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SOME RELEVANT ASPECTS OF FOOTBRIDGE VIBRATIONS

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Abstract. *Considering the contemporary structural materials that are becoming more resistant, having higher strength to weight ratio, and the fact that live load of footbridges is low, the design based on static analysis only, respecting ultimate limit states requirements, leads to slender bridge structures for pedestrian and cycle track use. As a consequence, stiffness and masses decrease, facing lively, easy to excite structures, with smaller natural frequencies. The excitation of a footbridge by a pedestrian passing over it can be unpleasant for a person walking or standing on the bridge, but usually not destructive for the structure itself. Recent experiences regarding dynamic behavior of slender footbridges have especially shown that vibration serviceability limit states are very important requirements in any structural design. We are presenting a general algorithm for analytical testing of dynamic parameters of structures, calculation of deflection, thus speed and acceleration of superstructure under human-induced excitation, as predicted by Eurocode, British and Canadian standards in use, since no Yugoslav code deals with the problem. The evaluated system is a footbridge in a system of a simply supported concrete girder. The presented model is used to show correspondence of results, obtained by the algorithm, with the results obtained using the simplified methods suggested by the Codes of Practice, since the latter exists only for certain structural systems.*

Key words: *Footbridge, human-induced excitation, forced vibrations, serviceability limit states.*

1. INTRODUCTION

The design of up-to-date bridges is a challenge, having in mind a trend of building elegant structures. Considering footbridges, regarding the fact that their live load is relatively low and that modern materials enable larger bearing power, by design based on

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static analyses, controlling ultimate limit states, one gets the structures with small stiffness and mass, which, therefore, are easy to excite. The excitations caused by walk can be classified as a question of user's comfort, thus serviceability, regarding sensibility of human body to vibrations, not large enough to cause structural damage, but large enough to cause walking disturbance. Therefore, it is especially important to fulfill the requirements of serviceability limit states of structural elements, in particular regarding vibrations induced by pedestrians, in vertical, as well as in horizontal transversal direction. The actual Yugoslav Codes do not treat this phenomenon, so that this analysis is proceeded according to some current codes in this field: Eurocode, British Standards, and Canadian standards.

In principle, there are two approaches in avoiding excessive vibrations of structures, caused by pedestrians:

– tuning fundamental natural frequency of the structure,
– calculation of forced vibrations, i.e. limitation of vertical acceleration of any part of superstructure under dynamic load caused by walking pedestrian.

Here, as a prerequisite for previously noted procedures, one can raise the following questions:

– establishing a relevant mathematical model of exciting force due to walking,
– defining acceptability limit for vibrations level.

1.1. Modeling of dynamic force due to walking

Observing the walk of a pedestrian, one can conclude that every step can be treated as one impulse, and series of steps as impulses along the way and shifted in time (Fig. 1). Therefore, load induced by walking can be assumed as sum of loads caused by continual steps, which further can be simulated with moving pulsating point load. With accurate assumptions (see [5]) that the load applied by every step is approximately of the same value, and that the time needed for transmission of pressure is constant for given walking pace, one can assume that this load is of periodic nature.

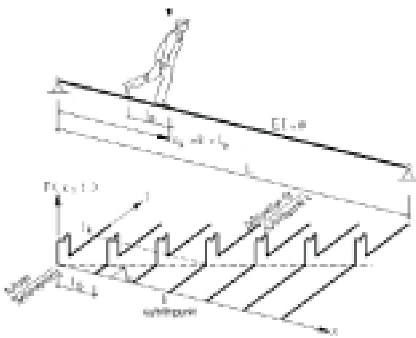


Fig. 1. Walking pedestrian load – change along trace and in time

Consequently, vertical load due to walking, as a sum of static components presenting weight G of pedestrian, with additional periodic components, can be presented in the form of Fourier series:

$$F_p(t) = G + G\cdot\alpha_1\cdot\sin(2\cdot\pi\cdot f_p\cdot t) + G\cdot\alpha_2\cdot\sin(4\cdot\pi\cdot f_p+\varphi_2) + \dots \tag{1.1}$$

where: t - time, f_p – walking frequency, α_n - so known factor of dynamic load for n^{th} load harmonics, φ_n – phase angle of n^{th} harmonic in relation to the first harmonic. One can get the factor of dynamic load usually from the analysis of response signal in the frequency domain, isolating the contribution of each harmonic.

The research has shown that during normal walk the pedestrians take approximately two steps in second, what has as a result the frequency of the first Fourier harmonic of about 2 Hz (see [2]). Therefore, theoretically speaking, the footbridges with natural frequency close to the first (≈ 2 Hz) or higher (≈ 4 Hz, ≈ 6 Hz, etc.) excitation harmonic's frequency, can experience too high vibrations under normal conditions of exploitation, assuming sufficient energy in particular harmonic. Although possible, it is generally assumed that other kinds of excitations, like running or jumping, are not occurring frequently enough to be taken into account during calculation of these structures.

In Eurocode, British, and Canadian standards, the load applied by pedestrian due to walking is described as harmonic pulsating point load $F(t)$ moving over the bridge with constant velocity. This load has a frequency f_p, which has to coincide the first natural frequency f_0 of unloaded bridge. The force $F(t)$, in Newton, is given as

$$F(t) = 180\cdot\sin(2\cdot\pi\cdot f_p\cdot t), \tag{1.2}$$

where: t – time. In equation 1.2 factor 180 (N) is amplitude of first harmonic of the load, i.e. product of pedestrian weight (assuming value 700 N) and dynamic factor $\alpha = 0.257$. The expression 1.2 has to be applied to footbridges which have fundamental natural frequency lower than 5 Hz (frequency range of first two harmonics). The comparison of expressions 1.1 and 1.2 shows that noted standards are taking in considerations only first dominant harmonic of pedestrian load.

1.2. Problem acceptability limit for vibrations level

The criterion of acceptability of vibrations level is more a problem of psychological effects on humans (mechanical, optical, acoustical effects), i.e. problem of serviceability, than a problem of exceeding ultimate limit states of structure (stresses, strains, fatigue). As such one, this criterion is rather difficult to define precisely. Because of that, different standards propose different values for acceptable level of vertical or horizontal vibrations (see [5]). Some aspects that have to be taken in consideration in criterions application are: frequency of appearance of certain vibrations (frequent, exceptional, rare appearance), desirable level of comfort, expected tolerance level of user.

Basically, the criterion of vibration acceptability is function of frequency and displacements, usually expressed in acceleration units. In case of vertical vibrations, accelerations from 0.5 do 1 m/s^2, i.e. 5-10 % gravitation acceleration of Earth, g, is acceptable. The humans are more sensitive to horizontal vibrations, so that the acceptable accelerations are of order 1 – 2 % g. It is also to be mention that amplitudes larger than 10 mm in vertical direction and 2 mm in horizontal direction can cause the phenomenon known as «lock-in» effect – synchronization in walk of certain number of pedestrians, having as a result significant excitation of structure.

Acceptability limit of vibrations adopted in Eurocode and British standard (BSI 1978), based on many experiments, is defined by the following expression:

$$a_{\lim} = 0.5\sqrt{f_v} \tag{1.3}$$

where: f_v (Hz) –frequency of structure, $a_{\lim}$ (m/s^2) – maximal acceleration. The standards cite that one can assume that f_v is fundamental natural frequency f_0 of the structure.

2. System vibrations

2.1. Natural frequency tuning

The approach accepted by European Commission for Concrete (Comité Euro-International du Beton - CEB 1993) considering problems of serviceability regarding vibrations is avoiding critical frequency range: the avoidance of vertical natural frequency of footbridges between 1.6 and 2.4 Hz (range of normal distribution of walk frequency) and between 3.5 and 4.5 Hz (range of second harmonic) is recommended.

Deriving natural frequency of structure and functions of oscillation modes is the very first step in algorithm. This procedure is very known in Structural Dynamics. Natural frequencies, i.e. eigenfrequencies are obtained as eigenvalues of matrix of frequency equation, and oscillation forms as their eigenfunctions, by means of programming procedure written in programming package - system Mathematica.

2.2. Dynamic response analysis

More precise approach to verify the state of serviceability of footbridges regarding vibrations is by means of dynamic response analysis of structure subjected to designed load (problem of forced vibrations), i.e. load causing the largest response, exciting the structure close to its fundamental natural frequency f_0 in vertical direction. As already mentioned in 1.1, it is of general opinion that normal walk, as considered in actual standards and described by expression 1.2, is the most appropriate form of excitation in order to find out the actual state of serviceability regarding the vibrations of footbridges. The response to dynamic load depends on factors as stiffness and damping of structure, and relation between frequency of applied force and fundamental natural frequency of structure.

With a possibility to be measured easily, the acceleration became the most width accepted parameter for check-out of vibration level. As a general approach, the acceleration has to be calculated in the design phase of structure or to be measured on the real bridge.

Forced vibrations of beam systems

In this part of the paper the theoretical background of the problem of forced vibrations is presented. Beam system excited by moving concentrated deterministic force is treated, suggesting the algorithm for its resolution, involving some appropriate assumptions (see [3]).

2.2.1. Theoretical background

Consider vertical vibrations of beam of length l (Fig. 2), with constant bending stiffness EI and constant mass μ per unit length. With corresponding assumptions, this vibrations can be described with the following differential equation 2.2.1.1:

$$\mu \frac{\partial^2 \upsilon(x,t)}{\partial t^2} + Fd\left(\frac{\partial \upsilon}{\partial t}, \frac{\partial^5 \upsilon}{\partial^4 x \partial t}\right) + EI \frac{\partial^4 \upsilon(x,t)}{\partial^4 x} = Q(x,t) \qquad (2.2.1.1)$$

With the boundary conditions, respecting the type of support:

$$\upsilon(x,0) = \upsilon_0(x); \quad \dot{\upsilon}(x,0) = \dot{\upsilon}_0(x) \qquad (2.2.1.2)$$

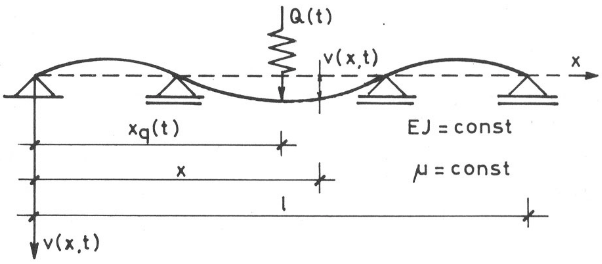


Fig. 2. Mathematical model

In equation 2.2.1.1 υ(x,t) is deflection at the point x in time t; *Q(x,t)* is deterministic excitation force; Fd is damping according to 2.2.1.3, as follows.

$$Fd\left(\frac{\partial \upsilon}{\partial t}, \frac{\partial^5 \upsilon}{\partial x^4 \partial t}\right) = \begin{cases} 2\beta\mu \frac{\partial \upsilon(x,t)}{\partial (t)} \quad \text{or} = EI\alpha \frac{d}{dt}\left(\frac{\partial^4 \upsilon(x,t)}{\partial x^4}\right) \quad \text{or} = 2\beta\mu \frac{\partial \upsilon(x,t)}{\partial (t)} + EI\alpha \frac{d}{dt}\left(\frac{\partial^4 \upsilon(x,t)}{\partial x^4}\right) \end{cases} \qquad (2.2.1.3)$$

Constants α and β are to be determinate experimentally, and they usually represent the weak point in the analysis before the building process, since not being easy predictable.

Applying modal analysis, $\upsilon(x,t)$ is modeled as series of natural mode functions $V_r(x)$, according to relations 2.2.1.4:

$$\upsilon(x,t) = \sum_{r=1}^{\infty} \eta_r(t) V_r(x) \;\ldots\text{(a)} \quad \eta_r(t) = \frac{1}{H_r}\int_0^l \upsilon(x,t) V_r(x) dx \;\ldots\text{(b)}, \quad H_r = \int_0^l V_r^2(x) dx \qquad (2.2.1.4)$$

Equation 2.2.1.1 can be presented as a system of *r*-independent equations of the following form:

$$\ddot{\eta}_r(t) + 2\xi_r \omega_r \dot{\eta}_r(t) + \omega_r^2 \eta_r(t) = \frac{1}{M_r} F_r(t) \qquad (r = 1,2,\ldots,\infty) \qquad (2.2.1.5)$$

where: $\eta_r(t)$ is the principal coordinate of mode *r*, ω_r is natural circular frequency of mode *r*, M_r and $F_r(t)$ are generalized mass of mode *r*

$$M_r = \mu H_r = \mu \int_0^l V_r^2(x)dx \tag{2.2.1.6}$$

$F_r(t)$ generalized force of mode r

$$F_r(t) = \int_0^l Q(x,t)V_r(x)dx \tag{2.2.1.7}$$

2.2.2. Problem solution

$$\upsilon(x,t) = \sum_{r=1}^{\infty} \eta_r(t)V_r(x)$$

The basic solution is the deflection function $\upsilon(x,t)$, expressed as in 2.2.1.4(a), and as its first and second derivative, velocity and acceleration functions can be obtained, for the optional exciting deterministic force $F(t)$ moving across the span at a constant speed c.

In order to simplify the solution, talking about slender structure, it is also acceptable to negligee the damping, since it is relatively small. In that case $\eta_r(t)$ can be found as the solution of differential equation:

$$\ddot{\eta}_r(t) + \omega_r^2 \eta_r(t) = \frac{1}{M_r} F_r(t) \quad (r = 1,2,...,\infty) \tag{2.2.2.1}$$

where: M_r is generalized mass of mode r, according to 2.2.1.6, $F_r(t)$ generalized force of mode r, determinate in 2.2.1.7, and $Q(x,t)$ is excitation force. For the observed pulsating point load $F(t)$ moving across the span of the structure at the constant speed c, exciting function $Q(x,t)$ can be presented with the relation:

$$Q(x,t) = F(t) \cdot \delta(x - ct) \tag{2.2.2.3}$$

using Dirac delta function, δ, which is determinate as:

$$\delta(x-ct) = \begin{cases} 0 \text{ za } x \neq ct \\ 1 \text{ za } x = ct \end{cases}$$

$$\int_0^l \delta(x-ct)dx = 1 \quad 0 \leq ct \leq \infty \tag{2.2.2.4}$$

$$\int_0^l F(x) \cdot \delta(x-ct)dx = F(ct) \quad 0 \leq ct \leq \infty$$

implying:

$$F_r(t) = \int_0^l F(t) \cdot \delta(x-ct) \cdot V_r(x)dx \quad \Rightarrow \quad F_r(t) = F(t) \cdot V_r(ct) \tag{2.2.2.5}$$

According to Eurocode the exciting load representing pedestrian is pulsating point load:

$$F(t) = F_0 \cdot \sin(2\pi f_p t) \text{ i.e. } F(t) = F_0 \cdot \sin(\nu t) \tag{2.2.2.6}$$

moving over the span with the speed of $c = 0,9 \cdot f_0$.

In the (2.2.2.6), F_0 is amplitude of the exciting force, $F_0 = \alpha \cdot G = 0.257 \cdot 700 = 180$ N, f_p is load frequency, assumed to equal to f_0 the first natural frequency of the system.

In British and Canadian Codes there is a simplified method for deriving maximal acceleration, applicative for single span or two-or-three-span continuous symmetric superstructures, of constant cross section. This calculation method is obtained form the numerical studies, including the application of equation 1.2 in respect of varying bridge structure parameters.

3. DESIGN EXAMPLE

Single span concrete footbridge

The evaluated system is a single span concrete footbridge (shown in figure 3), 15.0 m in span, constant box cross-section, and constant weight of 33.0 kN/m. The natural frequency of the system is f_0 = 3.75 Hz. According to the fact that structural system of single span bridge may have certain enclosed form solutions for solutions for dynamic properties, the aim of this design example is to show the accordance of the algorithm.

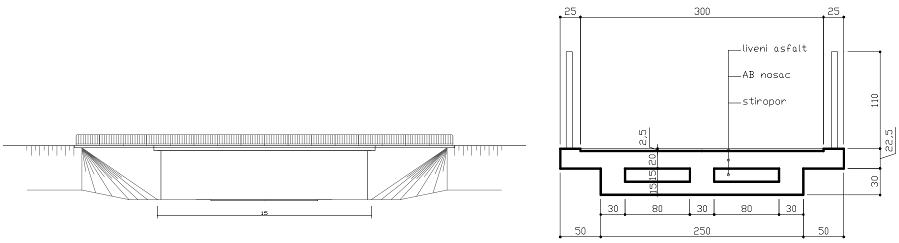
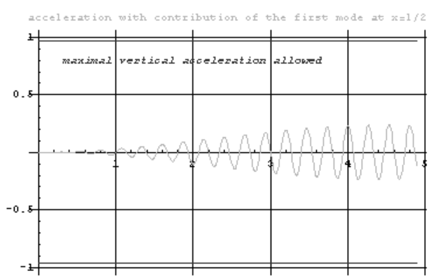
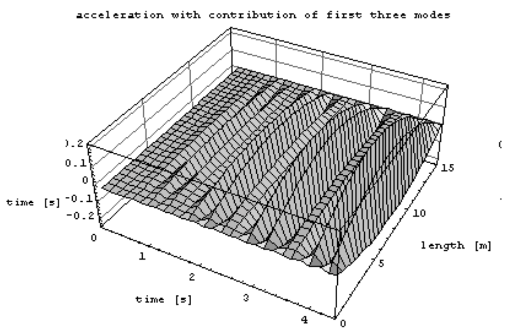


Fig. 3. Appearance and cross-section of the bridge

Dynamic response analysis

Applying the *Mathematica* procedure, written according to algorithm explained in part 2.2.2, the obtained maximal acceleration value is a = 0.22 m/s^2.





Simplified calculation method

According to British Standard BS 5400, Part 2, maximal vertical acceleration should be calculated as: a = $4\pi^2 f_0^2 y_s k\psi$, where f_0 = 3.75 Hz, is fundamental natural frequency, y_s = 0.0000517, is static deflection at the half span, caused by point force of 0.7 kN in the same cross-section, k = 1, is the structural system factor for single span girder, ψ = 0.6, is dynamic response factor for adopted logarithmic damping decrement $\delta = 0.05$ for reinforced concrete structures. The calculated acceleration value is a = 0.172 m/s^2.

The requirement $a < a_{\lim}$,where $a_{\lim}$ is determined according to equation 1.3, should be satisfied.

$$\Rightarrow a = 0.172 \text{ m/s}^2 < a_{\lim} = 0.968 \text{ m/s}^2$$

4. Conclusion

The paper presents algorithm for calculation of important dynamic parameters of pedestrian bridges subjected to human induced loading, allowing to predict serviceability behavior. The authors are aware of assumptions made in this approach, considering them acceptable. The numerical example of the simply supported beam structure shows satisfying accordance of results obtained by this algorithm with the results obtained by using simplified methods presented in relevant Codes of practice.

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NEKI RELEVANTNI ASPEKTI VIBRACIJA PEŠAČKIH MOSTOVA

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Prateći trend izgradnje elegantnih konstrukcija, posebno mostovskih, susrećemo se sa novim izazovima u pogledu projektovanja i proračuna. Kada je reč o pešačkim mostovima vibracije prouzrokovane ljudima mogu se kalsifikovati kao pitanje stanja upotrebljivosti iz razloga što je ljudsko telo vrlo osetljivo na vibracije, a nivo vibracija koji prouzrokuje smetnju pešacima je nedovoljan da prouzrokuje konstrukcijska oštećenja. Imajući u vidu činjenice da je korisno opterećenje pešačkih mostova relativno malo, a da savremeni materijali omogućavaju veću *nosivost, proračunom zasnovanim na isključivo statičkoj analizi, poštujući granična stanja nosivosti, dobijamo konstrukcije male krutosti i mase, koje se, dakle, lako pobuđuju. Stoga je naročito važno ispuniti uslove graničnih stanja upotrebljivosti konstrukcijskih elemenata, posebno granična stanja upotrebljivosti u pogledu vibracija indukovanih pešacima, kako u vertikalnom tako i u bočnom pravcu. Važeći jugoslovenski propisi ne tretiraju ovaj fenomen, te je u radu sprovedena analiza prema nekoliko aktuelnih svetskih standarda za ovu oblast: Evrokod, Britanski standardi i Kanadski standardi. Prikazan je algoritam za određivanje dinamičkih karakteristika konstrukcija i sračunavanje ugiba, odnosno brzine i ubrzanj, tačaka konstrukcije pod pokretnim harmonijskim opterećenjem. Testiran je model pešačkog mosta sistema proste grede, sa ciljem provere tačnost algoritma sobzirom na postojeća približna rešenjia za ovaj sistem u navedenim standardima.*

Ključne reči: *Pešački most, pobudjenje pešacima, prinudne vibracije, granična stanja.*