

ANALYSIS OF TIMBER FRAMES WITH LOCALIZED NONLINEARITIES

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Abstract. *A methodology of nonlinear analysis of timber frame structures with flexible connections has been developed in this paper. The methodology is based on the results of experimental investigation of connections behavior under static and cycling loading. The adopted numerical method is based on the concept of localized nonlinearity which facilitates the reproduction of the experimentally determined connections behavior in the analysis of frame structures. The methodology is also applicable to other types of timber structures.*

1. INTRODUCTION

The traditional analysis of timber structures is based on the assumption that elements connections are either rigid or hinged. This is an actually simplified approach and it excludes the possible cases of the real behavior of the connections. However, in order to obtain the actual stress and deformations distribution in timber structures, the real deformation characteristics must be included in the analysis. If the relation moment-rotation is known for each connection in a structure, the exact solution for the stresses and deformations can be obtained by a nonlinear analysis.

The real nonlinear analysis of a structure is possible only in the time domain (time history analysis). If we assume in the analysis of timber structures that the nonlinear deformations occur only in connections, then the stresses in a structure are linearly dependent on the deformation for the most part of the degrees of freedom, and nonlinear response occurs in a relatively small number of degrees of freedom. Because the points of a structure in which the nonlinear deformations occur are known in advance, an analysis can be carried out by the so call Fast Nonlinear Analysis developed by Wilson [7].

2. MATRIX MODIFICATION OF THE RIGID CONNECTIONS TO FLEXIBLE CONNECTION

The stiffness matrix of a prismatic member of the length L with rigid end connections and generalized displacement in a plane defined in Fig. 1 is given by the Eq. 1.



Fig. 1. The generalized displacement of a straight member

$$[K] = \frac{EI}{L} \begin{bmatrix} A/I & 0 & 0 & -A/I & 0 & 0 \\ 0 & 12/L^2 & 6/L & 0 & -12/L^2 & 6/L \\ 0 & 6/L & 4 & 0 & -6/L & 2 \\ -A/I & 0 & 0 & A/I & 0 & 0 \\ 0 & -12/L^2 & -6/L & 0 & 12/L^2 & -6/L \\ 0 & 6/L & 2 & 0 & -6/L & 4 \end{bmatrix} \quad (1)$$

However, if the connection at the member ends can deform, the stiffness matrix given by Eq.1 can be modified to take into account the effects of the connection deformations in the corresponding degrees of freedom. The modified stiffness matrix can be determined by the direct method based on the clear geometrical and static meaning of its elements [4]. Because the problem of bending is independent of the problem of axial deformation, the matrices of transversal and axial stiffness can be treated separately.

2.1 Matrix of transversal stiffness

The stiffness matrix of a bar with incompletely fixed end, whose rotational stiffness are S_{rj} and S_{rk} (see Fig.2), can be determined by giving the unit generalized displacement in different degrees of freedom taking into account the possible relaxation in the rotational direction, as illustrated by Eq. 2.



Fig. 2. A member with flexible end connection

$$[K^*] = [A]^{-1}[K]; [A] = [I] + [K][S]; [S] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{r2}^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{r4}^{-1} \end{bmatrix} \quad (2)$$

The obtained matrix is given by Eq. 3

$$[K^*] = \frac{EI}{L(1+\beta_3)} \begin{bmatrix} \frac{12+\beta_1+\beta_2}{L^2} & \frac{6+\beta_2}{L} & -\frac{12+\beta_1+\beta_2}{L^2} & \frac{6+\beta_1}{L} \\ \frac{6+\beta_2}{L} & 4+\beta_2 & -\frac{6+\beta_1}{L} & 2 \\ -\frac{12+\beta_1+\beta_2}{L^2} & -\frac{6+\beta_2}{L} & \frac{12+\beta_1+\beta_2}{L^2} & -\frac{6+\beta_1}{L} \\ \frac{6+\beta_1}{L} & 2 & -\frac{6+\beta_1}{L} & 4+\beta_1 \end{bmatrix} \quad (3)$$

where

$$\beta_1 = \frac{12EI}{S_{ij}L}; \quad \beta_2 = \frac{12EI}{S_{ik}L}; \quad \beta_3 = \frac{4EI}{S_{ij}L} + \frac{4EI}{S_{ik}L} + \frac{12}{S_{ij}S_{ik}} \left(\frac{EI}{L} \right)^2 = \frac{\beta_1}{3} + \frac{\beta_2}{3} + \frac{\beta_1\beta_2}{12} \quad (4)$$

The form of the matrix given by Eq.3 is general, because it can be also used for fixed or hinged bar ends, by simply letting the corresponding stiffness S_{ri} tend to infinity or to zero.

2.2 Matrix of axial stiffness

The stiffness of a bar connection can be expressed through the reduced cross-section area of a bar A^* . In the model shown in Fig.3 it is displayed by the classic springs with stiffness S_{ai} .



Fig. 3. The model of axial stiffness

The reduced cross-section area is then given by Eq. 5

$$A^* = \frac{A}{1 + \frac{EA}{L} \left(\frac{1}{S_{a1}} + \frac{1}{S_{a2}} \right)} \quad (5)$$

and the modified axial stiffness matrix has the form

$$[K^*] = \frac{EA^*}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (6)$$

for the degrees of freedom defined in Fig. 3, the complete stiffness matrix obtained by the superposition of matrices given by Eq. 3 and Eq. 6 is

$$[K^*] = \frac{EI}{L(1+\beta_3)} \begin{bmatrix} \alpha_0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & \frac{12+\beta_1+\beta_2}{L^2} & \frac{6+\beta_2}{L} & 0 & -\frac{12+\beta_1+\beta_2}{L^2} & \frac{6+\beta_1}{L} \\ 0 & \frac{6+\beta_2}{L} & 4+\beta_2 & 0 & -\frac{6+\beta_2}{L} & 2 \\ -\alpha_0 & 0 & 0 & \alpha_0 & 0 & 0 \\ 0 & -\frac{12+\beta_1+\beta_2}{L^2} & -\frac{6+\beta_2}{L} & 0 & \frac{12+\beta_1+\beta_2}{L^2} & -\frac{6+\beta_1}{L} \\ 0 & \frac{6+\beta_1}{L} & 2 & 0 & -\frac{6+\beta_1}{L} & 4+\beta_1 \end{bmatrix} \quad (7)$$

where

$$\alpha_0 = \frac{A^*(1+\beta_3)}{L} \quad (8)$$

The modified stiffness matrix is also developed for the analysis of buckling and for the application of the *Theory of second order and secondary effects*.



Fig. 4. A member with incompletely fixed ends under compression

$$[K^*] = [A']^{-1} [K'] \quad (9)$$

where $[K^*]$ – modified stiffness matrix in accordance with the Theory of second order,

$[A']^{-1}$ – modification matrix,

$[K']$ – original stiffness matrix of the Theory of second order.

When the Eq. 9 is applied one obtains

$$[K^*] = \frac{EI}{L(1+\beta_3)} \begin{bmatrix} \frac{2c'-\omega^2+\beta_1+\beta_2-\gamma_1-\gamma_2-\gamma_3}{L^2} & \frac{c'+\beta_2}{L} & -\frac{2c'-\omega^2+\beta_1+\beta_2-\gamma_1-\gamma_2-\gamma_3}{L^2} & \frac{c'+\beta_1}{L} \\ \frac{c'+\beta_2}{L} & a'+\beta_2 & -\frac{c'+\beta_2}{L} & b' \\ \frac{2c'-\omega^2+\beta_1+\beta_2-\gamma_1-\gamma_2-\gamma_3}{L^2} & \frac{c'+\beta_2}{L} & \frac{2c'-\omega^2+\beta_1+\beta_2-\gamma_1-\gamma_2-\gamma_3}{L^2} & -\frac{c'+\beta_2}{L} \\ \frac{c'+\beta_1}{L} & b' & -\frac{c'+\beta_2}{L} & a'+\beta_1 \end{bmatrix} \quad (10)$$

where

$$\beta_1 = \frac{EI}{LS_{r2}}(a^2 - b^2); \beta_2 = \frac{EI}{LS_{r4}}(a^2 - b^2); \beta_3 = \frac{1}{\omega^2}(\gamma_1 + \gamma_2 + \gamma_3);$$

$$\gamma_1 = \frac{\omega^2 a EI}{LS_{r2}}; \gamma_2 = \frac{\omega^2 a EI}{LS_{r4}}; \gamma_3 = \frac{\omega^2}{S_{r2} S_{r4}} \left(\frac{EI}{L} \right) (a^2 - b^2); \quad (11)$$

2.3 Application to the nonlinear analysis

The nonlinear behavior of timber frames with flexible connections is mainly the result of the nonlinear deformations of connections and the second order effects by large deformations. If the analysis is limited to the small deformations, then the stiffness matrix given by Eq. 7 can be applied. In that case, the nonlinear behavior in a structure is limited to the connections, those may be termed as the localized nonlinearities. In order to carry out the analysis, the force-displacement relation, like one shown in Fig. 5, must be defined for all stiffness coefficients S_i in the stiffness matrix given by Eq.7.

An analysis of nonlinear structures is usually carried out by so called incremental method. The loading is divided into a series of small partial loading or increment which can be different in the general case. Within each load increment the analysis is carried out with the assumption of linear behavior of structures with the stiffness matrix modified in accordance with the achieved level of deformation.

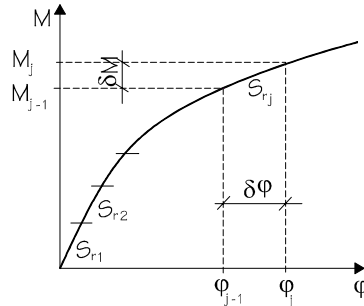


Fig. 5. An arbitrary moment-rotation relationship of a flexible connection

$$\delta M = S_{rj} \cdot \delta \phi, \quad S_{rj} = (M_j - M_{j-1}) / (\phi_j - \phi_{j-1}) \quad (12)$$

In order to achieve a faster convergence of the solution, the method is combined with an iterative procedure in each increment (Newton-Raphson modified procedure, at et.). If by an analysis, the secondary effects (geometrical nonlinearity) are also considered, the stiffness matrix of a bar given by Eq. 10 is used. The geometrical nonlinearity in timber structures originates not only from the deformation of slender compressed members, but also from the significant deformation of connections.

3. MODELING OF CONNECTIONS WITH NONLINEAR FINITE ELEMENTS

In order to avoid the nonlinear behavior of connections with a single spring with complex force-displacement relationship, several nonlinear elements with simple force-deformations characteristics are employed, and they can be found in some commercial program for the structural analysis based on Finite element method. This approach is based on the concept of localized nonlinearities, which is in particular suitable for the analysis of timber structures because the nonlinear behavior occurs generally only in connections, while beams and columns stay in the range of linear deformations.

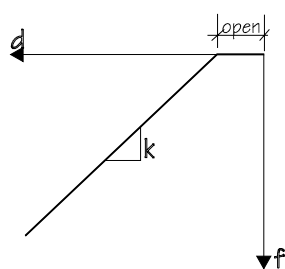
Such nonlinear elements can be found, for instance, in the computer program SAP2000n, and are termed as Nlink elements [7]. Basically, they are springs with a zero length and can be located between the end joint of the structural elements or between structural elements and supports.

3.1 Nlink elements

In this paper, three types of elements were used: Gap, Hook and Plastic 1. Each element can deform independently in the all six degrees of freedom.

3.1.1 Gap element

The nonlinear force-deformation relationship is given in Fig. 6.



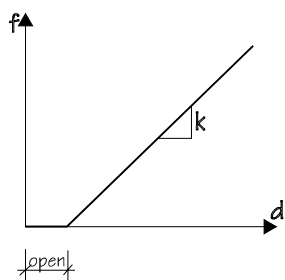
k - spring constant,

open - initial gap opening, which must be zero or positive.

Fig. 6. Gap element - force-deformation relationship

3.1.2 Hook element

The nonlinear force-deformation relationship is given in Fig. 7.



k - spring constant,

open - initial hook opening, which must be zero or positive.

Fig. 7. Hook element - force-deformation relationship

3.1.3 Plastic 1 element

The nonlinear force-deformation relationship is given in Fig. 8.

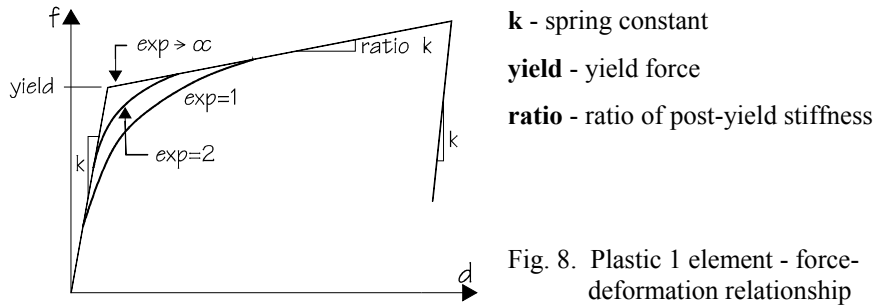


Fig. 8. Plastic 1 element - force-deformation relationship

3.2 Modeling of a connection with Nlink elements [5]

It was found that the actual behavior of a connection between a beam and a column in the timber frame structures determined by the experiments can be modeled with a series of four Nlink elements placed between end joints of a beam and a column and one fictitious joint between the actual joints, as shown in Fig. 9. The fictitious joint is required so that a good reproduction of the actual connection behavior could be achieved.

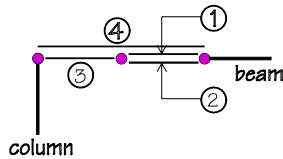


Fig. 9. A joint of a timber frame modeled with four Nlink elements

In this way, a single spring is replaced by four Nlink elements, which for a given degree of freedom give the same force-displacement relationship as a single spring. In Fig. 10 is shown the force-displacement relationship of the model connection under a cyclic loading.

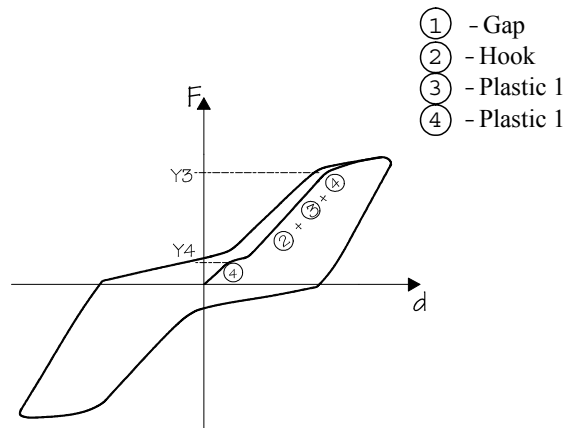


Fig. 10. Hysteretic behavior of the proposed model connection

The above figure showed that the transfer of loading through a connection by the engagement of the element ④, whose initial stiffness is equal to the initial stiffness of the connection, coming mainly from the friction between the connected members. When the loading overcomes the friction (point Y4) the element ④ begins to yield and yields until the opening in Hook element closes, that is until the timber engages connectors. After that the elements ④, ③ and ② are engaged. The beginning of the plastification of the connection coincides with the beginning of yielding of element ③ (point Y3). The model behaves analogously when a loading changes direction, the only difference being that instead of Hook, the Gap element gets engaged. It should be clear that the force-displacement relationship can be easily adjusted to different connections and loading by simply changing the parameters of the Nlink elements.

When modeling connections behavior with Nlink elements, it is possible to take into analysis the deformations of a connection in all directions simultaneously (rotations and translations), and their joint effect on the behavior of a complete timber structure. So, if the deformation characteristics of a connection are known (through experiments), its modeling becomes simple and allows a detailed analysis of a structure for a time-varying loading.

4. SOLUTION OF EQUATION OF MOTION BY TREATING THE COUPLING ELEMENTS AS A PSEUDO-LOADING

This approach is convenient for solving equations of motion which are only partially coupled and in which only a small part of mass-matrix and stiffness matrix elements are not constant [2]. Those parts of mass, stiffness and damping matrices are transferred to the right side of equation and in the farther analysis treated as a pseudo-loading, leaving on the left side of the equations matrices with constant coefficients with which a standard modal analysis can be carried out.

The modes computed from those mass and stiffness matrices are then used to uncouple the transformed equations of motion. The solution of equations of motion is obtained by iteration. If the pseudo-loading is small in comparison to the actual loading, their effect on the response

of a structure is also small and the convergence of iteration process is fast. If the pseudo-forces are significant the iteration is slower.

In the continuation of this presentation only the procedure only for the case when the coupling of generalized coordinates is caused by the nonlinear stiffness matrix will be illustrated. If the response of a structure is determined by step-by-step integration, in each step the parts of the stiffness matrix causing the coupling of the equations of motion must be transferred to the right-hand side of the equations. This requires that the time increments selected should be small enough, so that the variation not only of actual loading, but also of pseudo-loading could be considered linear. In each time step the modal displacements are determined assuming linear behavior of a structure and linear variation of loading and pseudo-loading, carrying out iteration until the modal forces on the left-hand side of the equations are set in equilibrium with the forces on the right-hand side within the specified accuracy.

4.1. Change of stiffness

The change in stiffness in the analysis of a structure is defined through the difference between the original linear stiffness and the stiffness at the achieved level of deformations. Then, the original elastic stiffness matrix is used to determine the Eigen modes, while the difference is in each step treated as pseudo-forces. In Fig. 11 this concept is illustrated for only one degree of freedom.

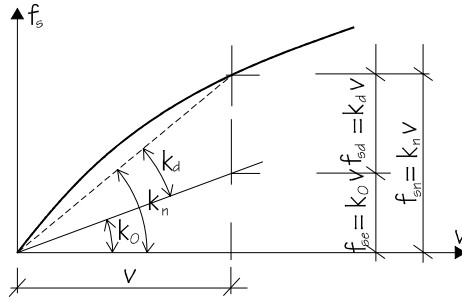


Fig. 11. Definition of original stiffness and pseudo-forces

As the original stiffness, an arbitrary stiffness k_0 is assumed, while the nonlinear stiffness is defined by an average slope k_n . The nonlinear force is then expressed as

$$f_{sn}(t) = k_n \cdot v(t) \quad (16)$$

However, in the procedure with pseudo-loading, this force must be expressed through the difference in respect to elastic force, that is

$$f_{sn} = f_{se} + f_{sd} \quad (17)$$

where

$$f_{se} = k_0 v \quad \text{and} \quad f_{sd} = k_d v$$

and

$$f_{sn}(t) = (k_0 + k_d) \cdot v(t) \quad (18)$$

When this approach is applied to a multi-degree-of-freedom system, the vector of nonlinear forces is expressed by the following equation

$$\{f_{sn}(t)\} = ([k_0] + [k_d]) \cdot \{v(t)\} \quad (19)$$

The equations of motion can be then written as follows

$$[m]\{\ddot{v}(t)\} + [c]\{\dot{v}(t)\} + [k_0]\{v(t)\} = \{p(t)\} - [k_d]\{v(t)\} \quad (20)$$

When the nonlinear change in stiffness is transferred to the right-hand side of the equations of motion, one obtains

$$[m]\{\ddot{v}(t)\} + [c]\{\dot{v}(t)\} + [k_0]\{v(t)\} = \{p(t)\} - [k_d]\{v(t)\} \quad (21)$$

The left-hand side is now standard formulation of the linear response of a system, while the nonlinear changes of stiffness appear on the right-hand side as pseudo-forces.

The nonlinear changes of stiffness $[k_d]$ are, of course, dependent on the displacement $\{v\}$. In the course of solving the equations of motion $[k_d]$ is being determined from the stress-strain diagram of a given material, or from the force-displacement relationship of a connection in the case of timber structure.

The equation (21) can be solved by a direct step-by-step integration, but it is much more efficient to uncouple the equation by means of the natural modes of the system defined by the left-hand side of the equation, and then to integrate them using the limited number of modes, that is $\{v(t)\} = [\Phi]\{Y(t)\}$. Using this transformation, the Eq. (21) obtains the following well known form:

$$[M]\{\ddot{Y}(t)\} + [C]\{\dot{Y}(t)\} + [K]\{Y(t)\} = \{P(t)\} - [F_{sd}(t)] \quad (22)$$

If the system is proportionally damped, the equation (22) is uncoupled and the integration is actually carried out on the equations

$$\ddot{Y}_n(t) + 2\xi_n\omega_n\dot{Y}_n(t) + \omega_n^2Y_n(t) = \{\phi_n\}^T\{p(t)\} - \sum_{p=1}^m K_{d_{np}}Y_p(t) \quad (23)$$

where $K_{d_{np}} = \{\phi_n\}^T[k_d]\{\phi_p\}$ are coefficients that couple generalized (modal) coordinates. For this reason the modal pseudo-forces are functions of all modal coordinates, that is

$$F_{sd_n}(t) = \sum_{p=1}^m K_{d_{np}}Y_p(t).$$

The equilibrium in one time step for one mode "n" in the "k" cycle of iteration is defined by the expression

$$\ddot{Y}_n^{(k)} + 2\xi_n\omega_n\dot{Y}_n^{(k)} + \omega_n^2Y_n^{(k)} = \{\phi_n\}^T\{p\} - \sum_{p=1}^m K_{d_{np}}Y_p^{(k-1)} \quad (24)$$

In each cycle of iteration the equations are integrated and the displacement and velocity at the end of a time step are determined, assuming that both, the modal loading $P_n(t)$ and the pseudo-force $F_{sdn}(t)$ vary linearly within the time increment.

4.2. Solution of equations of motion using Ritz-vectors

The investigations have shown that a dynamic analysis based on load-dependent Ritz-vectors are more accurate than an analysis carried out with the same number of natural modes [2]. The reason is that the Ritz-vectors are derived taking into account the actual distribution of a dynamic loading, while the natural modes depends only on the physical characteristics of a system.

The basic steps in deriving each Ritz vector are:

- solution of a system of linear equations of equilibrium in order to determine the deformation of a system due to inertial forces corresponding to the previously defined vector,
- application of Gram-Schmidt procedure to make the derived vector orthogonal to all previously derived vectors with respect to mass of the system,
- normalization of the vector to obtain the unit modal mass.

The first Ritz vector is derived as the vector of static deformation due to gravitational loading of a system.

4.3. Quasi-static nonlinear analysis

Response of a structure in a natural made in each time increment can be divided in two parts:

- response to the loading proportional to the modal loading, and
- transient response which is oscillatory and depends on the displacement and velocity at the beginning of the time increment.

If the natural modes have high frequency (short periods) the inertial effects during time increment are small and can be neglected. Therefore, by a nonlinear time dependent analysis of a timber structures.

5. 5. COMPARISON OF THE EXPERIMENTALLY DETERMINED AND MODELED WITH NLLINK ELEMENTS BEHAVIOR OF THE CONNECTION [5]

5.1 Under dynamic loading

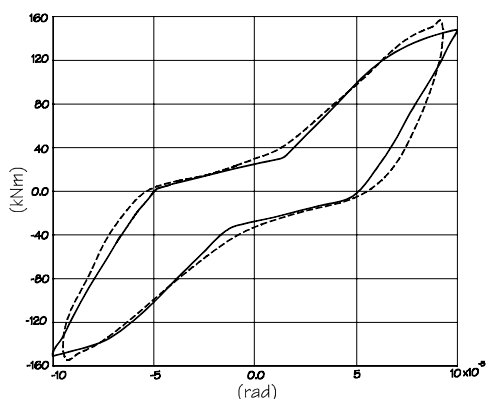


Fig. 12. Moment-rotation relationship in the joint of a frame for one cycle of the cyclic loading

5.2 Under static loading

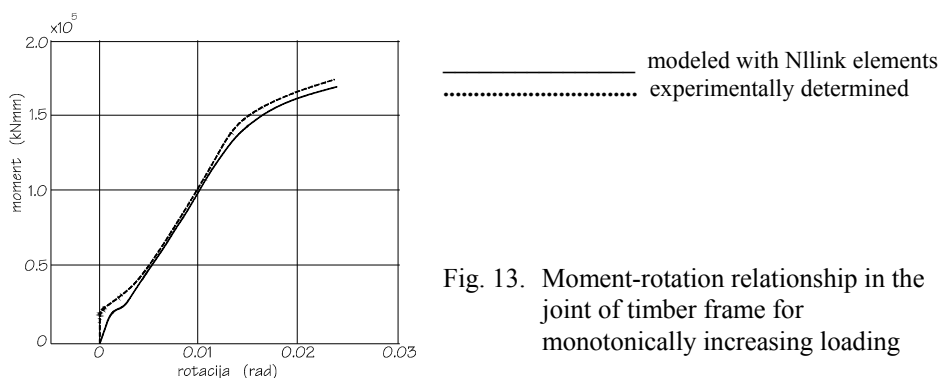


Fig. 13. Moment-rotation relationship in the joint of timber frame for monotonically increasing loading

6. EFFECTS OF CONNECTION DEFORMABILITY IN A TIMBER FRAME FOR STATIC LOADING [5]

The developed methodology has primarily been applied to the analysis of the certain type of timber frame structures whose results of tests of connection behavior under static loading were available to the author.

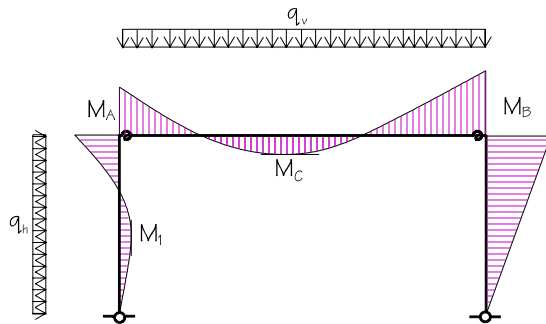


Fig. 14. Loading and bending moments in a timber frame for the monotonically increasing static loading

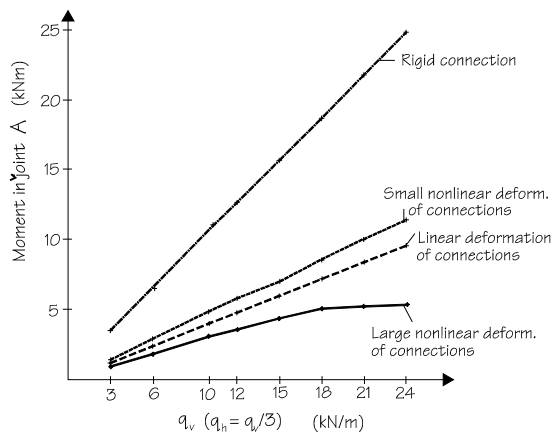


Fig. 15. Dependence of the bending moment in joint A on the monotonic increase of loading shown in previous figure

In terms of simple frame, it has been established that the distribution of section forces (on the first place bending moments) does not significantly differ from the distribution resulting from the estimation with flexible connection behavior defined with current regulations, applied both within the country and elsewhere (Eurocode 5). These differences are, however, significant in the case of frames of a more complex geometry. Final analysis, however, require an overall parameter study, which would be the subject of a separate research.

7. EFFECTS OF CONNECTION DEFORMABILITY
ON THE DYNAMIC RESPONSE OF A TIMBER FRAME [5]

The developed numerical model and method of analysis has been also used to analyze the behavior of timber frames under dynamic loading.

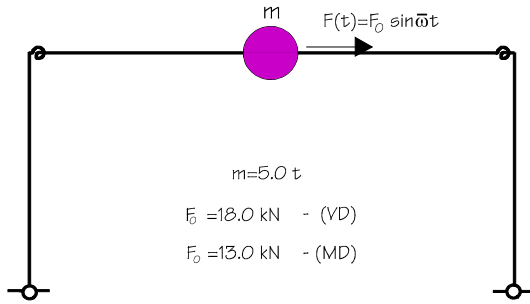


Fig. 16. Dynamic model of the frame with harmonic loading

Determination of the relative damping from the harmonic response curves of the frame with joints deforming in the range of large nonlinear deformations and with the assumption that joints deform linearly.

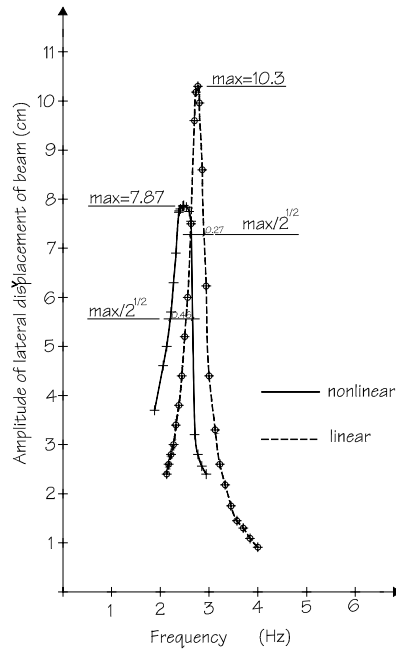


Fig. 17. Response spectra of the frame with linearly and nonlinearly deformable connections to harmonic loading

It has been found that the coefficient of the relative damping suggested in literature and indirectly introduced even in Eurocode 8 has been overestimated. The reason for this lies in the fact that the deformation of connections, even in the case of the strongest seismic load, remains within the range of small inelastic deformations, so that the dissipation of energy is lower than the one found by experiments with large connection deformations.

8. CONCLUSION

It has been found that the coefficient of the relative damping suggested in literature and indirectly introduced even in Eurocode 8 has been overestimated. The reason for this lies in the fact that the deformation of connections, even in the case of the strongest seismic load, remains within the range of small inelastic deformations, so that the dissipation of energy is lower than the one found by experiments with large connection deformations.

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ANALIZA RAMOVA OD DRVETA SA LOKALNOM NELINEARNOŠĆU

Esad Mešić

U ovom radu je izneta metodologija nelinearne analize ramova od drveta sa pomerljivim vezama u čvorovima. Metodologija je bazirana na rezultatima eksperimenta veza usled statičkog i cikličkog opterećenja. Primenjeni numerički metod je baziran na konceptu lokalne nelinearnosti kojom se potvrđuju eksperimentalno određeno ponašanje veza u analizi ramovskih konstrukcija. Metodologija je primenjiva i na druge tipove drvenih konstrukcija.