

DESIGN METHODS OF A TIMBER-CONCRETE T-CROSS-SECTION

UDC 624.016:624.011.1+624.012.3(045)=20

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Abstract. *This paper deals with the composite timber-concrete structures.. By combining timber and concrete in a new type of composite material and using the best properties of both materials (the high tensile strength of timber and the high compressive strength of concrete) a new type of composite structure is obtained, which can have many applications, depending on the different building conditions, due to the certain advantages it has over concrete or steel structures. Here, the design procedures according to the theory of elasticity based on the exact method and approximate method are given in order, and particularly according to the regulations and recommendations of the modern concept for design of timber structures and concrete structures, given in Eurocode 5 and based on the limit states of bearing capacity and usability of structures.*

1. INTRODUCTION

The analysis of composite timber-concrete beams requires knowledge of a relationship between stress and deformations for all three components, timber, concrete and shear connectors.

The complexity of problems lies in determination of this relationship and requires an introduction of a large number of parameters, which complicate the calculations.

For practical calculations, certain simplifications can be made, and certain assumptions to facilitate reaching the solution in a relatively easy way. The approximate calculation method for semi rigid structures is more appropriate for use in engineering because the calculus procedure is simpler.

The calculations of the design of the rigid composite structure are given, considering there is no relative slip in the interface, via the transformed section method. In this method, the concrete section is transformed in a timber section, and the neutral line remains in the same original position. The width of the cross-section depends on the rate E_c/E_t . On the other side the section with two materials in semi rigid structures will show

yield neutral lines. Depending on the connection stiffness, we can introduce a reduction of the effective inertia moment "I" and compare it to the momentum of inertia of the rigidly jointed section.

2. THE EXACT METHOD

The theoretical analysis, based on equations of equilibrium, is done according to *exact method*, [15] which makes possible the accurate determination of the strengths, flows and displacements. On the other side, the *approximated method*, which incorporates simplifications in the problem and facilitates both the calculations and design procedures, and shows that the simpler equations can be applied. The slip between mechanically connected timber and concrete is taken into account in structural models by means of the slip modulus.

The basic assumptions to composite structures, for example, concrete-wood T-beams, are the following:

- Displacements owing to bending are small and, therefore, the small displacements theory is valid;
- Displacements owing to shear deformations are negligible in each element;
- Bernoulli-Navier's hypothesis about plane sections remain plane and perpendicular to deformed axis of the section after deformation is not valid along the whole cross-section, but it is individually valid for both timber cross-section and concrete cross-section.
- Timber and concrete are isotropic elastic materials and Hook's law is valid;
- Load-slip relationship for the connector can be approximates to elastic-linear. Connectors are placed at certain distance and can be regarded as equivalent continuous connection.

If a beam is subjected to any transversal loading with intensity "q" that may vary along its axis, if the boundary conditions of the beam do not have the concrete meaning, in regards to the assumed stress and deformation of coupled cross section (see figure 1.), using both the principles of static equilibrium and displacements compatibility, we can obtain the differential equation that defines the phenomenon depending on displacement "w" [16]:

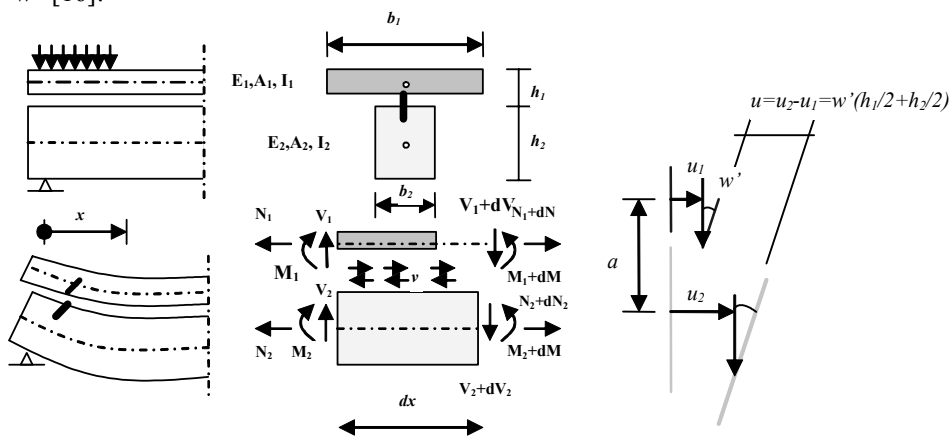


Fig. 1. System, cross section, deformation, stress, element dx.

$$w''' - \alpha^2 w'' = -\frac{M''}{(EI)_0} + \alpha^2 \frac{M}{(EI)_\infty} \quad (2.1)$$

where:

$$\alpha^2 = k \left(\frac{1}{E_1 A_1} + \frac{1}{E_2 A_2} + \frac{r^2}{(EI)_0} \right) \quad (2.2)$$

$$(EI)_0 = E_1 A_1 + E_2 A_2 \quad (2.3)$$

$$(EI)_\infty = (EI)_0 + \frac{E_1 A_1 E_2 A_2}{E_1 A_1 + E_2 A_2} r^2 \quad (2.4)$$

$(EI)_0$ the bending stiffness of uncoupled cross section,
 $(EI)_\infty$ the bending stiffness of coupled cross section,
 E_1 modulus elasticity of a concrete
 E_2 modulus elasticity of a timber,
 A_1 cross-section area of a concrete part,
 A_2 cross-section area of a timber part,
 M, N, V adequate internal forces of cross section elements (parts),
the others geometrical signs are according to figure 1.

If we know the solution "w" to a specific set of boundary conditions, the internal forces for the whole section and each element of section are given by:

$$M = \frac{(EI)_\infty}{\alpha^2} w''' - (EI)_\infty w'' - \frac{(EI)_\infty}{\alpha^2 (EI)_0} q \quad (2.5)$$

$$V = \frac{(EI)_\infty}{\alpha^2} w'''' - (EI)_\infty w''' - \frac{(EI)_\infty}{\alpha^2 (EI)_0} q' \quad (2.6)$$

$$N_1 = -\frac{M + (EI)_0 w''}{r} \quad (2.7a)$$

$$N_2 = \frac{M + (EI)_0 w''}{r} \quad (2.7b)$$

$$M_1 = -E_1 I_1 w'' \quad (2.8a)$$

$$M_2 = -E_2 I_2 w'' \quad (2.8b)$$

$$V_1 = M_1' + vr_1 \quad (2.9a)$$

$$V_2 = M_2' + vr_2 \quad (2.9b)$$

$$v = \frac{M' + (EI)_0 w'''}{r} = \frac{V + (EI)_0 w''}{r} \quad (2.10)$$

When we know the stress from equation (2.2) to (2.10) it is possible to determine normal stress and shear stress.

The normal stresses are given in timber and concrete, separately observed, by:

$$\sigma_1(x, y) = \frac{M_1(x)}{I_1} y + \frac{N_1(x)}{A_1} \quad (2.11)$$

$$\sigma_2(x, y) = \frac{M_2(x)}{I_2} y + \frac{N_2(x)}{A_2} \quad (2.12)$$

where y is the distance between the center of the considered element and the fiber whose stress we want to determine.

In order to analyse the shear stress, we can observe the efforts in an elementary segment of timber from a composite timber-concrete T-beam in figure 2:

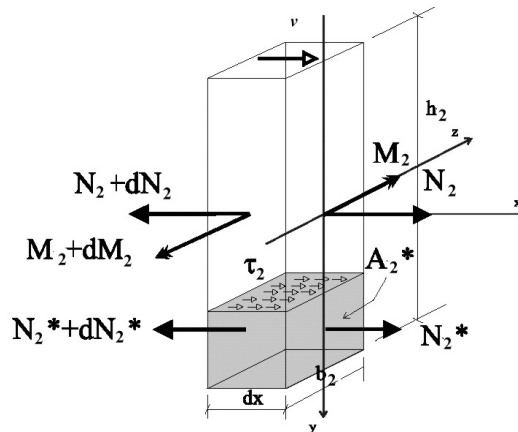


Fig. 2. The efforts in an elementary segment of timber

It is possible to write

$$N_2^* = \int_{A_2^*} \sigma_2(x, y) dA_2^* \quad (2.13)$$

where:

N_2^* resulting normal action of the stress that acts in the smaller element;

A_2^* transversal section area of the smaller element.

Using equations (2.12) and (2.13) and by calculating the equilibrium of the emphasized elementary segment in figure 2, we can find:

$$\tau_2(x, y) = \frac{V_2 S_2^*}{b_2 I_2} + \frac{v}{b_2} \left(\frac{A_2^*}{A_2} - \frac{S_2^*}{I_2} r_2 \right) \quad (2.14)$$

The equation (2.14) makes it possible to determine the shear stress in any one point on the timber cross-section. Analogously, the expression to the evaluation of the shear stress in the concrete section can be obtained by:

$$\tau_1(x, y) = \frac{V_1 S_1^*}{b_1 I_1} + \frac{v}{b_1} \left(\frac{A_1^*}{A_1} - \frac{S_1^*}{I_1} r_1 \right) \quad (2.15)$$

3. APPROXIMATE METHOD

This method provides approximate analytic solutions[29]. The basic assumptions in this method are the same as in *exact method* presented above.

The connection of timber and concrete can be made by means of many types of metal connectors: nails, steel dowels, rings, connected perforate metal plates, etc. The glued joints are regarded as rigid connections.

The slip between mechanically connected timber and concrete is taken into account in structural models by means of the slip modulus

$$K = \frac{F_s}{u} \quad (3.1)$$

where F_s is the shear force in the mechanical fastener and u is the slip in the connection.

The shearing flow " v " that appears on interface of the materials is yielded by:

$$v = \frac{F_s}{s} \quad (3.2)$$

where " s " is the spacing between connectors.

Using equations (3.1) and (3.2), we find:

$$v = ku \quad (3.3)$$

$$k = K / s \quad (3.4)$$

where K is the equivalent slip modulus in the joint.

According to the elastic principles of bending theory:

$$N_1 = E_1 A_1 u_1' \quad (3.5a)$$

$$N_2 = E_2 A_2 u_2' \quad (3.5b)$$

$$M_1 = -E_1 I_1 w'' \quad (3.6a)$$

$$M_2 = -E_2 I_2 w'' \quad (3.6b)$$

$$V_1 = -E_1 I_1 w''' \quad (3.7a)$$

$$V_2 = -E_2 I_2 w''' \quad (3.7b)$$

$$v = ku = (u_2 - u_1 + w'a) = u_2 - u_1 + w'(h_1/2 + h_2/2) \quad (3.8)$$

From the equilibrium of the two elements (timber and concrete) in both longitudinal and axial directions, we obtain:

$$N_1' + v = 0 \quad (3.9a)$$

$$-N_2' + v = 0 \quad (3.9b)$$

$$M_1' = V_1 - v h_1 / 2 \quad (3.10a)$$

$$M_2' = V_2 - v h_2 / 2 \quad (3.10b)$$

$$V_1' + V_2' = -p = V' \quad (3.11)$$

where "p" is a generic loading applied to beam. Adding equations (3.10a) to (3.10b), taking into consideration the equations (3.11) and consequently differentiating it in relation to x, we find:

$$M_1'' + M_2'' + v'a + p = 0 \quad (3.12)$$

If using the elastic principles changes both internal forces and moments, the following system of differentiable equation is yielded:

$$E_1 A_1 u_1'' + k(u_2 - u_1 + w'a) = 0 \quad (3.13)$$

$$E_2 A_2 u_2'' + k(u_2 - u_1 + w'a) = 0 \quad (3.14)$$

$$(E_1 I_1 + E_2 I_2) w'' - k(u_2' - u_1' + w'a) = p \quad (3.15)$$

In this way equations (3.9a), (3.9b) and (3.12) are formulated in function of the displacements u_1 , u_2 and v .

The practical application of a system of differentiable equation will be shown at the example of simply supported beam with a sinusoidal distribution loading as shown in figure 3, so a simple analytical solution can be achieved. This is due to deformation forms in the directions of the axis which agrees with both sinusoidal and cosine functions [29].

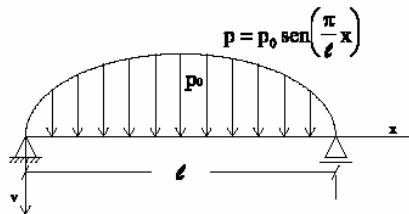


Fig. 3. Sinusoidal distribution loading.

We can get:

$$p = p_0 \sin\left(\frac{\pi}{l}x\right) \quad (3.16)$$

$$u_1 = u_{10} \cos\left(\frac{\pi}{l}x\right) \quad u_2 = u_{20} \cos\left(\frac{\pi}{l}x\right) \quad w = w_0 \sin\left(\frac{\pi}{l}x\right) \quad (3.17a,b,c)$$

where: u_{10} and u_{20} are the horizontal displacements at both concrete and timber centroid, respectively at the ends of the beam. The maximum vertically displacement w_0 is at midpoint.

These terms, when substituted in Equations (3.13), (3.14) and (3.15), produce a system of equations with the constants u_{10} , u_{20} , w_0 , whose solution allows to determine the stress at both concrete and timber centers:

$$\sigma_1 = -\frac{\gamma_1 E_1 M_0 a_1}{(EI)_{ef}} \quad (3.18)$$

$$\sigma_2 = -\frac{\gamma_1 E_2 M_0 a_2}{(EI)_{ef}} \quad (3.19)$$

where:

$$\gamma_1 = \frac{1}{1+k} \quad (3.20)$$

$$k = \frac{\pi^2}{l^2} \frac{E_1 A_1}{K} \quad (3.21)$$

$$a_2 = \frac{\gamma_1 E_1 A_1}{\gamma_1 E_1 A_1 + E_2 A_2} a \quad (3.22)$$

$$a_1 = \frac{E_2 A_2}{\gamma_1 E_1 A_1 + E_2 A_2} a \quad (3.23)$$

Stress at the outer fibers of both concrete and timber:

$$\sigma_{1,1} = -\frac{1}{2} \frac{E_1 M_0}{(EI)_{ef}} h_1 \quad \sigma_{1,2} = \frac{1}{2} \frac{E_1 M_0}{(EI)_{ef}} h_1 \quad (3.24a,b)$$

$$\sigma_{2,1} = -\frac{1}{2} \frac{E_2 M_0}{(EI)_{ef}} h_2 \quad \sigma_{2,2} = \frac{1}{2} \frac{E_2 M_0}{(EI)_{ef}} h_2 \quad (3.25a,b)$$

Although the deductions had been made for midpoint, the expressions for stress calculations can be extended to others cross-sections along the length of the coupled beam, being enough to change M_0 to $M(x)$.

The shearing flow along the length of the coupled beam can be calculated by expression:

$$v = \frac{\gamma_1 V E_1 A_1 a_1}{(EI)_{ef}} \quad (3.26)$$

The elastic line is given by:

$$v''(x) = -\frac{M(x)}{(EI)_{ef}} \quad (3.27)$$

This means that, considering a sinusoidal distribution loading, we attained a differential equation to elastic line.

The equation (3.27), is well-known from bending theory, wherein we replace the bending stiffness EI of homogeneous beam by the bending stiffness $(EI)_{ef}$ of the case of coupled beams. The solution of this equation is much simpler than expression (3.1).

4. DESIGN METHOD ACCORDING TO EUROCODE 5

The design of the composite beams is regulated in the appendix B of the Eurocode 5. The stress calculation for timber and concrete and the calculation of the connectors is to be performed in accordance with the theory of the elastic compound.

According to recommendations from the appendix B of the Eurocode 5, in consistence with what has been said, we can calculate the geometrical properties, stresses, and characteristics of connection of the cross section shown by figure 4, according to next steps:

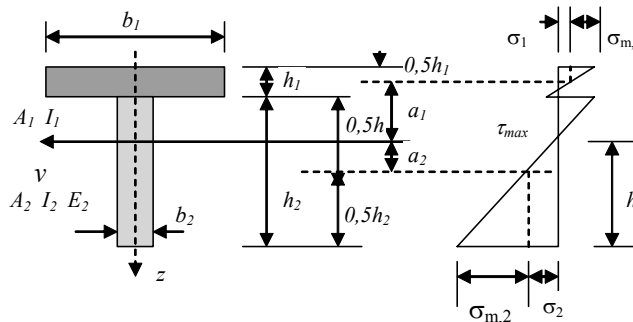


Fig. 4. Geometrical properties and stresses

The effective bending stiffness will be calculated as follows:

$$(EI)_{ef} = \sum_{i=1}^n (E_i I_i + \gamma_i E_i A_i a_i^2), \quad (4.1)$$

where: i number of elements consisting composite (complex) cross section. In case of T- cross section, that is 2.

E the average value of modulus of elasticity for concrete and timber, respectively

$$A_i = b_i h_i, \quad (4.2a)$$

$$I_i = b_i h_i^3 / 12, \quad (4.2b)$$

$$\gamma_2 = 1, \quad (4.2c)$$

$$\gamma_i = \left[1 + \pi^2 E_i A_i s_i / (K_i \ell^2) \right]^{-1} \text{ for } i = 1 \text{ and } i = 3, \quad (4.2d)$$

$$a_2 = \frac{\gamma_1 E_1 A_1 (h_1 + h_2) - \gamma_3 E_3 A_3 (h_2 + h_3)}{2 \sum_{i=1}^3 \gamma_i E_i A_i}. \quad (4.2e)$$

For T-cross sections, $h_3 = 0$.

The normal stresses are given by equations

$$\sigma_i = \gamma_i E_i a_i M / (EI)_{ef} \quad (4.3a)$$

$$\sigma_{m,i} = 0,5 E_i h_i M / (EI)_{ef} \quad (4.3b)$$

The shear stress has the maximum magnitude at the point where normal stresses are equal to zero. Maximum shear stress at a certain point along the height of the timber element of cross section should be calculated according to the expression:

$$\tau_{2,max} = (\gamma_3 E_3 A_3 a_3 + 0,5 E_2 b_2 h^2) V / (b_2 (EI)_{ef}). \quad (4.4)$$

The load of the fastener should be calculated according to expression

$$F_i = \gamma_i E_i A_i a_i s_i V / (EI)_{ef} \quad (4.5)$$

with $i=1$ and 3 , where $s_i = s_i(x)$ distance between fasteners determined in B1.3 and $V = V(x)$.

REFERENCES

1. A. Ceccotti, Timber-concrete composite structures, Structural Timber Engineering Proceedings 2 (STEP 2), lecture E13, 1995;
2. H. J. Blaß und M. Schlager, Trag- und Verformungsverhalten von Holz-Beton-Verbundkonstruktionen-Teil 1, Bauen mit Holz 5/96, pages: 392-399, 1996;
3. H. J. Blaß, M. van der Linden und M. Schlager, Trag- und Verformungsverhalten von Holz-Beton-Verbundkonstruktionen-Teil 2, Bauen mit Holz 6/96, pages: 472-477, 1996;
4. ENV 1995-1-1. Eurocode 5: Design of timber structures, Part 1.1: General rules and rules for building. European Committee for Standardisation. 1993;
5. ENV 1992-1-1. Eurocode 2: Design of concrete structures, Part 1.1: General rules and rules for building, European Committee for Standardisation 1993;
6. R. Cvetković, Behaviour of Composite Timber-Concrete Structures with Bending Actions Masters thesis, Department of Reinforced Concrete and Prestressed Concrete Structures, Ruhr University Bochum, Germany, 2002.
7. B. Stevanovic, Analysis of Composite Timber-Concrete Structures, Doctoral thesis, Faculty of Civil Engineering, Belgrade, 2003;
8. Demarzo Mauro, Tacitano Marcelo: "Semirigid composite wood-concrete T-beams" Proceedings of World Conference on Timber Engineering (WCTE 2000), P47, British Columbia, Canada, 31. Jul-3. August 2000.
9. Girhammar U. A. and Gopu K. A. 1993. Composite beam-columns with interlayer slip - exact analysis. Journal of Structural Engineering, New York, v.119, n.4, p. 1265-1282, Apr.

METODE PRORAČUNA SPREGNUTIH KONSTRUKCIJA OD DRVETA I BETONA T PRESEKA

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U radu je data analiza spregnutih konstrukcija tipa drvo-beton. Povezivanjem drveta i betona i korišćenjem najboljih svojstava jednog i drugog materijala dobija se novi spregnuti tip konstrukcije za koji se, zavisno od različitih uslova građenja, može naci mnogo razloga za primenu s obzirom na određene prednosti u odnosu na beton i čelik. Ovde su, redom, date proračunske procedure prema teoriji elastičnosti zasnovane na tačnom i približnom metodu i posebno, prema pravilima i preporukama modernog concepta za proračun drvenih i betonskih konstrukcija datih u Evrokodu 5 i zasnovanih na graničnim stanjima nosivosti u upotrebljivosti.