, would be k' (A',B',C',N_∞), and in that way the segments $A'B' \equiv \overline{A'}\,\overline{B'}$ and $B'C' \equiv \overline{B'}\,\overline{C'}$ are equal. If two segments on the series are identical to the appended segments on the appended series, then all the other appended segments are equal.

One of the ways to determine the identical appended series of points ρ is to apply rotation and translate the bundle of straight lines ($\overline{V_\infty}$) of the field $\overline{P}$, bring into the respective position with the pencil of the straight lines (V) of the field P. The perspective axis of the perspective pencils ($\overline{V_\infty}$) and (V) will be the straight line on which the series of points are identical. In the process, the attention must be paid that not any pair of the appended rays may be coincided, but only the ones forming the same angle to the vanishing lines.

In bundle of the straight lines ($\overline{V_\infty}$) in the field $\overline{P}$ (Fig. 2.) there is ∞^1 of rays forming the angle ρ with the vanishing line $\overline{n}$. In the pencil of the straight lines (V), in the field P, there are two rays forming the angle ρ with the vanishing lined ${}_1n$, those being the rays g and j. To them, the rays $\overline{g}$ and $\overline{j}$ are projectively appended in the bundle ($\overline{V_\infty}$) of the field $\overline{P}$.

By coinciding the rays $g \equiv \overline{g}$, two perspective positions may be obtained, with two perspective axes. The one is k straight line coincided with the straight line $\overline{k}$, where the series $k(A^o,B^o,C^o,N_\infty)$ is identical with the series $\overline{k}(\overline{A^\circ},\overline{B^\circ},\overline{C^\circ},\overline{N_\infty})$. The other perspective axis is the straight line k_1, coincided with the straight line $\overline{k_1}$, where the series k_1 (A_1°, B_1°, C_1°, N_∞) is identical with the series $\overline{k_1}(\overline{A_1^\circ},\overline{B_1^\circ},\overline{C_1^\circ},\overline{N_\infty})$.

By coinciding the rays $j \equiv \overline{j}$, two perspective positions may be obtained, them being the same axes of perspective, straight lines $k \equiv \overline{k}$ and $k_1 \equiv \overline{k_1}$ on which series of points are identical.

Since the field P is not moving, the position of the series (k) and (k_1) is determined by the previously described procedure. The appended series (k) and (k_1) in the field P, that is moving, can be found by supplementing the projective series AD, BC and $\overline{A}\,\overline{D}$, $\overline{B}\,\overline{C}$ by utilizing the perspective pencils (S) and ($\overline{S}$) and the perspective axis p_1 (BC) and p_2 (AD), (Fig. 2. and Fig. 4a.).

This implies that in the field P there are series of 1^{st} order points (k) and (k_1), which have two appended identical series of 1^{st} order points ($\overline{k}$) and ($\overline{k}_1$) in the set of ∞^1 identical series in the field $\overline{P}$.

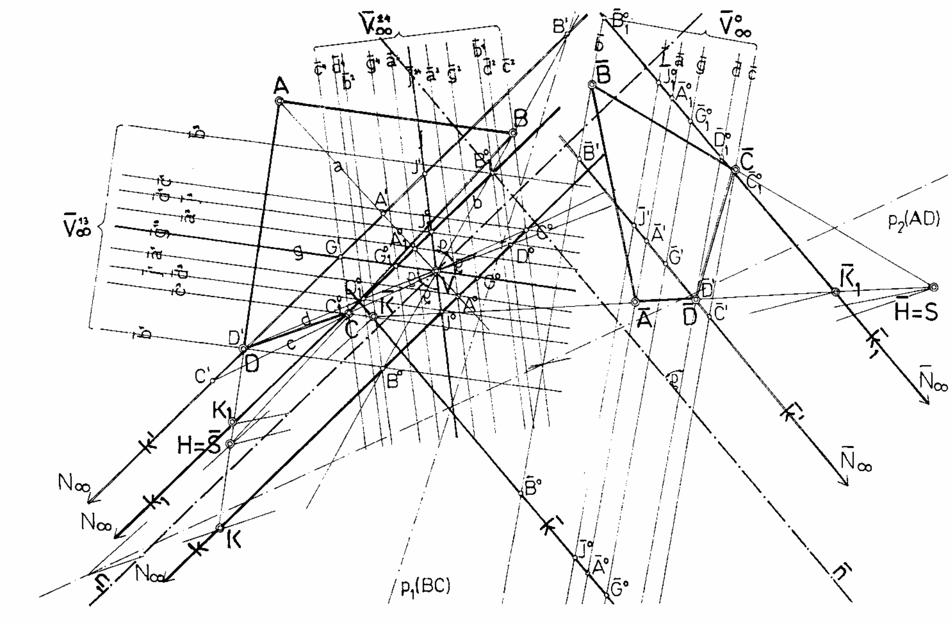


Fig. 2.

3.2. Identical segments on the series of points ($\overline{k'}$)

If the perspective center ∞^1 of the series in the field $\overline{P}$, is point $\overline{V_\infty}$, and two segments on one randomly chosen series of points $\overline{k'}(\overline{A'}, \overline{B'}, \overline{C'}, \overline{N'})$ are identical $\overline{A'}\,\overline{B'} \equiv \overline{B'}\,\overline{C'}$, then the series appended to it k' (A', B', C', N_∞) in the field P, will have two segments of identical to $A'B' \equiv B'C'$. The set of ∞^1 similar series in the file P will have two equal segments. Only two of ∞^1 similar series in the field P, will have the series with segments equal to the randomly chosen $\overline{A'}\,\overline{B'} \equiv \overline{B'}\,\overline{C'}$. Those are the series $k(A^o, B^o, C^o, N_\infty)$ and $k_1(A_1^\circ, B_1^\circ, C_1^\circ, N_\infty)$ in the field P, to whom the series $\overline{k}(\overline{A}^\circ, \overline{B}^\circ, \overline{C}^\circ, \overline{N}_\infty)$ and $\overline{k}_1(\overline{A}_1^\circ, \overline{B}_1^\circ, \overline{C}_1^\circ, \overline{N}_\infty)$ are appended in the field $\overline{P}$, in such a manner that the segments $A^oB^o \equiv B^oC^o \equiv \overline{A}^\circ\,\overline{B}^\circ \equiv \overline{B}^\circ\,\overline{C}^\circ \equiv A_1^\circ B_1^\circ \equiv B_1^\circ C_1^\circ \equiv \overline{A}_1^\circ\,\overline{B}_1^\circ \equiv \overline{B}_1^\circ\,\overline{C}_1^\circ$ are equal, (Fig. 4b).

4. SPECIAL PROCEDURE

Up to now, the general procedure of determination of the appended identical series in two general-collinear fields has been considered. However, there is a possibility to choose the special procedure which is graphically more convenient that the general one.

Out of ∞^1 points that may be taken as the centers of the perspective of the identical series of points in the field $\overline{P}$, the point $\overline{{}_1O_\infty}$, (Fig. 3) can be chosen, so that the rays of perspective (straight lines of the bundle ($\overline{{}_1O_\infty}$)) are normal to the vanishing line in $\overline{n}$. To the point $\overline{{}_1O_\infty}$ on the fictitious straight line $\overline{{}_1n_\infty}$ in the field $\overline{P}$ the point ${}_1O$ is appended on the vanishing line ${}_1n$ in the field u P. To the bundle of straight lines ($\overline{{}_1O_\infty}$) in the field $\overline{P}$, the pencil of straight lines (${}_1O$) is appended in the field u P.

4.1. Different segments on the series of points ($\overline{k'}$)

The only ray in the field P, in the pencil (${}_1O$), which is normal to the vanishing line ${}_1n$ is the main normal line n_g. The straight line appended to it, in the bundle ($\overline{{}_1O_\infty}$) of the field $\overline{P}$, is the main normal line $\overline{n}_g$ in the field $\overline{P}$, normal (perpendicular) to the vanishing line $\overline{n}$.

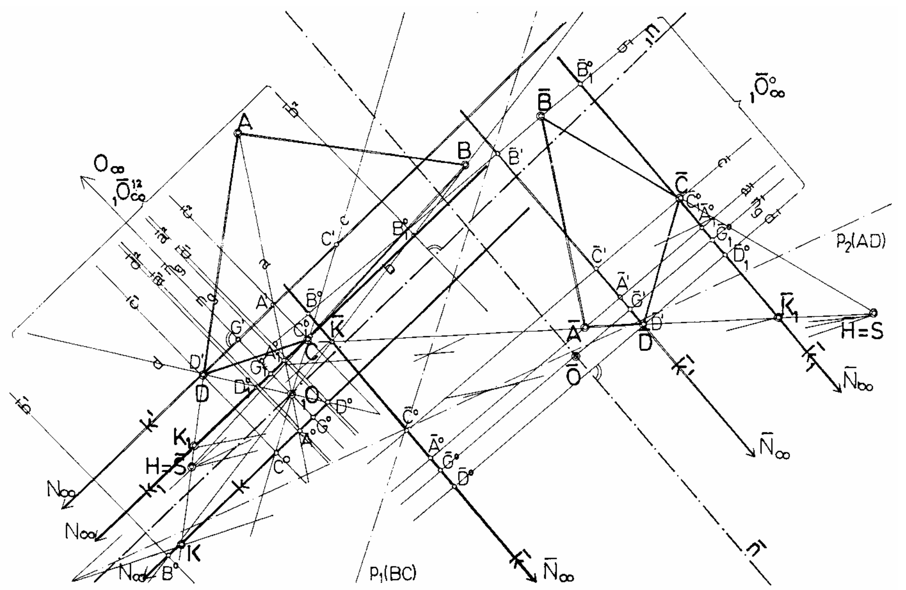


Fig. 3.

By coinciding the main normal lines $n_g \equiv \overline{n}_g$, the pencil (${}_1O$) and the bundle ($\overline{{}_1O_\infty}$), can be brought in the perspective position in two ways, and thus two axes of perspective are obtained. One of the is the straight line $k \equiv \overline{k}$, and the other $k_1 \equiv \overline{k}_1$. The series of points on these straight lines are identical, because the appended segments are equal: $A^{\circ}B^{\circ} \equiv \overline{A^{\circ}}\ \overline{B^{\circ}}$, $B^{\circ}C^{\circ} \equiv \overline{B^{\circ}}\ \overline{C^{\circ}}$, $A_1^{\circ}B_1^{\circ} \equiv \overline{A_1^{\circ}}\ \overline{B_1^{\circ}}$, $B_1^{\circ}C_1^{\circ} \equiv \overline{B_1^{\circ}}\ \overline{C_1^{\circ}}$, (Fig. 3. and Fig. 4c).

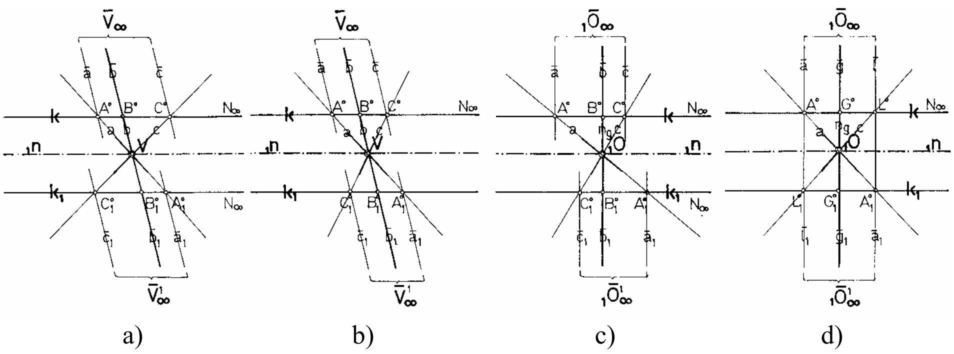
In this way, two straight lines k and k_1 in the field P, are obtained, and the series of points on them are identical to the appended straight lines of the series of points on the straight lines $\overline{k}$ and $\overline{k}_1$ in the field $\overline{P}$. The points on the series (k) and (k_1) are in the central symmetry in respect to the $_1O$ on the vanishing line $_1n$, while the points on the series ($\overline{k}$) and ($\overline{k}_1$) are orthogonally symmetrical in respect to the vanishing line $\overline{n}$ (and vice versa).

4.2. Equal segments on the series of points ($\overline{k'}$)

If in the bundle of straight lines ($\overline{_1O_\infty}$), two equal segments $\overline{A'}\ \overline{G'} \equiv \overline{G'}\ \overline{L'}$ are taken on the random series of points $\overline{k'}(\overline{A'}, \overline{G'}, \overline{L'}, \overline{N_\infty})$, then, in the pencil of the straight lines ($_1O$) the appended series of points k' (A', G', L', N_∞) will map in the same ratio, so the segments A'G' ≡ G'L' are equal.

Since the point $\overline{G'}$ is taken on the main normal line $\overline{n}_g$, in the field $\overline{P}$, then the point G' that is appended to it, in the field P, on the main normal line n_g. The rays in the pencil of the straight lines ($_1O$) through the points A' and L' form with the normal line n_g the same angle (they are symmetrical in respect to n_g).

In order to identify the identical series of points on the straight lines that intersect the pencil ($_1O$) and the bundle ($\overline{_1O_\infty}$), it is necessary to bring ($_1O$) and ($\overline{_1O_\infty}$) into the perspective position by coinciding the main normal lines and to determine the axis of perspective for one position. For another perspective position, the perspective axis is symmetrical in respect to the vanishing line $_1n$, (Fig. 4d).



a) b) c) d)

Fig. 4.

CONCLUSION

In the collocal, general-collinear fields, there are two pairs of appended identical series of 1st order points. Those are the series of the points (k) and (k_1), in the field P, and the series of points ($\overline{k}$) and ($\overline{k}_1$) are appended to them, in the field $\overline{P}$. The points on the series

(k) and (k_1), are in central symmetry in respect to ${}_1O$, and the points on the series ($\bar{k}$) and ($\overline{k_1}$) are orthogonally symmetrical in respect to the vanishing line $\bar{n}$ (and vice versa).

All this implies that the appended identical series of points as invariants may be used for bringing of collocal general-collinear fields from a general into the perspective position.

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IDENTIČNI PRIDRUŽENI NIZOVI TAČAKA KAO INVARIJANTE U KOLOKALNIM OPŠTE-KOLINEARNIM POLJIMA

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Da bi se kolokalna kolinearna polja dovela iz opšteg u perspektivni položaj, potrebno je da se pronadju pridruženi identični nizovi tačaka. Zbog osobina koje zavise od projektiviteta zadatog četvorkom pridruženih tačaka (pravih) pridruženi identični nizovi tačaka I vrste se ubrajaju u invarijante opšte-kolinearnih i perspektivno-kolinearnih polja. Postupak odredjivanja pridruženih identičnih nizova tačaka sastoji se u tome da se u skupu od ∞^1 perpektivnih sličnih nizova u jednom polju (čiji je centar perspektiviteta tačka na nedoglednici), pronadju oni koji su identični sa svim nizovima u skupu od ∞^1 perspektivnih identičnih nizova tačaka u drugom polju (čiji je centar perspektiviteta tačka na beskonačno dalekoj pravoj). Pri tom se polazi od pridruženih sličnih nizova, dobijenih opštom metodom. Način na koji se mogu dobiti pojednostavljuje se uvodjenjem specijalno uzetih sličnih nizova tačaka. Zaključak je: U opšte-kolinearnim poljima postoje dva pridružena para identičnih nizova tačaka.

Ključne reči: *kolokalna opšte-kolinearna polja, pridruženi identični nizovi tačaka*