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DETERMINATION OF INTERSECTING CURVE BETWEEN TWO SURFACES OF REVOLUTION WITH PARALLEL AXES BY USE OF AUXILIARY PLANES AND AUXILIARY SPHERES

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Ratko Obradović

Faculty of Technical Sciences, Institute for Mathematics and Physics in Engineering
Trg Dositeja Obradovića 6, 21121, Novi Sad, Serbia, Serbai and Montenegro
E-mail: obrad_r@uns.ns.ac.yu

Abstract. *In this paper the space intersecting curve between two surfaces of revolution with parallel axes of surfaces have been determined. Two mathematical models for determination of intersecting curve between two surfaces of revolution have been formed: auxiliary planes have been used in the first mathematical model and auxiliary spheres have been used in the second model (Obradović 2000). In the first case each auxiliary plane intersected with each surface of revolution on circle and two points of intersecting curve are obtained as intersecting points between these two circles. In the second case centres of two locks of auxiliary spheres are put on axes of surfaces of revolution (centre of first lock is on axis of the first surface of revolution and centre of second lock is on axis of the second surface of revolution) on same z coordinate (when axes of surfaces of revolution are parallel with z axis of coordinate system). First lock sphere intersects the first surface of revolution on* w_1 *parallels and second lock corresponding sphere intersects the second surface of revolution on* w_2 *circles. It is possible to find a relationship that for selected radius of the first lock sphere can determine the radius of second lock sphere and real points of intersecting curve have been determined by use of these two spheres. The points of intersecting curve between two surfaces of revolution are obtained by intersection between* w_1 *circles from the first surface with* w_2 *circles from the second surface (Obradović 2000).*

Key words: *Surfaces of Revolution, Auxiliary plane, Auxiliary Sphere, Descriptive Geometry, Computer Graphics.*

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1. AUXILIARY PLANES

Surfaces are given by meridians and axes which are in plane Oxz and axis of one surfaces is coincident with z axis of coordinate system (Fig. 1). In this case the lock of auxiliary planes whose pencil line is horizontal at infinity can be used (Obradović 2000), that is all auxiliary planes are parallel with the horizontal plane (plane Oxy). The intersection of surfaces of revolution with horizontal plane on the height $z=z_P[i]$ is observed. Plane intersects the first surface on the circle $p_1[i]$ and the other surface on the circle $p_2[i]$, where

$$p_1[i]: \quad x^2+y^2=R_1^2[i]$$

$$p_2[i]: \quad (x-x_{O_2})^2+y^2=R_2^2[i]$$

where are

$$R_1[i]=x_1[i]; \qquad R_2[i]=x_2[i]-x_{O_2}$$

These circles touch or intersect each other on condition that

$$R_1[i]+R_2[i]\ge x_{O_2}$$

Two unknown coordinates x and y of intersecting points of two horizontal circles can be found on condition that the previous requirement is satisfied and then

$$x_1[i]=x_2[i]=\frac{R_1^2[i]-R_2^2[i]+x_{O_2}^2}{2x_{O_2}}; \quad y_{1,2}[i]=\pm\sqrt{R_1^2[i]-x_1^2[i]}$$

Intersecting points are $K_1[i](x_1[i],y_1[i],z_P[i])$ and $K_2[i](x_2[i],y_2[i],z_P[i])$.

2. AUXILIARY SPHERES

In this case the same solution can be found by using of two locks of auxiliary spheres, with centres O_1 and O_2, which are respectively on axes of surfaces of revolution and these centres are on the same height z. For observed circles (parallels) on height $z=z_P[i]$ radius of sphere of the first surface is given by

$$R_{L_1}^2[i]=R_1^2[i]+z_P^2[i]=x_1^2[i]+z_P^2[i]$$

Radius of the other sphere is

$$R_{L_2}^2[i]=R_2^2[i]+z_P^2[i]=(x_2[i]-x_{O_2})^2+z_P^2[i]$$

Last two equations give a relation which describes a relation between radiuses of two spheres from the different locks (Obradović 2000):

$$R_{L_1}^2=R_{L_2}^2+x_1^2[i]-(x_2[i]-x_{O_2})^2 \tag{1}$$

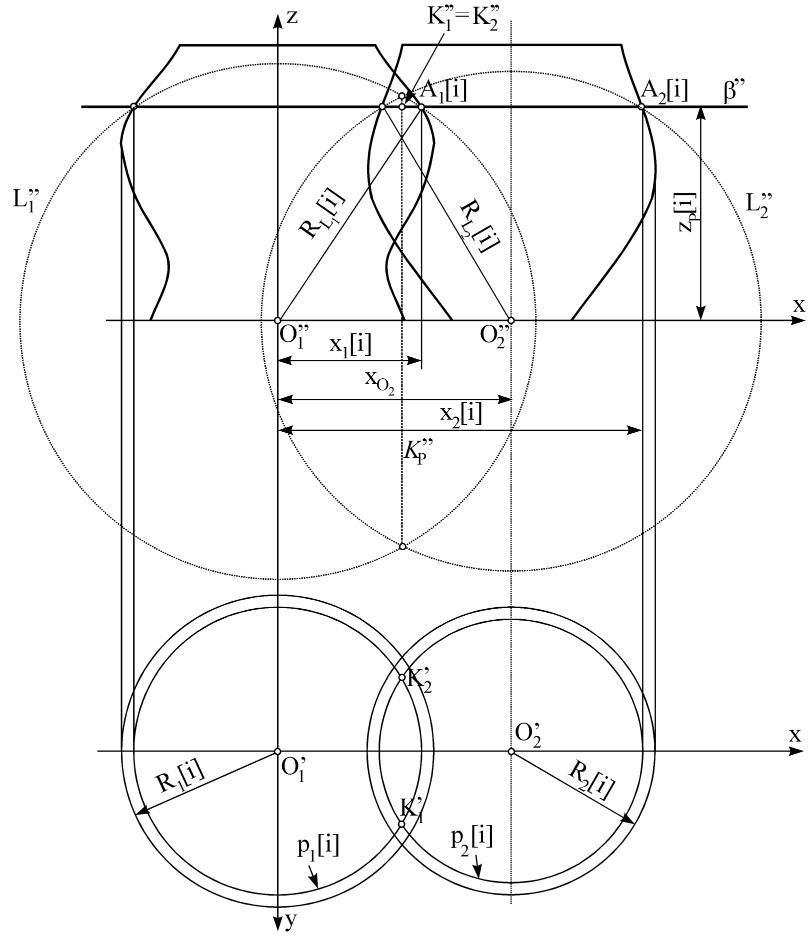

Fig. 1. Parallel axes

On condition that previous requirement is satisfied, the spheres intersect surfaces on parallels which are on same height *z* and if they (parallels) intersect these points will be points of intersecting curve of two surfaces. A pair of spheres is given by the equations

$$x^2 + y^2 + z^2 = R_{L_1}^2[i]$$

$$(x - x_{O_2})^2 + y^2 + z^2 = R_{L_2}^2[i]$$

These two spheres intersect on the profile parallel whose *x* coordinate is yielded by a combination of the last two equations

$$x_1[i] = x_2[i] = \frac{R_{L_1}^2[i] - R_{L_2}^2[i] + x_{O_2}^2}{2x_{O_2}}$$

In intersection of the profil parallel with the horizontal plane $z = z_P[i]$ y coordinates of points K_1 and K_2 are determined

$$y_{1,2}[i] = \pm\sqrt{R_{L_1}^2 - x_1^2[i] - z_P^2[i]}$$

Thus, when surfaces of revolution with parallel axes of revolution are observed, then intersecting curve can be obtained by using horizontal auxiliary planes or by using two locks of auxiliary spheres where the centre of one lock is on axis of one surface and centre of the other lock is on the axis of the other surface while both centres are at same height. Radiuses of each pair of spheres from two locks are different but in defined relation (1), i.e. for selected radius of sphere from one lock, the radius of sphere from the other lock is known and these two spheres give real points of intersecting curve. Procedure for using of two locks of auxiliary spheres is as follows (Obradović 2000):

1. To select parameter from interval $i \in [i_0, i_{max}]$;
2. From parametric equations of both meridians are calculated x and z coordinates of points A_1 and A_2 on these meridians for selected value of parameter i;
3. To calculate values of radiuses R_{L_1}, R_{L_2};
4. To determine x coordinate of profil parallel in intersection of two spheres;
5. To determine y coordinates of intersecting points in intersection of profil circle with horizontal plane $z_1[i] = z_2[i]$.

In Fig. 2 in a pair of projections the intersection between two surfaces of revolution is shown, with parallel axes (curve is presented by 348 points), and in Fig. 3 the solution of same problem is presented in 3D for parallel projecting lines.

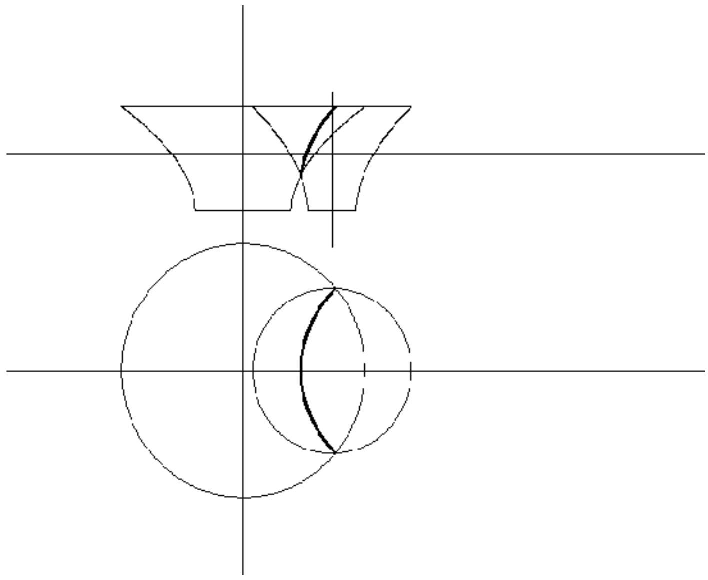

Fig. 2. Two orthographic projections of an intersecting space curve of two surfaces of revolution with parallel axes

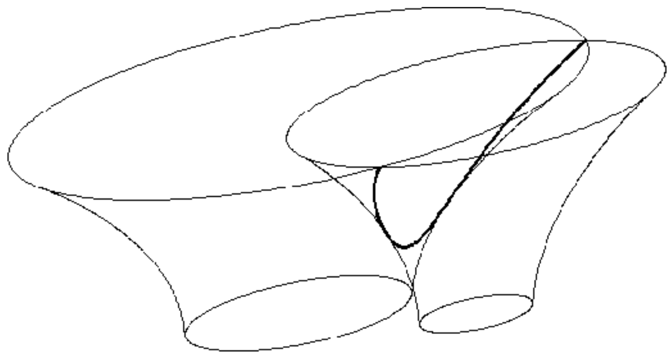

Fig. 3. Intersection between two surfaces of revolution
whose axes are parallel in case when projecting lines are parallel

3. CONCLUSION

The quality of solution which is based on the descriptive geometrical approach is seen in using the intersection of original surfaces of revolution and not by using their approximate surfaces, i.e. patches where each surface is approximated by the system of triangles or plane quadrangle or in any other way known in computer graphics (Obradović 2000). In this case it is possible to make correct determination of coordinates of points of intersecting curve of two surfaces of revolution. We can emphasize an important fact that we avoided the usual problem of order in connecting of intersecting points because in this approach the intersecting curve of two surfaces of revolution is represented by a group of three-dimensional points. The surface is presented by a contour line and in that way the observed surface has a realistic appearance.

In the future research there we could try to determine curve visibility, blocking of one surface by the other one, and the determination of shadowing of one surface on the other one.

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ODREĐIVANJE PRESEKA DVEJU ROTACIONIH POVRŠI ČIJE SU OSE PARALELNE KORIŠĆENJEM POMOĆNIH RAVNI I POMOĆNIH LOPTI

Ratko Obradović

U ovom radu određena je prostorna presečna kriva dveju rotacionih površi čije su ose rotacija paralelne. Formirana su dva matematička modela za određivanje presečne krive dveju rotacionih površi, u prvom su korišćene pomoćne ravni, a u drugom parovi pomoćnih lopti (Obradović 2000). Kod prvog matematičkog modela svaka pomoćna ravan je sekla svaku rotacionu površ po paraleli, a u preseku tih parova paralela za posmatranu pomoćnu ravan dobijene su dve tačke presečne krive. Kod matematičkog modela koji je baziran na korišćenju pomoćnih lopti, centri dva pramena pomoćnih lopti postavljeni su na osama rotacionih površi (centar prvog pramena je na osi prve rotacione površi, a centar drugog pramena na osi druge rotacione površi) na istoj z visini (kada su ose rotacionih površi paralelne sa z osom koordinatnog sistema). Lopta iz prvog pramena seče prvu rotacionu površ po w_1 *paralela, a odgovarajuća lopta iz drugog pramena seče drugu rotacionu površ po* w_2 *paralela. Moguće je uspostaviti relaciju kojom se za izabrani poluprečnik lopte iz prvog pramena može odrediti poluprečnik lopte iz drugog pramena, a da te dve lopte zajedno dovode do realnih tačaka presečne krive. Tačke presečne krive dveju rotacionih površi dobijaju se u preseku* w_1 *paralela prve površi sa* w_2 *paralela druge površi (Obradović 2000).*

Ključne reči: *rotaciona površ, pomoćna ravan, pomoćna lopta, deskriptivna geometrija, kompjuterska grafika.*