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REACTIVE LOADING FUNCTION ON TUNNEL EXCAVATION CONTOUR IN ROCK MASS

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Abstract. *Investigation of the stress field around the cavity that is loaded or partially loaded at the inner surface by the rotationally symmetric loading is still the contemporary problem in the theory of elasticity.*
As the contribution to the similar investigations, the paper introduces the new function of loading in the form of the infinite sine series. Besides the definition of the mentioned loading function as the boundary condition on inner surface of the cavity, the paper presents the comparative analysis of the new function with the previously used ones.
The said loading function at the inner surface of a cavity is internal loading between rock mass and supporting structure, for the case of homogeneous isotropic elastic medium as the first approximation of the real situation.

Key words: *cavity, loading function, Fourier series, Fourier integrals, infinite sine series*

1. INTRODUCTION

The treated problem stemmed from the need to obtain, in an analytical way, the internal loading between a supporting structure and the surrounding rock mass. The solution in its first approximation is presented for conditions of elastic homogeneous and isotropic medium and analytically defines the shapes of a cavity.

The solution of the basic equation of the theory of elasticity $\nabla^2\nabla^2\Psi=0$ requires the analysis of boundary condition functions which would ensure the existence of closed form solutions. The first requirement for such solutions of the biharmonic equation is that the boundary condition function, representing the internal loading, has to be continuous

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within the loaded area. Another necessary requirement is that the loading function has to be continuous at the ends of the loaded area i.e. the loading has to be defined at its ends without abrupt changes.

In their book [3], considering the state of stress around the spherical cavities, Jaeger & Cook have introduced a loading with axial symmetry defined by Fourier series. This definition of internal loading has been used as the boundary loading condition.

The boundary conditions on the internal surface of infinite cylinder cavity were defined in publication [2], by use of Fourier integrals.

The boundary conditions on the internal surface of oblong spheroid cavity, spherical cavity, and infinite cylindrical cavity, were defined in publication [4], in the form of infinite sine series. The elaboration of particular properties of loading functions for cavities having all mentioned shapes, is presented here below.

Apart from the description of loading functions used so far as the boundary conditions on the internal surface of cavities, including those presented in publications [4] and [1], this presentation includes the comparative study of their properties, with particular attention made to explain advantages and drawbacks of the mentioned functions which have been used to define cavity internal loading with axial symmetry.

2. Definition of functions

2.1. Functions defined by Fourier series (Jaeger & Cook)

In order to resolve the stress state around a spherical cavity the boundary condition at its internal side i.e. the loading with axial symmetry has been defined [3], in the form of Fourier series

$$p(\varphi) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\varphi \tag{1}$$

where the coefficients are:

$$a_0 = \frac{2\beta p}{\pi} \qquad a_n = \frac{2p}{n\pi}[1+(-1)^n]\sin n\beta \tag{2}$$

and: p – is the constant amplitude of loading

β – is the angle which defines the loaded area (−β, +β)

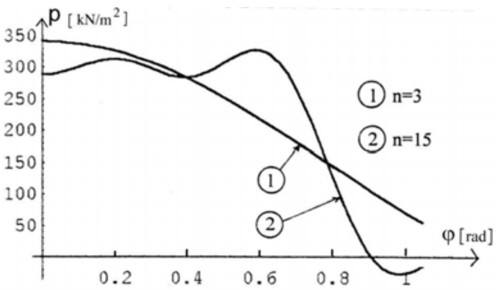

Fig.1. Function of Cook-Jaeger (p=300 kN/m″, β=π/4)

The loading function (1) is presented in Fig. 1, for two chosen numbers, n=3 and n = 15, of series representation. On the graphs of the said functions one can find out rather large dispersions around the central value of the loading function. Also, at the ends of the loading area, for the case $\beta = \pi/4$, the functions obtain the values that are different from zero, which is a serious important drawback.

2.2. Functions defined by Fourier integrals

The state of stress around an infinite cylindrical cavity, with an internal axially symmetrical loading, has been elaborated, see [2], in a general form on the basis of loading defined in the form of Fourier integral:

$$p\left(\frac{z}{r}\right) = \frac{2}{r\pi}\int_0^\infty \cos\left(\frac{\alpha_1 z}{r}\right) d\alpha_1 \int p \cdot \cos\left(\frac{\alpha_1 z}{r}\right) dz \tag{3}$$

where: r – is the cylinder radius
p – is the constant amplitude of loading

In the case of a partially loaded cavity, within the limits: $-B/2 \le z \le B/2$ or $0 \le z \le a$, the loading function has been defined, see [2], in the following form:

$$p\left(\frac{z}{r}\right) = \frac{2p}{\pi}\int_0^\infty \frac{1}{\alpha_1} \sin\left(\frac{\alpha_1 B}{2r}\right) \cos\left(\frac{\alpha_1 z}{r}\right) d\alpha_1 \tag{4}$$

Substituting $x = \alpha_1 B/2r$ into (4), after some mathematical transformations, one can obtain the loading function in the following form:

$$p\left(\frac{z}{r}\right) = \frac{2p}{\pi}\int_0^\infty \frac{\sin x \cos Ax}{x} dx \tag{5}$$

where: $$A = 2z/B \tag{6}$$

The integral (5) is well known in the literature and the loading can be obtained from (4) and (5) as:

$$\begin{aligned} p\left(\frac{z}{r}\right) &= p \quad \text{for} \quad z < \left|\frac{B}{2}\right| \\ p\left(\frac{z}{r}\right) &= \frac{p}{2} \quad \text{for} \quad z = \left|\frac{B}{2}\right| \\ p\left(\frac{z}{r}\right) &= 0 \quad \text{for} \quad z > \left|\frac{B}{2}\right| \end{aligned} \tag{7}$$

From the relationships (7) one can conclude that the loading function has the constant value inside the interval $(-B/2, B/2)$, but at the end points $-B/2$, and $B/2$ (i.e. 0 and a) it has the discontinuity of the first kind, and the final loading value of $p/2$. Outside these points the value of the function is zero. This is the reason to note the so far unresolved problem of the existence of the solution of the basic equation $\nabla^2 \nabla^2 \Psi = 0$ in the vicinity of these end points.

2.3. Functions defined by infinite sine series

Considering already described problems at the ends of the loaded areas, particularly in respect to the existence of the solution of the basic equation, a new loading function defined by infinite sine series has been introduced in [4]. This improvement stemmed from the aim to obtain unbiased mathematical solution of the basic equation $\nabla^2\nabla^2 \Psi=0$. It is to be emphasized that the loading is defined by the introduced function only within the limited loaded area (interval), and that outside the loaded area the loading function value is zero. In order to obtain the state of stress around a spherical cavity the loading function is defined in the following form:

$$\begin{aligned} p(\varphi) &= p\sum_{n=0}^{\infty}\frac{4}{(2n+1)\pi}\sin\frac{(2n+1)\pi}{2\beta}(\beta-\varphi); \quad -\beta \le \varphi \le \beta;\ (\pi-\beta<\varphi<\pi+\beta) \\ p(\varphi) &= 0 \quad - \text{on the unloaded part of the cavity} \end{aligned} \tag{8}$$

The graph of the loading function between limits (-β, β), for β= π/4, has been presented in Fig.2. The value of the function outside the defined limits is equal to zero. On the basis of the graphs obtained for the different numbers of series members, one can conclude that the function has a rather small dispersion around its central value, with fast convergence of the sine functions to zero at the end points of the loaded area.

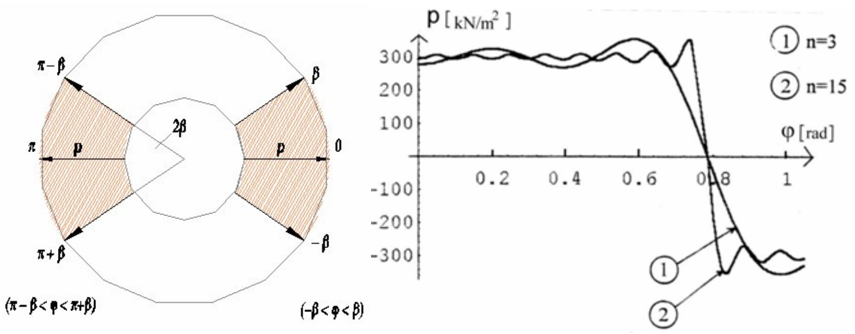

Fig. 2. Function in the form of the infinite sine series (p=300 kN/m″, β=π/4) - sphere

For determination of the state of stress around infinitely long cylindrical cavity, the loading function has been defined, see [1], in the following form:

$$\begin{aligned} p(z) &= p\sum_{n=0}^{\infty}\frac{4}{(2n+1)\pi}\sin\left[\frac{(2n+1)\pi}{a}(a-z)\right] & 0 \le z \le a \\ p(z) &= 0 & z \le 0 \wedge z \ge a \end{aligned} \tag{9}$$

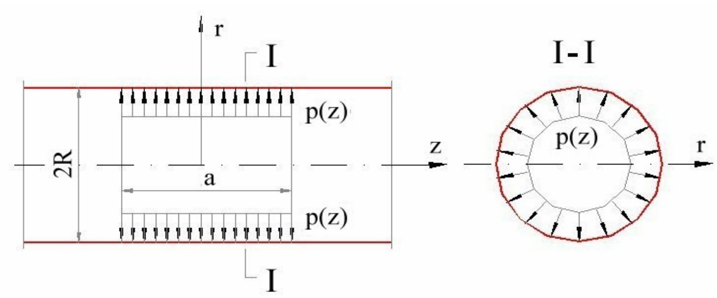

Fig. 3. Partially distributed rotationally symmetric loading

As may be seen in [4], particular attention has been paid to the definition of loading functions inside oblong spheroid cavity, considering two cases of loading area. The first case of loading area has been defined by the sine series in the form:

$$p_1(\varphi) = p\sum_{n=0}^{\infty}\frac{4}{(2n+1)\pi}\sin\left[\frac{(2n+1)\pi}{\pi-2\beta}(\varphi-\beta)\right]$$
$$\text{where}\quad \beta \le \varphi \le \pi-\beta, \qquad \pi+\beta \le \varphi \le 2\pi-\beta \tag{10}$$
$$p_1(\varphi) = 0 \qquad \text{outside the loaded area}$$

where β is the angle that defines the limits of the loaded area (Fig.4).

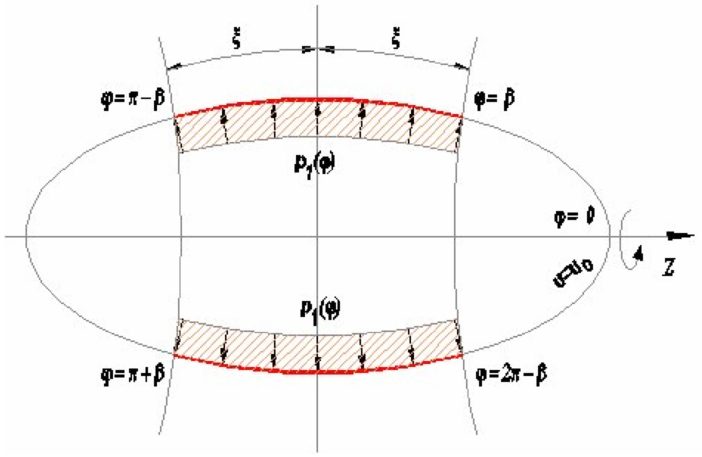

Fig. 4 Loading on contour $p_1(\varphi)$

The part of function (10) defining loading with axial symmetry is shown in Fig. 5:

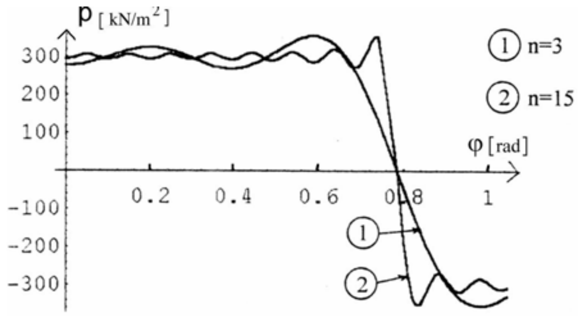

Fig. 5. Function in the form of the infinite sine series (p=300 kN/m″, β=π/4) - ellipsoid

It can be seen that the dispersion around function central value is rather small for the reasonable number of series members, and the fast convergence to its constant value at the end points of the loaded area could be noted too.

The second case of loading area is defined by:

$$p_2(\varphi) = p\sum_{n=0}^{\infty}\frac{4}{(2n+1)\pi}\sin\left[\frac{(2n+1)\pi}{2\beta}(\varphi+\beta)\right] \quad -\beta<\varphi<\beta$$
$$p_2(\varphi) = 0 \qquad \text{outside the loaded area} \tag{11}$$

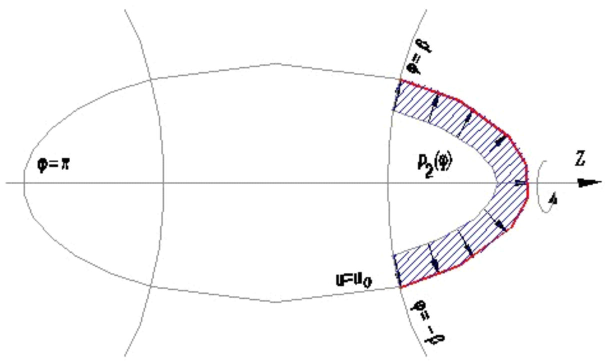

Fig.6. Loading on contour $p_2(\varphi)$

By introducing the said loading functions, it is possible to define boundary conditions in the form of infinite sine series, which enables the straightforward determination of unknown constants in the process of finding the solution of the basic differential equation. The particular importance lies in the fact that the loading function is defined by only one parameter, which enables simpler procedure of rather complex mathematical derivations.

3. Comparative comments

The selection of loading functions in the said problem may be considered from different aspects, depending on the goals sought. In this presentation, the comparative study has been made with the aim to compare the previously used loading function and the recently introduced ones. This comparison is shown by the graphical presentations given in Figs.7 and 8.

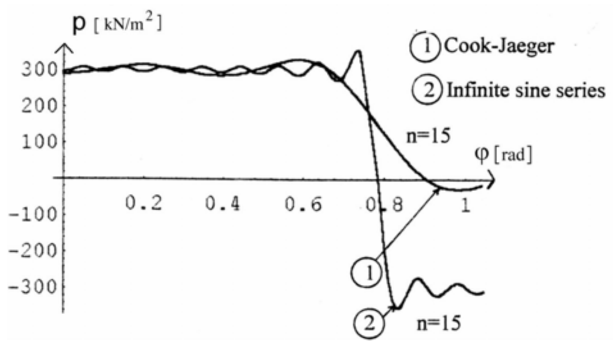

Fig. 7. Comparative presentation of functions (p=300 kN/m″, $\beta=\pi/4$) - sphere

After the comparison of the function given by Jaeger & Cook (in the form of Fourier series) and the new one given in the form of infinite sine series, see Fig. 7, it may be concluded as follows:

- the function defined by Fourier series at the loaded area ends has the values that are different from zero, i.e. the abrupt change in loading value exists at these points, while the loading function defined in the form of sine series provides the continuity of loading within whole loaded area;
- the function in the form of sine series has the fast convergence to its central value, and it is more useful for application with a smaller number of series members;
- the Gibson's effect is present, see [5], for both functions at the ends of the loaded area, but this is not in conflict with the existence of the solution.

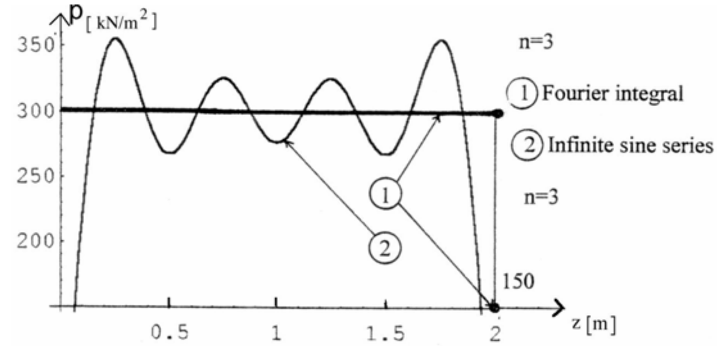

Fig. 8. Comparative presentation of functions (p = 300 kN/m″, a = 2 m) - cylinder

By comparing the graphs of functions based on Fourier integrals (on Fig. 8) which has the discontinuity of the first kind at interval ends, to the loading function based on sine series, which is continuous, one may conclude that it is always more prudent for the mathematical considerations to use the continuous function over all loaded area and its ends.

4. Conclusions

In the present paper, and also in [4], it has been demonstrated that the recently introduced single-parametric loading function in the form of infinitive sine series, may be very useful for further studies of the states of stress around cavities. In addition to that, some advantages in the use of such a function have been noted. The main advantages of this type of loading function are:

- the absence of the discontinuous transition to zero value at the ends of loaded area,
- the fast convergence to the central value of the loading within loaded area, and
- the fast convergence to zero at the ends of the loaded area.

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FUNKCIJA REAKTIVNOG OPTEREĆENJA NA KONTURI TUNELSKOG ISKOPA U STENSKOJ MASI

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Istraživanje naponskih stanja oko šupljine opterećene ili delimično opterećene sa unutrašnje strane rotaciono simetričnim opterećenjem je uvek aktuelan problem Teorije elastičnosti.

Kada se razmatra naponsko stanje za bilo koji oblik šupljine, definisanje funkcije koja predstavlja unutrašnje opterećenje odnosno granične uslove na konturi, je jedan od osnovnih zadataka. U ovom radu se najpre razmatraju najčešće korišćeni oblici funkcija pri izučavanju naponskih stanja: oko sferne šupljine u obliku reda Furie i oko šupljine oblika beskonačnog cilindra u obliku integrala Furie. Kao doprinos dosadašnjim istraživanjima uvodi se novi oblik funkcije opterećenja kao beskonačni sinusni red.

Napred definisane funkcije unutrašnjeg opterećenja šupljine predstavljaju reaktivno opterećenje podgrade. Pri tome, stenska masa se usvaja kao homogena, izotropna i elastična sredina, kao prva aproksimacija stvarnog stanja.