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MODELING CONOID SURFACES

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Abstract. *In this paper we consider conoid surfaces as frequently used surfaces in building techniques, mainly as daring roof structures. Different types of conoids are presented using the programme package Mathematica.*
We describe the generation of conoids and by means of parametric representation we get their graphics. The geometric approach offers a wide range of possibilities in the research of complicated spatial surface systems.

Key words: *Catalana surface, conoid, hyperbolic paraboloid, helicoid, Plucker's conoid, Mathematica.*

1. INTRODUCTION

Ruled surfaces and, specially, conoid surfaces are widely applied in civil engineering and architecture. The simplicity of production and very rich spectrum of shapes are the main reasons for application as roof structures but also for some other functions. Warped surfaces have a great advantage for shell structures because they may be formed from straight form boards even though they are surfaces of double curvature. This type of shell structure can be built to what appears to be the ultimate in lightness of construction, minimum reinforcing and ease of moving forms.

This is a reason for having a need of describing and investigating completely in different ways this kind of surfaces.

The main aim of this work is to point out the use of programme package *Mathematica* for investigating conoid surfaces.

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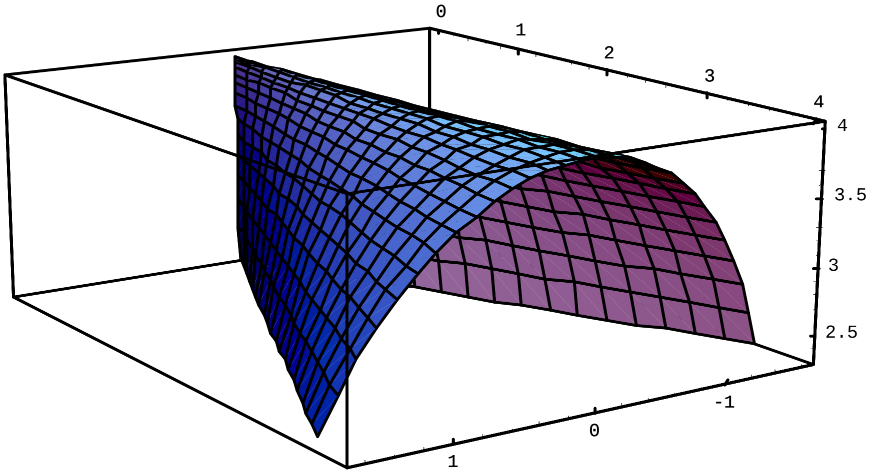

Fig. 1

Ruled surface is a surface which can be swept out by a moving a line in space and therefore has a parameterization of the form

$$\vec{r}(u,v)=\vec{R}(u)+v\vec{l}(u), u,v\in R \tag{1}$$

where $\vec{R}=\vec{R}(u)$ is called the directrix (also called the base curve) and the straight lines $\vec{l}(u)$ themselves are called rulings. The rulings of a ruled surface are asymptotic curves. Gaussian curvature on a ruled regular surface is everywhere non-positive.

Ruled surfaces were investigated first by Monge (in his Applications [1]). Catalana [2] pointed the class of ruled surfaces parallel to a constant plane, after him called **Catalana surfaces.** In case of conoid rulings intersect constant line –axes of conoid.

A ruled surface is called a **conoid** if it can be generated by moving a straight line intersecting a fixed straight line-axis of conoid. If rulings are $\vec{l}(u)=\vec{i}\cos u+\vec{j}\sin u$ the conoid is

$$\vec{r}(u,v)=v(\vec{i}\cos u+\vec{j}\sin u)+f(u)\vec{q} \tag{2}$$

where $\vec{q}$ is a vector of the axis of conoid. Conoid is right if the axis is perpendicular to the plane and to the rulings. Taking the perpendicular plane as the xy-plane and the line to be the x-axis gives the parametric equations

$$\vec{r}(u,v)=v(\vec{i}\cos u+\vec{j}\sin u)+f(u)\vec{k}. \tag{3}$$

2. THE MAIN TYPES OF CONOID SURFACES

The main types of conoid surfaces are hyperbolic paraboloid, helicoid, Plucker's conoid, and conical edge of Wallis.

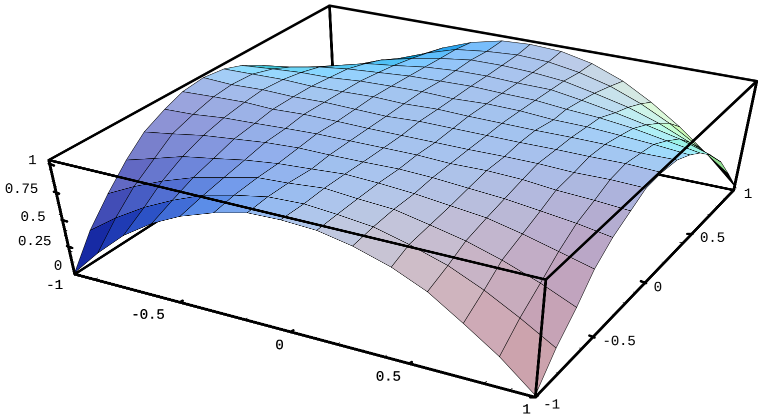

Fig. 2

The hyperbolic paraboloid is the simplest conoid. The line moving parallel to a fixed plane intersects two axes. Geometric and constructive characteristics of hyperbolic paraboloid were considered in [4] and [5].

The helicoid is generated by a line attached orthogonal to an axis such that line moves along and also rotates, both at constant speed. It is a minimal surface having a helix as its boundary. It is the only ruled minimal surface other than the plane (Catalan [2] 1842). For many years, the helicoid remained the only known example of a complete embedded minimal surface of finite topology with infinite curvature. However, in 1992 a second example, known as Hoffman's minimal surface and consisting of a helicoid with a hole, was discovered (Sci. News 1992). The helicoid is the only non-rotary surface that can glide along itself (Steinhaus 1999). In case of helicoids we get

$$\vec{r}(u,v) = v(\vec{i}\cos u + \vec{j}\sin u) + au\vec{k}.$$

from (3) for $f(u) = au, a = const.$

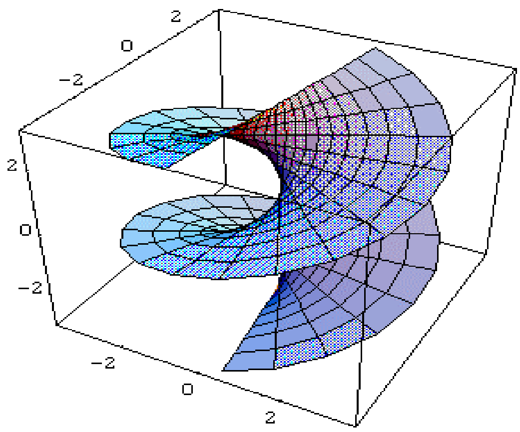

Fig. 3

Plucker's conoid or cylindroid is an often used shape of conoid surfaces in constructional geometry. Cylindroid is an inversion

Non-parametrically it is defined by

$$z = \frac{2xy}{x^2 + y^2}.$$

The Monge parameterization of this surface is

$$\vec{r}(u,v) = (u, v, \frac{2uv}{u^2 + v^2}).$$

The same surface at polar parameterization is

$\vec{r}(\rho,\theta) = (\rho\cos\theta, \rho\sin\theta, 2\cos\theta\sin\theta) = (0,0,2\cos\theta\sin\theta) + \rho(\cos\theta, \sin\theta, 0).$

It shows that z-axis is the base curve and the circle $(\cos\theta, \sin\theta, 0)$ director curve. In case of polar parameterization we can see the rulings passing through the z-axis (Fig.4).

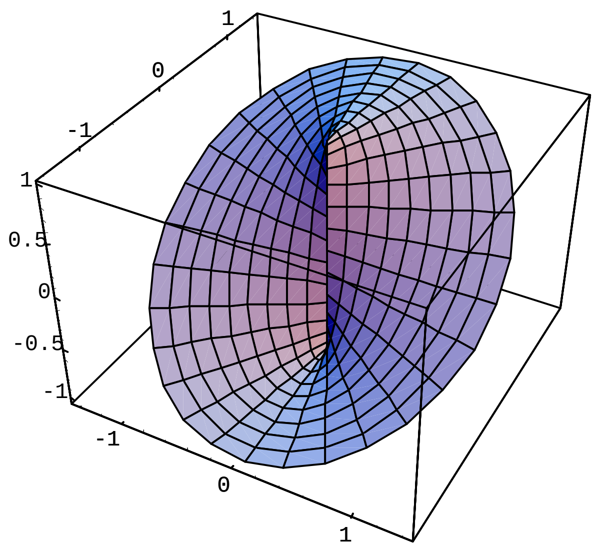

Fig. 4.

A generalization of Plucker's conoid (Fig. 6) has n = 4 folds instead of two (Fig. 5)

$$\vec{r}[n](\rho,\theta) = (0,0,\sin n\theta) + \rho(\cos\theta, \sin\theta, 0).$$

Different parameterizations of this surface illustrate this surface in different ways. In case of polar parameterization, if we want to see the rulings we get the following interesting shapes using Mathematica. Option *ParametricPlot3D* enables good insights of the surface in the neighborhood of singular point. A consideration of geometric characteristics of this surface (rigidity, characteristic lines…) is possible with [3].

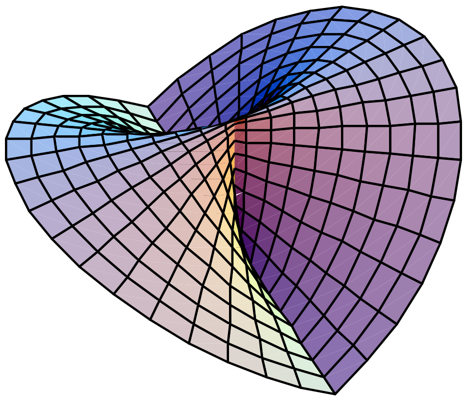

Fig. 5

We will here mention a type of conoids called **conical edge of Wallis**, defined by

$$\vec{r}(u,v) = (v\cos u, v\sin u, c\sqrt{a^2 - b^2\cos^2 u}).$$

Rulings are parallel to xOy plane and the axis of conoid is z-axis (Fig. 7) or the rulings are parallel to yOz plane and intersect x-axis (Fig. 6).

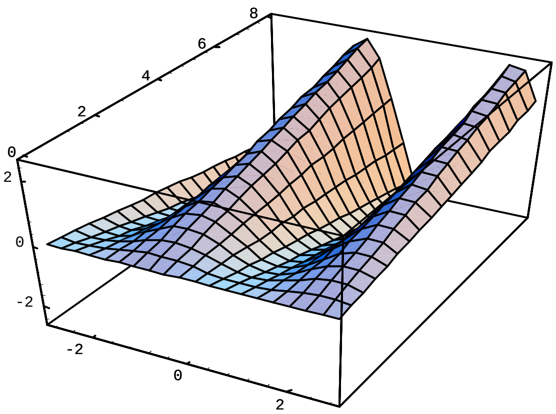

Fig. 6

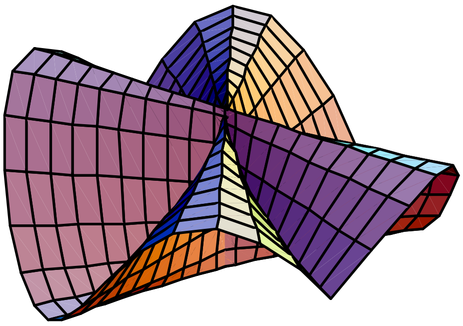

Fig. 7

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MODELIRANJE KONOIDNIH POVRŠI

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U radu se razmatraju konoidne površi koje su često u upotrebi u gradjevinskoj tehnici, uglavnom kao smele krovne konstrukcije. Različiti tipovi konoida se razmatraju uz upotrebu programskog paketa Mathematica .

Opisuje se nastajanje konoida I pomocu parametarskog predstavljanja dobijaju se njihovi grafici. Geometrijski pristup nudi široke mogućnosti za istraživanje komplikovanih prostornih površinskih sistema.

Ključne reči: *Površ Katalana, konoid, hiperbolicki paraboloid, helikoid, Plikerov konoid, Mathematica.*