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COMPARISON OF THE ANALYTIC AND SYNTHETIC METHODS FOR TRANSFORMATION OF THE *NTH* ORDER CURVES INTO THE *2NTH* ORDER CURVES

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Biserka Marković

Faculty of Civil Engineering and Architecture, Niš, Aleksandra Medvedeva 14, Serbia

Abstract. *This paper analyses the quadratic transformation, as known in the literature, which is effected by the analytic geometry procedures in trilinear coordinate system, as well as the graphic construction based on it. Also, the square transformation, based on the methods and the graphic procedures of the synthetic geometry is shown, and it has been realised in the pencils of the polar fields, set by a pair of fundamental conics, with the autopolar triangle. By the comparison of these two methods, the similiraties and differences are presented, and the invariants of both methods were established. The paper shows that the analytic and synthetic method, though using different means and procedures, are mutually intertwined, complemented and congruent, and strive towards the common goal: obtaining the variety of different families and forms of curves.*

Key words: *analytic-synthetic geometry; trilinear coordinate system – autopolar triangle, square transformation, form of curves, pencil of polar fields.*

1. HISTORICAL BACKGROUND

An interest in the study and invention of new curves has never ceased. The topicality of dealing with curves is constantly stimulated by the practical human needs in the various areas of his activities, but they are also significant for the "joy at observing the forms"[1] that is human aesthetic and artistic need. In different historical periods, scientists changed the approach and the method of research, but human knowledge has constantly been enriched with new types and forms of curves.

1.1. Review of the methods of the curve lines derivation

The science of the curve lines was established by the ancient Greek mathematicians. The method of obtaining the curve lines by intersecting the geometrical surfaces with a

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[1] Klein Felix (1849-1925)

plane originates from those mathematicians. The knowledge of mechanical properties curve lines (kinematics), defines the way of derivation curve lines.

The development of analytic geometry and utilization of coordinates establishes the general methods of setting and studying the lines. The analytic expression of a curve is its equation, and the visual representation is the graph of the function represented by the equation.

The presence of the coordinate system, with which the curve is not naturally related is a drawback to analytic research. The coordinate system with its properties, significantly affects the treatment of the curve line. Striving towards freeing from the influence of the coordinate system develops the algebraic geometry and the algebraic curves theory. The subject of research are the invariant properties of lines, with respect to the different choice of the coordinate system and its characteristics.

The deficiency of the analytic method is comprised in the fact that it does not show the generation of the curve and the geometrical construction stemming from that. This fact makes numerous researchers turn to the old, synthetic geometry, which opens the wide opportunities of deriving and studying the new curves by the numerous methods of mapping (projective, correlative, tangential), and then joining.

The topologic method is the specific way of obtaining and studying of the curve lines, applicable for the most complex forms.

The modern application of the vector method is also significant. Computer graphics are its powerful tool in representing the variety of the different forms of curves.

2. Coordinate systems in plane

The classification of curves is problematical, because the nature of curves does not only depend on the equation, but also on the coordinate system within which they are analyzed. The curve can be algebric in one coordinate system, and in the other it can be transcendental. The circle can be both algebric and transcendent curve, for example in the polar coordinate system.

The curve does not change its character in respect to the translations and rotations of the system in the Cartesian coordinate system, with the rectangular axes. Cartesian coordinate system, because of that, is suitable for the classification of the lines.

Cartesian (Newton's) coordinate system with classical, inhomogeneous coordinates is used in normal, Euclidean space.

A coordinate system with homogenous coordinates, (Moebius' Grassman's, Pliker's) is more suitable for the projection geometry, and is called the projective or trilinear coordinate system.

2.1. Trilinear coordinate system

Projective coordinates of a point in plane are given by an ordered set of number: $(\lambda_1, \lambda_2, \lambda_3)$. Geometrically, they represent proportionate distance of that point from three fixed straight lines determined by the equations in non-homogenous coordinates:

$$\begin{aligned} \mathbf{a_1x + b_1x + c_1 = 0} \\ \mathbf{a_2x + b_2x + c_2 = 0} \\ \mathbf{a_3x + b_3x + c_3 = 0,} \end{aligned}$$

which satisfy the condition:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$$

These three straight lines from the triangle, $O_1\ O_2\ O_3$, of the projective coordinated system called the trilinear coordinate system.

Projective coordinates: λ_1, λ_2, λ_3, are proportional to the linear functions of the Cartesian coordinates: **x**, **y**, by definition:

$$\begin{aligned} \rho\lambda_1 &= a_1x + b_1x + c_1 \\ \rho\lambda_2 &= a_2x + b_2x + c_2 \\ \rho\lambda_3 &= a_3x + b_3x + c_3 \end{aligned} \tag{1}$$

where $a_i, b_i, c_i, (i = 1, 2, 3)$ are numbers satisfying the condition:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0,$$

where ρ is the random number, other than zero: $\rho \neq 0$.

From condition (1) it follows that equations:

$$\lambda_1 = 0, \quad \lambda_2 = 0, \quad \lambda_3 = 0,$$

determine the sides of the trilinear coordinate system, whose vertexes are in the following points:

$$O_1(1; 0; 0), \quad O_2(0; 1; 0), \quad O_3(0; 0; 1).$$

Projective, trilinear coordinate system has practical advantages over the rectilinear, Cartesian coordinate system. In the projective coordinate system, the infinitely distant points in plane are determined by the finite coordinates, so that it is more suitable for the projective transformations and examination of the behavior of the curves in infinity. Apart from that, the projective coordinate system is more general than Cartesian and enables wider scope of research of most varied forms of equations.

3. Square transformation

The trilinear coordinate system with homogenous, projective coordinates is most suitable for the representation of square transformation with following equations:

$$\begin{aligned} \rho\,\lambda_1 &= \lambda'_2 \cdot \lambda'_3, & \rho_1\lambda'_1 &= \lambda_2 \cdot \lambda_3, \\ \rho\,\lambda_2 &= \lambda'_1 \cdot \lambda'_3, & \rho_1\lambda'_2 &= \lambda_1 \cdot \lambda_3, \\ \rho\,\lambda_3 &= \lambda'_2 \cdot \lambda'_1, & \rho_1\lambda'_3 &= \lambda_2 \cdot \lambda_1, \end{aligned} \tag{2}$$

where are: coordinates of point P - λ_1, λ_2, λ_3, of P'- λ'_1, λ'_2, λ'_3, ρ and ρ_1 constant indeterminate factors.

Equations (2) are transformation which is rational and square, in both directions; that is why it is called birational square transformation.

From formulae (1) it follows that for $\boldsymbol{\lambda_1 = 0}$:

$$\lambda'_2 = 0 \text{ and } \boldsymbol{\lambda'_3 = 0,}$$

meaning that each point et vertex of the coordinate triangle has a corresponding point at vertex opposite the sideline.

(A)

If equations (2) are applied to the equation of any straight line:

$$\boldsymbol{a\lambda_1} + b\boldsymbol{\lambda_2} + \boldsymbol{c\lambda_3} = 0,$$

the following equation of the joined 2^{th} curve (conic) is obtained the:

$$\boldsymbol{a\cdot\lambda'_2}\cdot\boldsymbol{\lambda'_3} + b\cdot\boldsymbol{\lambda'_1}\cdot\boldsymbol{\lambda'_3 = 0}.$$

On this basis, a general conclusion can be drawn:

Square transformation of curve **k** of the **n** order joins the curve of the **k'**, of the **2n^th^** order which in the vertexes of the triangular coordinate system has **n**-fold points. [14]; pg.116.

(B)

3.1. Graphic interpretation of square transformation

A geometrical construction, resulting from such analytically formulated square transformations, is based on the linking (intersecting) of joined elements of transformation in coordinate triangle.

Two straight lines are joined by transformation: to the straight line c_1, which passes through the apex **A(O$_1$)**, of the coordinate triangle, straight line c_2, through the same apex **A** and the opposite sideline **a**, to the apex **A**. Here, the curve of the **2n** order which was created by the transformation of the straight line c_1, which passes though apex **A**, is disintegrated into two straight lines: **a** and **c$_2$**. Straight line c_2 is easily graphically determined. It is symmetrical to the straight line **c$_1$**, in respect to the line of symmetry of the apex **A** angle of the coordinate triangle **ABC (O$_1$ O$_2$ O$_3$)**, on the basis of the transformation characteristics (Fig. 1.).

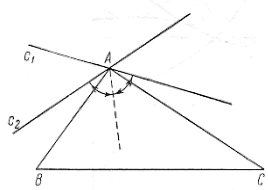

Fig. 1. [14] fg.35. pg.118.

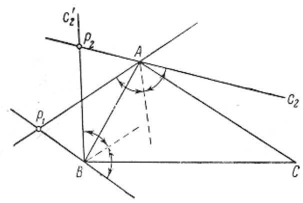

Fig. 2. [14] fg.36. pg. 118.

If for the given point **P**, which is being transformed, one should find the point **P'**, point **P** is connected to any of the apices of he coordinate triangle, for example with A, so for this straight line **c$_1$ = PA**, a symmetric straight line c_2,is determined, in respect to the line of symmetry of the apex **A** angle of the **ABC** triangle.

The described procedure is repeated in respect to any other apex of the coordinate triangle, for example, in respect to **B**. The straight line $\mathbf{c'_2}$, is obtained, symmetrical to the straight line $\mathbf{c'_1 = PB}$, in respect to the line of symmetry of the **B** apex angle of the coordinate triangle. [14]; pg.118.

Applying this graphic procedure to a series of points on the curve line **k** of $\mathbf{n^{th}}$ order gives by the transformation an appended series of points on the curve line **k'**, of the $\mathbf{2^{th}}$ order.

Each sideline of the coordinate triangle meets, for exaple a circle, at two points, so that the curve **k'**, is obtained by the transformation has three double points (with maximal number of double points). It follows that the product of transformation, **k'**, is a rational curve of the 4^{th} order.

*The pair of intersecting points of circle **k** and any sideline of coordinate triangle give, in opposite vertex, double points of curve **k'** of $\mathbf{4^{th}}$ order, like:*

1. *a node (points are real and separated),*
2. *a cusp (points are real and coincidenced),*
3. *a acnode (a pair of conjugate-imaginary points)* [Fig: 3].

(C)

In the figure 3. there is a square transformation of the circle **k**, which touches the sideline of the triangle **b** in the points **1 = 2**, intersects sideline **c** in the points **4** and **7**, an it also has a pair of common conjugate-imaginary points with the sideline **a**. Curve line **k'** of $\mathbf{4^{th}}$ order, obtained as a result of this transformation, has a cusp in vertex **B**, in **C** it has a node, and in the vertex **A** an acnode point. /14/;pg.116.

In literature it is analytically proved that by the square transformation of the infinitely distant straight line a circle is obtained, through three apex points, **ABC**, of the coordinated triangle. [14]; pg.118.

(D)

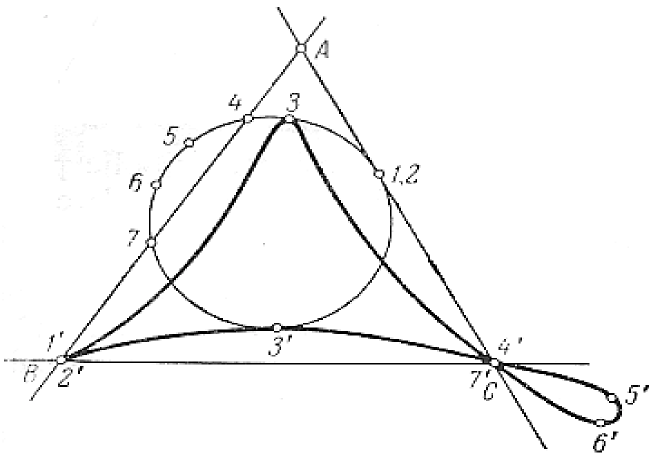

Fig. 3. [14]; pg.116.

This characteristic of the square transformation **(D)** is significant for the analytic, but also for the graphic determination of number and character of infinitely distant points of curve line k', which is obtained as the result of the transformation.

4. Transformation in the Pencil of the Polar Fields

The pencil of the conics defines the pencil of the polar fields and is set by fundamental elements:

– couple of any two conics $\mathbf{k_1}$, $\mathbf{k_2}$, of the conic pencil;

– by fundamental points **ABCD,** which make autopolar quardrangle, which determines the autopolar triangle **PQR** (common for all the conics of the pencil).

In the pencil of the polar fields, appending is effected – mapping of point to point, straight line to conic, curve line of n Order into a curve line of 2n order. This mapping is a square transformation, done in two steps. The first step, the polar mapping with element changing, the series of n order points determines two projectively appended pencils of straight lines of n order, with respect to the given conic of the pencil of conics. In the second step, with intersection the couples of projectively appended rays, these two pencils of the straight lines of **n.** order, a series of points of **2n**. order is determined [4]. page.64.

The way of setting the pencils by the choice of fundamental elements defines the transformation model which defines the terms of transformation on one hand, but on the other hand also the characteristics of the transformation results [7].

The choice of the type and mutual relationship of the fundamental elements represents the specialization f the transformation model [10].

Two circles, for example, are chosen to be the fundamental elements of the conic pencils represent the specialization of the "transformation model"[2] (condition) which facilitates the structural simplicity. Two appended pencils of the straight lines are determined by the construction of fundamental circles polars. These are favourable conditions for transformation, however, with such specialization of the transformation model, the range of the transformation product is reduced to the circular curve lines which pass through the absolute points of the plane [5], [6], [9].

The adoption of the more general transformation model gives an opportunity to analyze different conditions of mapping and finding more varied families of the curve, by the specialization of such transformation model.

4.1. Graphic projective mapping of transformations in pencil polar fields

The transformation model with the simple graphic procedure for polar mapping is investigate, the procedure being not exclusive, in order to achieve obtaining of widest possible range of diversified families of curves.

Two degenerated conics, $\mathbf{k_1}$ i $\mathbf{k_2}$, of the pencil of the conic, through the four fundamental points, **ABCD**, the apices of the autopolar with four apices, determine the model of transformation which satisfies the requirements. Such adopted model of transformation can be further specialized by the change of the reality of the couples of fundamental points, or their position and relationship to the infinitely distant straight line plane.

Each pencil of the conic comprises three degenerated conic of the pencil: $\mathbf{k_1}$, $\mathbf{k_2}$ and $\mathbf{k_3}$. They mutually intersect in the fundamental points **ABCD**, four autopolar vertexes, which is common for all the conics of the pencil [2] pg.383. Degenerated conics are divided into couples of straight lines which intersect in points $\mathbf{P(k_1)}$, $\mathbf{Q(k_3)}$ and $\mathbf{R(k_2)}$. That

[2] "Model of transformation" define chose the conditions and with alone that the productions the transformation.

is why **P, Q,** and **R** can be called self-intersecting points for the couples of straight lines which determine the degenerated conics k_1, k_2 and k_3. These three points, **PQR**, are the vertexes of the autopolar triangle, common for all the conics of the pencil.

Any vertex (**P,Q,R**) is the pole of the opposite sideline (p,r,q), which is its polar.

The polars of the series of points on any straight line a, are made of projectively appended pencil of straight lines through the pole **A**, of the straight line **a**.

(I)

Each point **X** on the straight line a determines the conjugated point **Y** on **a**, where the polar **x**, of the **X** point, intersects the straight line **a**.

(I.a)

From the characteristics of the polarity (**I, I.a**) follows that mapping of **X** to **Y** is projectivity **(** $\barwedge$ **):** $\mathbf{X} \barwedge \mathbf{x} \barwedge \mathbf{Y}$

(I.1)

The polar of any point E, with respect to one of the fundamental degenerated conics $\mathbf{k_1}$ **(R = p x q)**, which is determined by the straight lines **p** and q, is the straight line e_2. It is harmonically conjugated to the straight line $\mathbf{e_1}$ **= ER** in respect to the p and q [3], pg.182. **E** point, in the same manner maps into $\mathbf{e'_2}$, in respect to the other fundamental, degenerated conic $\mathbf{k_2}$, of the conic pencil, divided into two straight lines **p** and **r**. The straight $\mathbf{e'_2}$ is harmonically conjugated to the straight line $\mathbf{e'_1}$ **= EQ**, touching point of the **E** point with self-intersecting point of **Q** conic $\mathbf{k_2}$ **(Q = p x r)**, in respect to the straight lines **p** and **r**.

Thus, to the **E** point, by the virtue of polar mapping in the polar fields pencil, with respect to the fundamental conics $\mathbf{k_1}$ i $\mathbf{k_2}$, the straight lines $\mathbf{e_2}$ and $\mathbf{e'_2}$ are appended. They intersect in **E'** point which is the product of the point **E** transformation.

The graphic interpretation of the square transformation and its simplicity is shown in the figures 4. and 5., in the ementary appendage of the E point to the E' point. E point can be outside the polar triangle PRQ (Fig. 4), or inside it (Fig. 5). During the analysis of the character of this transformation, one starts from the characteristics of the polarity (especially I.1). It follows that the polars of the common point E are mutually projectively appended straight lines: $e_2 \barwedge e'_2$.

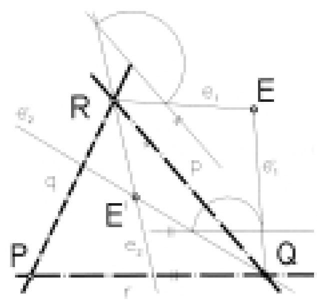

Fig. 4.

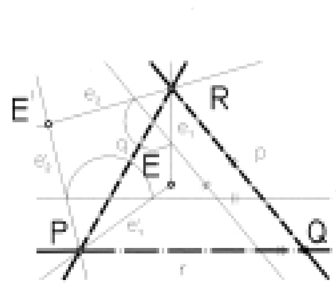

Fig. 5.

Intersecting these two straight lines, **E'** point is obtained, projectively appended to the **E** point.

(I.2)

Analogous to this, if the series of poles E_i 1^{st} order, on the straight line e,in the pencil of the polar fields, given by the couple of the degenerated conics k_1 and k_2, through the fundamental points ABCD, then two projectively appended pencils of the 1^{st} order polars, $[e_2]$ in respect to the conic k_1 and $[e'_2]$ with respect to the conic k_2, of the conic pencil. From (I.2) it follows:

$$[e_2] \overline{\wedge} [e'_2].$$

(II)

These two projectively appended pencils of the 1st order polar (II), by the mutual intersection of the couples of appended rays, determine a series of E'_i points, 2^{nd} order, on the conic k', appended to the series of points E, on the straight line e.

(III)

The result of the analysis (II, III) is the evidence that in the pencil of the polar fields a square transformation is performed, which, in the general case, maps a curve line k of the n order into a k' curve line of 2n order.

4.2. Autopolar triangle as a trilinear coordinate system

Pencil of the polar fields set by two degenerated conics, $\mathbf{k_1}$ $\mathbf{k_2}$, of the conic pencil, determines the autopolar with four apices **ABCD** and autopolar with three vertexes **PQR**, as the common elements of all the conics of the pencil.

The autopolar triangle is significant for the exploration of the properties of curve line **k'**, of **2n** order, which is obtained by the transformation of the curve line **k**, of **n.** order, but also for determination of interdependencies of those curve lines on the conditions of transformation (choice of type and position of the curve line k in respect to the autopolar triangle).

Each vertex of the autopolar triangle (**P,Q,R,**) is a pole of the opposite sideline (**p, q ,r**), where two tops (for example **Q,R**) of each sideline (**p**), are the conjugated points, and each two sidelines (q,r) through the opposite vertex (**P**) are conjugated straight lines.

(IV)

From the characteristics of polarity (I) it follows that the series of poles $\mathbf{P_i}$, on the sideline **p**, of the autopolar triangle **PQR**, appended polars p_i pass through vertex **P**, opposite to the sideline **p**. On the basis of this and previous assertions (I-IV) it follows that:

The curve line **k**, of **n**. order, which is transformed in the pencil of the polar fields and intersects any sideline (**p, q ,r**) of the autopolar triangle in **n** points, yields as the result of transformation the curve line **k'**, of **2n**. order which has **n**–fold point in the opposite vertex (**PQR**) of that sideline.

(V)

On the basis of explanation (V) and (B) the identical role of the autopolar triangle **PQR** and coordinate triangle **ABC** of the trilinear coordinate system is obvious ($\mathbf{O_1}$, $\mathbf{O_2}$,$\mathbf{O_3}$).

Since "analytic geometry is a model of synthetic geometry" an analytic connection is established between these two triangles. The coordinate triangle is "at the same time both the autopolar triangle, in the special case when the polarity is reduced to canonical or normal form" [14] pg.147. and pg.156.

This is realized in the pencil of the polar fields, at autopolar triangle, common for all the pencil conics. If, at that, the curve line **k**, which is being transformed, is a circle, then, on the basis of assertions I-V and explanation of the single role of triangles (autopolar **PQR** and coordinate **ABC**) the same conclusions may be restated **C(1,2,3)**, given in the section 3.1. Illustration on the figure 3. Would be common for this type of transformation in the polar fields pencils and for analytically founded square transformation.

The conic of the centers is geometrical place of the poles of all pencil conic for the infinitely distant polar. Therefore, the infinitely distant straight line of the plane is appended, by the transformation in the pencil of the polar fields, to the conic of centers of conic pencils.

For the general case of setting the fundamental points **ABCD**, as real separated finite points, the points of the center conics are determined from the harmonic relationship of the conjugated points on the sides of the autopolar with four apices **ABCD**, with infinitely distant points of each sideline.

The conic of the centre passes through the apices of the autopolar triangle **PQR** and in the general case it is the hyperbola. /4/ pg. 36-40.

The relationship of curve line **k**, of n. order, which is transformed in the polar fields pencil, towards the conic of the centers, determine both number and the character of infinitely distant points of curve line **k'**, of **2n**. order, produced by the transformation.

(V)

4.3. Graphic illustration of the transformation done in the polar fields pencils

Line **k**, in the general case of **n**. order (**n = 1,2,3,…**), which is being transformed is given as a circle.

For the illustrative examples of the graphic procedure of square transformation, circle **k** is placed in respect to the conic of the centers that it has no common points with it. Transformed curve **k'**, of **4th** order does not have because of that, infinitely distant points..

A symmetrical model of transformation is chosen because of the rationality of the constructive processing of the **k'** curve line, of **4th** order.

We aim to present the general principle of the constructive processing, varying of the character of double points of the **k'** curve of the **4th** order as the products of the transformation, choice of the different relationship of the **k** circumference, which is being transformed, in relation to the sidelines of the autopolar triangle **PQR**.

The example in the figure 6 shows obtaining of the rational curve line **k'** of **4th** order, with nodal (self-intersecting) point in **R** apex and two cusps in vertexes **P** and **Q**, because of the conditions that the circcle k intersects the sideline **r**, of the autopolar triangle **PQR**, in two real separated points, and it touches the sidelines **p** and **q**.

The rational curve line **k'** of **4th** order obtained by the transformation in the figure 7 has one cuspidal point in vertex **R** and two isolated points in vertices **P** and **Q**. This is achieved by the specialization of the circumference **k** position within the autopolar triangle. It is the condition that **k** circlee, touches the sideline **r**, and sidelines **p** and **q** of the autopolar triangle PRQ, intersect in two pairs of conjugate- imaginary points.

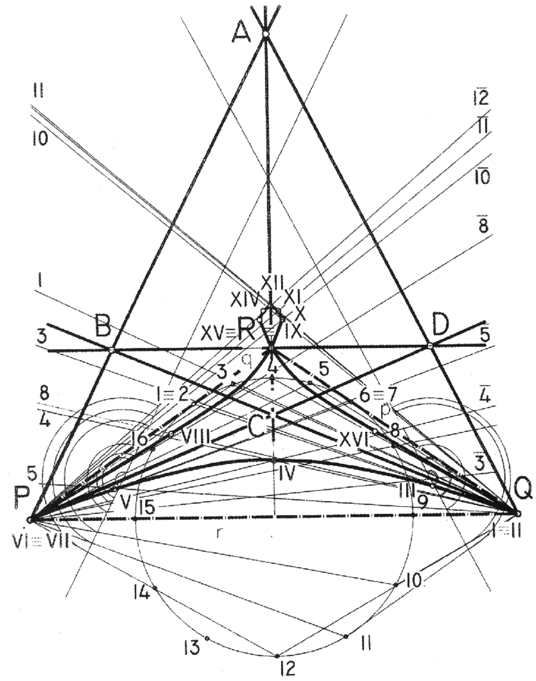

Fig. 6.

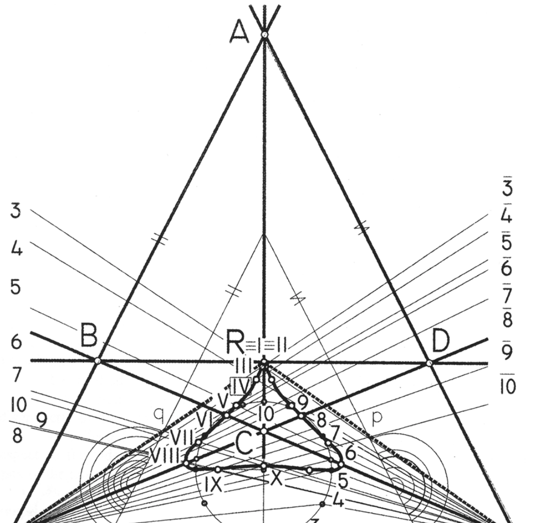

Fig. 7.

5. Conclusion

This paper has shown that in the polar field pencil, set by a couple of fundamental conics of the pencil, the mapping is realized, which is by its results equal to the square transformation. Mapping is based on the methods and procedures of synthetic geometry.

The square transformation developed by the analytic procedure with the appropriate graphic construction is analyzed.

By comparing the graphic procedures and the obtained results, the most general conclusion is drawn: both considered methods belong to the square transformations, because they map a line of n order into a line of 2n order. Transformations have mutual similiarities, but also differences.

The basic similarity is the construction of the triangle, and its functional meaning is invariant in both methods. The triangle is:

– trilinear coordinate system, in the analytic method.,

– autopolar with three apices in the synthetic geometry method.

This elementary graphic procedure consists of, in both methods, two steps. In each of those steps, there is an "incident" and "reflected" ray, towards to the appropriate vertex of triangle and from it.

The determination of the incident ray is identical in both cases: point E which is being transformed is connected to any apex of the triangle.

The method of determination of the "reflected" ray is different:

– "reflected" ray is symmetrical to the "incident" ray in respect to the line of symmetry of the corresponding apex angle of the triangle (analytic method),

– "reflected" ray is harmonically conjugated to the "incident" ray, in respect to the sidelines of the autopolar with four apices, of the corresponding apex of the autopolar triangle (synthetic method)

The basis of the procedure and metrics of determination of the "reflected" ray comprise the essential difference of two analyzed methods.

These differences do not affect the origin and position of the double points of the curve line of **2n**. order which is obtained by the transformation. Double points are the invariant to both graphic procedures and both methods..

The origin number, and characteristics of infinitely distant points of the **2n**. order curve line which is obtained s the same in both methods. The conclusion is that the infinitely distant points are invariants of both analyzed transformations

The difference is in the fact that the infinitely distant straight line has the appended:

– circumference circumscribed around the triangle in analytic method,

– general conic circumscribed around the autopolar with four apices, in synthetic method.

This paper shows that the analytic and synthetic method of obtaining and nevertheless researching of the curve lines, using different means and procedures, are mutually intertwined, complemented and overlapping while striving towards the common goal.

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POREDJENJE ANALITIČKE I SINTETIČKE METODE ZA TRANSFORAMACIJU KRIVIH LINIJA N. REDA U KRIVE 2N. REDA

Biserka Marković

U ovom radu analizurana je kvadratna transformacija, poznata iz literaure, ostvarena postupcima analitičke geometrije u trilinarnom koordinatnom sistemu i na njoj zasnovana grafička konstrukcija. Isto tako, prikazana je kvadratna transformacija, utemeljena na metodama i grafičkim postupcima sintetičke geometrije, realizovana u pramenu polarnih polja, zadatih parom degenerisanih temeljnih konika, sa autopolarnim trouglom. Poredjenjem ove dve metode istaknute su sličnosti i razlike i ustanovljene invarijante obe metode. Pokazano je da se analitička i sintetička metoda, služeći se različitim sredstvima i postupcima, medjusobno prožimaju, dopnjavaju i preklapaju, na zajedničkom cilju: dobijanju obilja različitih familija i formi krivih linija.