FACTA UNIVERSITATIS
Series: **Architecture and Civil Engineering** Vol. 2, N^o 2, 2000, pp. 117 - 129

DETERMINATION OF INTERSECTING CURVE BETWEEN TWO SURFACES OF REVOLUTION WITH INTERSECTING AXES BY USE OF AUXILIARY SPHERES

UDC 514.18:681.3.06(045)

Ratko Obradović

PhD, Assistant Professor, Faculty of Technical Sciences,
Institute for Mathematics and Physics in Engineering, Trg Dositeja Obradovića 6,
21121, Novi Sad, Serbia, Yugoslavia, Email: obrad_r@uns.ns.ac.yu

Abstract. *A descriptive geometrical method of auxiliary spheres has been used for the determination of intersecting curve between two surfaces of revolution with intersecting axes of surfaces (Dovniković 1994, Anagnosti 1984). Namely, if the centre of all auxiliary spheres is put on intersecting points between axes of surfaces then in general each auxiliary sphere intersects each surface of revolution on k circles-parallels (k=2m because circle is second order's curve and the starting curve of surface of revolution is order m). These circles lie in planes orthographic on axis of the surface of revolution. In case when sphere intersects both surfaces on two circles then the pair of circles are intersected at most 8 points. These intersecting points of circles are points of intersecting space curve between two surfaces for this sphere. New circles and new points of intersecting curve have been determined by radius changing of auxiliary spheres (Obradović 2000).*
It is possible to make the mathematical form and procedure for this idea and in this case the sequence for the connection of intersecting curve points will not be important because procedure will be realized by computer. With a sufficiently large number of auxiliary spheres it will be possible to avoid connecting problem of intersecting curve points (Obradović 2000).

Key words: *Surface of Revolution, Auxiliary Sphere, Descriptive Geometry, Computer Graphics.*

1. REFLECTION

Each surface of revolution is given by an axis of revolution and a meridian which is coplanar with the axis. When we wish to find the reflecting meridian we can use modern knowledge from computer graphics and each complex problem we can show as group of elementary problems (Rogers, Adams 1990).

Received June 04, 2002

Meridian and axis of revolution light in plane *Oxz* are given. We can find the equation of reflecting meridian from the given meridian about the axis of surface. We can find the solution to this problem by combining several two-dimensional transformations (Fig. 1). These transformations are (Obradović 1999, Obradović 2000):

1. The translation of meridian and axis to the place where the axis is passing through the origin. We can make this translation in arbitrary direction but the most practical way is through the coordinate axes (x or z). In this case we will translate in the direction of axis z. We can term meridian and axis an object;
2. The rotation of object until the axis becomes coincident with the axis of the coordinate system. There are two possibilities in this case: that the axis of the surface is coincident with x or z axis of the coordinate system. In this paper the axis is coincident with x axis;
3. The reflection of meridian through an x axis;
4. Apply the inverse rotation about the origin;
5. Undo translation.

Matrixes for these two-dimensional transformations are (Obradović 1999, Obradović 2000):

1. The first step is translation of object for dimension n where the axis of surface is given by $z = mx + n$ (Fig. 1b).

$$[x^* \quad z^* \quad 1] = [x \quad z \quad 1]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -n & 1 \end{bmatrix} = [x \quad z-n \quad 1]$$

2. The next step is the rotation of the object about the origin (Fig. 1c) until the line has been coincident with x axis for an angle $\theta = \mathrm{arctg}\,|\,n/x_0\,|$, where is $x_0 = x\{y = 0\}$. The matrix of the last transformation is

$$[T] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Reflecting through the x axis (Fig. 1d)

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. The rotation about the origin (Fig. 1e)

$$[T] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. The translation (Fig. 1f)

$$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & n & 1 \end{bmatrix}$$

The resulting matrix is (Obradović 1999, Obradović 2000).

$$[T]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & -n & 1\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & n & 1\end{bmatrix}$$

$$[T]=\begin{bmatrix}2\cos^2\theta-1 & 2\sin\theta\cos\theta & 0\\ 2\sin\theta\cos\theta & 1-2\cos^2\theta & 0\\ -2n\sin\theta\cos\theta & 2n\cos^2\theta & 1\end{bmatrix}$$

The reflection is determined by the next equation

$$[X^*]=[x^* \quad z^* \quad 1]=[x \quad z \quad 1][T]=[x \quad z \quad 1]\begin{bmatrix}2\cos^2\theta-1 & 2\sin\theta\cos\theta & 0\\ 2\sin\theta\cos\theta & 1-2\cos^2\theta & 0\\ -2n\sin\theta\cos\theta & 2n\cos^2\theta & 1\end{bmatrix}$$

$$[X^*]=[2x\cos^2\theta+2(z-n)\sin\theta\cos\theta-x \quad 2(n-z)\cos^2\theta+2x\sin\theta\cos\theta+z \quad 1] \tag{1}$$

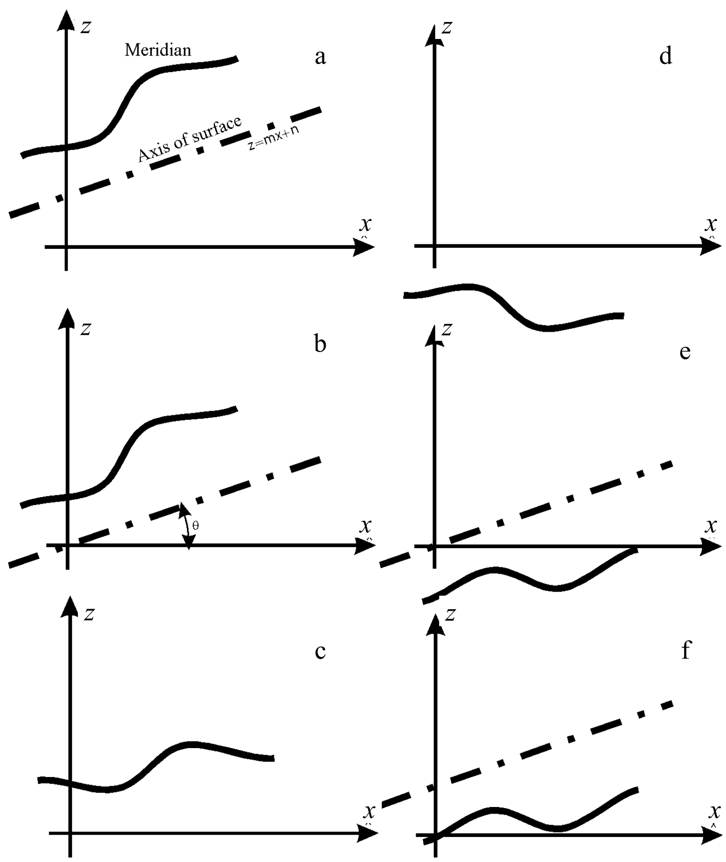

Fig. 1. Reflection

2. DETERMINATION OF SPHERES WITH MINIMAL AND MAXIMAL RADIUS

Meridians and axes of revolution of two surfaces of revolution are given in plane *Oxz* (Fig. 2). Equations of meridian and axis of revolution for the first surface are

$$x_1 = x_1(t);\ z_1 = z_1(t)$$
$$z = m_1 x + n_1$$

For the second surface

$$x_2 = x_2(t);\ \ z_2 = z_2(t)$$
$$z = m_2 x + n_2$$

By using the equation (1) we can determine equations of the reflecting meridian, i.e. we can find x_1^*, z_1^* and x_2^*, z_2^*. Intersecting points of contour meridians are (Fig. 2):

Table 1. Intersecting points of contour meridians

POINT	CURVES IN INTERSECTION
$1\,(x_1, z_1)$	$\begin{Bmatrix} x_1 = x_1(t) \\ z_1 = z_1(t) \end{Bmatrix} \cap \begin{Bmatrix} x_2 = x_2(t) \\ z_2 = z_2(t) \end{Bmatrix}$
$2\,(x_2, z_2)$	$\begin{Bmatrix} x_1 = x_1(t) \\ z_1 = z_1(t) \end{Bmatrix} \cap \begin{Bmatrix} x_2^* = x_2^*(t) \\ z_2^* = z_2^*(t) \end{Bmatrix}$
$3\,(x_3, z_3)$	$\begin{Bmatrix} x_1^* = x_1^*(t) \\ z_1^* = z_1^*(t) \end{Bmatrix} \cap \begin{Bmatrix} x_2 = x_2(t) \\ z_2 = z_2(t) \end{Bmatrix}$
$4\,(x_4, z_4)$	$\begin{Bmatrix} x_1^* = x_1^*(t) \\ z_1^* = z_1^*(t) \end{Bmatrix} \cap \begin{Bmatrix} x_2^* = x_2^*(t) \\ z_2^* = z_2^*(t) \end{Bmatrix}$

2.1. The procedure for the determination of intersecting points of meridians

Meridians of two surfaces are given by parametric equations

$$x_1, z_1, x_1^*, z_1^* \in [k_0, ..., k_{\max}];\ \ x_2, z_2, x_2^*, z_2^* \in [l_0, ..., l_{\max}]$$

The procedure for the determination of an intersecting point 1 is (Obradović 2000)

$m := 1; \Delta_{\min} := const;$

For k:=k_0 to $k_{\max}$ do

Begin

For l:=l_0 to $l_{\max}$ do

Begin

$\Delta_1[k,l] = |x_1[k] - x_2[l]; |\Delta_2[k,l] = |z_1[k] - z_2[l]|$

$$\Delta_{sr}[k,l] = \frac{\Delta_1[k,l] + \Delta_2[k,l]}{2}$$

If $\Delta_{sr}[k,l] > \Delta_{\min}$ Then GoTo 1

Else GoTo 2;

2: $\Delta_{\min} := \Delta_{sr}\, p\,[k,l]$

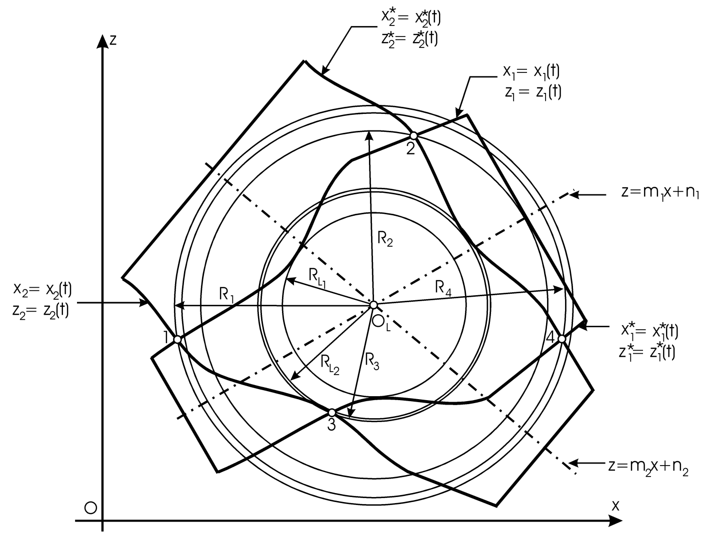

Fig. 2. Surfaces of revolution

$x_P[m] := x_1[k]; z_P[m] := z_1[k];$
m:=m+1;
1: End;
End;

The intersecting point of two axes is given by the determination of the system of two equations

$$z = m_1 x + n_1 \text{ and } z = m_2 x + n_2$$

Point $O_L(x_L, z_L)$

$$x_L = \frac{n_2 - n_1}{m_1 - m_2}; \qquad z_L = \frac{m_1 n_2 - m_2 n_1}{m_1 - m_2} \tag{2}$$

Radiuses of spheres (in plane Oxz these are circles) with the centre in the point O_L have dimensions which are equal to the distance between O_L and points 1,…,4 and they are determined by the expression

$$R_i = \sqrt{(x_L - x_i)^2 + (z_L - z_i)^2}; \; i = 1,...,4 \tag{3}$$

From these four radiuses it is necessary to find the biggest, and we can do this by using the following procedure (Obradović 2000):

$R_{\max} := R[1]$
for i= 2 to 4 do
Begin
if $R[i] > R_{\max}$ then $R_{\max} = R[i]$
else
End;

Now it is necessary to determine radiuses of spheres with the common centre O_L, if these spheres concern the surfaces of revolution. First, we will make a connection between global *Oxz* and the local coordinate system *Cuv* (Fig. 3).

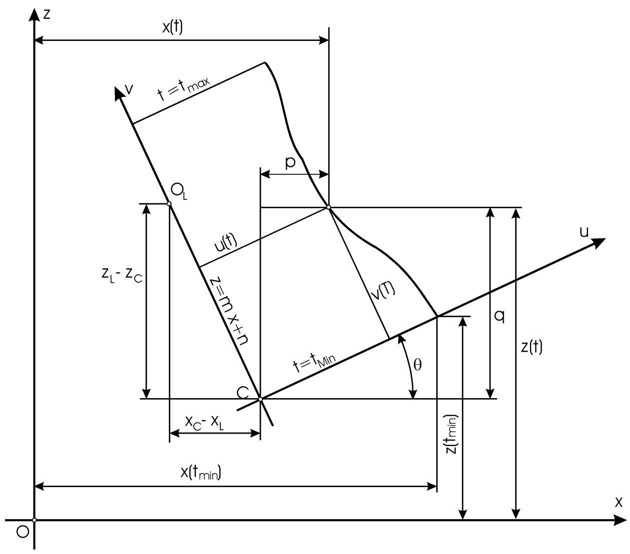

Fig. 3. The transformation of the coordinate system

3. THE TRANSFORMATION OF THE COORDINATE SYSTEM

The axis *v* of the local coordinate system is coincident with the axis of revolution $z = mx + n$, and because the axis *u* is orthogonal to axis *v*, the equation of the line coincident with axis *u* is

$$z = -\frac{1}{m}x + k$$

On *u* axis there is a point of meridian with coordinates $(x(t_{\min}),z(t_{\min}))$, in the global system *Oxz*. The substitution of coordinates of this point in the last equation gives us

$$k = z(t_{\min}) + \frac{1}{m}x(t_{\min})$$

The point *C* is the origin of system *Cuv*, and its coordinates in system *Oxz* will be formed by combining next equations

$$z = mx + n \text{ and } z = -\frac{1}{m}x + k$$

Point $C(x_C,z_C)$

$$x_C = \frac{k-n}{m^2+1}m;\ y_C = mx_C + n$$

The local system *Cuv* could be found by the translation and rotation of the system *Oxz*. First, the system *Oxz* will be translated for vector *OC*, and next it will be rotated for angle $\theta = \alpha - \pi/2;\ \alpha = arctg(m)$. We can take into consideration the two new parametric functions

$$p(t) = x(t) - x_C;\ q(t) = z(t) - z_C \tag{4}$$

From figure 3 we can see

$$u(t) = p(t)\cos\theta + q(t)\sin\theta;\ \ v(t) = -p(t)\sin\theta + q(t)\cos\theta \tag{5}$$

The coordinates of the point in the global and local coordinate system are being by using presented equations connected.

4. The determination of the tangent sphere

It is necessary to determine the sphere with the centre O_L which is on axis v of the local coordinate system if the radius of the sphere is determined from the condition that the sphere touches the meridian. The centre of the sphere in system *Cuv* is defined by the expression

$$O_L = (0, v_L); \qquad v_L = \sqrt{(x_L - x_C)^2 + (z_L - z_C)^2}$$

For the centre of an auxiliary sphere next equations are valid (Štulić 1994, Obradović 1999, 2000)

$$\begin{aligned} &v_L = v(t) - u(t)\frac{n_v(t)}{n_u(t)}; \qquad DS = v(t) - u(t)\frac{n_v(t)}{n_u(t)} \\ &n_u(t) = -\frac{dv(t)}{dt}; \quad n_v(t) = \frac{du(t)}{dt} \end{aligned} \tag{6}$$

Because $t_{\min} \le t \le t_{\max}$, and v_L is known, the radius of auxiliary sphere which concerns meridian and has the centre O_L, could be found by substituting values for parameter t (from $t_{\min}$ and if it is necessary to $t_{\max}$) in equations (6) which represented values of components of normal vector $n_u(t)$, $n_v(t)$, and in addition the right side of equation DS can be solved and if $v_L = DS$, then parameter t for the searching sphere is determined (Obradović 2000). Let t_L be the value of the parameter t for the tangent sphere. Now

$$R_L = \sqrt{u(t_L)^2 + (v_L - v(t_L))^2} \tag{7}$$

For one surface of revolution (i.e. one meridian and one axis of revolution) the radius of the tangent sphere R_{L1} is determined, and for another surface the radius is R_{L2}. The minimal radius, which touches one sphere and intersects another is given by

$$R_{\min} = \max(R_{L1}; R_{L2}) \tag{8}$$

5. The intersection of surfaces of the revolution with an auxiliary sphere

The surfaces of revolution and sphere project in the plane *Oxz*, i.e. the space problem of surface-surface intersection is treated as the plane problem. The sphere with $O_L(0,0,z_L)$ (Fig. 4), of the radius R in the plane *Oxz* is the frontal circle determined with

$$x^2+(z-z_L)^2=R^2$$

and from this

$$x=\pm\sqrt{R^2-(z-z_L)^2}$$

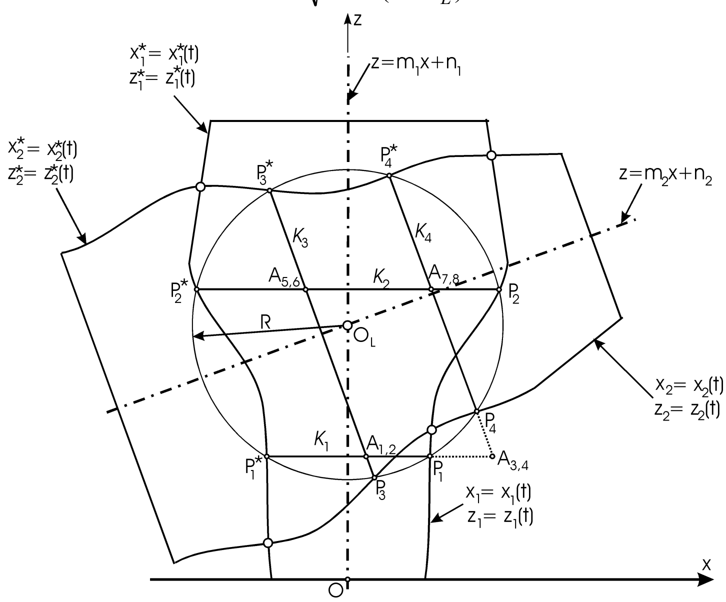

Fig. 4. Arbitrary auxiliary sphere

The circle intersects the given meridian x_1, z_1 of the first surface in points P_1 and P_2, and meridian x_2, z_2 of the second surface in points P_3 and P_4, when meridians are plane curves of the second order. All points P_1, ..., P_4 are determined by the unique procedure, which for the point P_1 looks (Obradović 2000):

$i{:=}1$; $t{:=}t_0$;
3: $z[t]{:=}z_1[t]$;
$x[t]=\sqrt{R^2-(z_1[t]-z_L)^2}$
If $|x[t]-x_1[t]|<\varepsilon$ then GoTo 1
Else GoTo 2;
2: $t{:=}t+1$;
If $t>t_{\max}$ Then GoTo 4
Else GoTo3;
1: $x_P[i]{:=}x_1[t]$; $z_P[i]{:=}z_1[t]$; $i{:=}i+1$;
GoTo2;
4:End;

Reflecting points $P_1^*(x_{P_1^*}, z_{P_1^*}),...,P_4^*(x_{P_4^*}, z_{P_4^*})$ are determined by the reflection of the points P_1, ..., P_4 about an axis of the corresponding surfaces of revolution. Circles K_1 and K_2 in plane Oxz are lines with equations

$$z = z_{P_1} \quad \text{and} \quad z = z_{P_2}$$

Intersecting circles K_3 and K_4, of the sphere and another surface of revolution, in plane Oxz are lines given with two points P_3, P_3^* and P_4, P_4^*.

The equations of these lines are:

$$z = a_3 x + b_3 \qquad \text{and} \qquad z = a_4 x + b_4$$

$$a_3 = -\frac{1}{m_2}; b_3 = z_{P_3} - a_3 x_{P_3} \qquad\qquad a_4 = -\frac{1}{m_2}; b_4 = z_{P_4} - a_4 x_{P_4}$$

By intersecting circles K_1 and K_3 points A_1 and A_2 were found, where x and z coordinates of the points were found by the intersection of the lines which are coincident with given circles in the plane Oxz:

$$z_{A_1} = z_{A_2} = z_{P_1}; \quad x_{A_1} = x_{A_2} = \frac{z_{A_1} - b_3}{a_3} \tag{9}$$

y coordinates are determined from the equation of the circle K_1:

$$y_{A_{1,2}} = \pm\sqrt{x_{P_1}^2 - x_{A_1}^2} \tag{10}$$

If these points are really intersecting points of surface-surface intersection the next condition must be satisfied

$$x_{P_1^*} \le x_{A_1} \le x_{P_1} \tag{11}$$

Coordinates of other points were determined analogically:

Table 2. Coordinates of the points of the intersecting curve for the arbitrary auxiliary sphere

Points A_3, A_4 on intersection circles K_1 and K_4:	Points A_5, A_6 on intersection circles K_2 and K_3:	Points A_7, A_8 on intersection circles K_2 and K_4:
$z_{A_{3,4}} = z_{P_1}$	$z_{A_{5,6}} = z_{P_2}$	$z_{A_{7,8}} = z_{P_2}$
$x_{A_{3,4}} = \frac{z_{P_1} - b_4}{a_4}$	$x_{A_{5,6}} = \frac{z_{P_2} - b_3}{a_3}$	$x_{A_{7,8}} = \frac{z_{P_2} - b_4}{a_4}$
$y_{A_{3,4}} = \pm\sqrt{x_{P_1}^2 - x_{A_3}^2}$	$y_{A_{5,6}} = \pm\sqrt{x_{P_2}^2 - x_{A_5}^2}$	$y_{A_{7,8}} = \pm\sqrt{x_{P_2}^2 - x_{A_7}^2}$
$x_{P_1^*} \le x_{A_3} \le x_{P_1}$	$x_{P_2^*} \le x_{A_5} \le x_{P_2}$	$x_{P_2^*} \le x_{A_7} \le x_{P_2}$

6. RESULTS

On next figure two surfaces of revolution were presented. They are given with axes and meridians (in plane Oxz) and their axes are in intersection. The four intersecting points on intersection between contour meridians are presented and two spheres with minimal and maximal radius are shown. Parametric equations of meridians are

$$
\begin{array}{ll}
x_1(t) = a(t - \sin t) & x_2(\varphi) = R\cos\varphi - 50 \\
z_1(t) = a(1 - \cos t) & z_2(\varphi) = R\sin\varphi \\
a = 35 & R = 120 \\
t \in [0, 2\pi] & \varphi \in \left[0, \dfrac{\pi}{2}\right]
\end{array}
$$

Meridian of the first surface is given by 360 points, and of the second with 90 points.

In Fig.6 it is shown the procedure for the determination of intersecting points of two surfaces of revolution with intersecting axes. For the arbitrary sphere whose radius is bigger than minimal, and smaller than maximal, pairs of parallel are determined in interest of auxiliary sphere with surfaces of revolution. In the intersection of these parallels points of 3D intersecting curve of two surfaces will be found, and in figure for this auxiliary sphere two points of the space curve are determined.

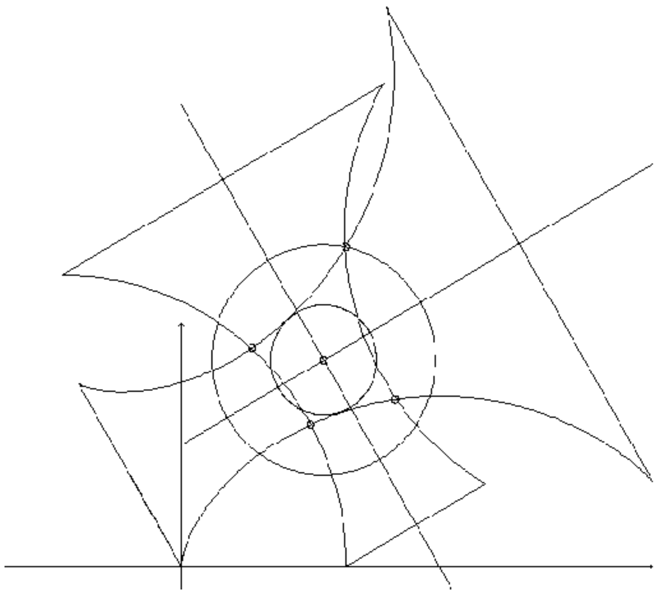

Fig. 5. Minimal and maximal sphere for two surfaces of revolution whose axes are intersect

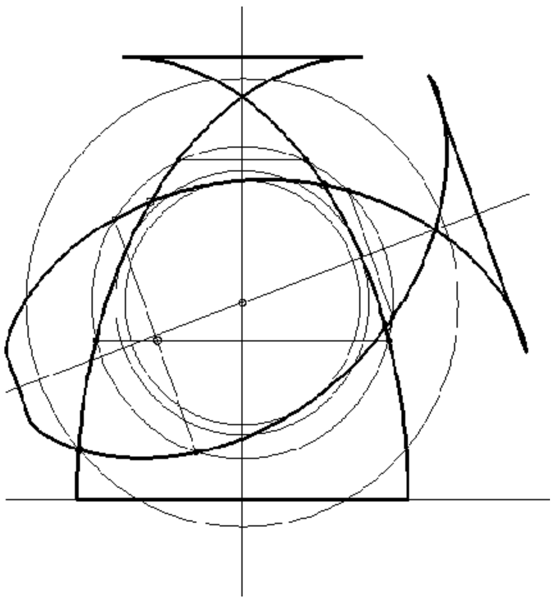

Fig. 6. Intersect between two surfaces of revolution whose axes are intersect with sphere with arbitrary radius, intersecting parallels of two surfaces with auxiliary sphere and intersecting points of space intersecting curve of two surfaces

In Fig. 7 in plane *Oxz* with 6040 points it is represented the intersecting curve of two surfaces of revolution with intersecting axes.

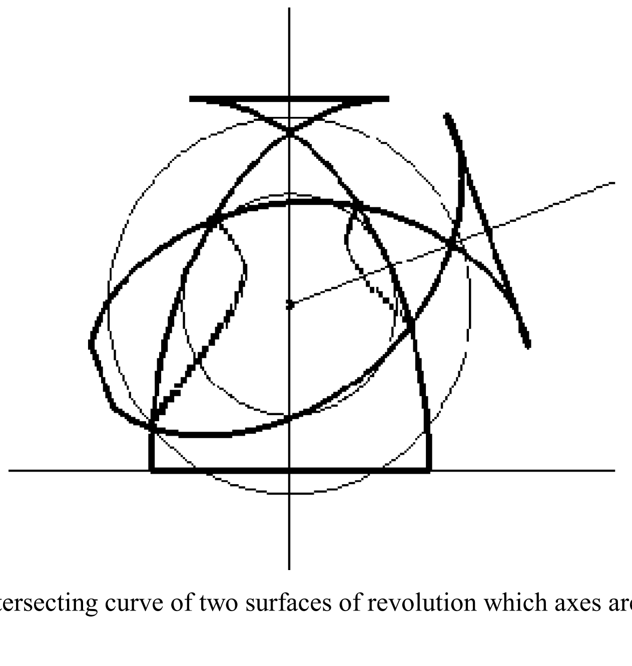

Fig. 7. Intersecting curve of two surfaces of revolution which axes are intersect

7. CONCLUSION

During the analysis of the problem three cases separated: axes of surfaces of revolution are intersected, they are parallel and they pass each other. In case when axes of surfaces intersect the easiest way for determining the intersecting curve of the two surfaces is using of auxiliary spheres, and the dominant characteristic is that 3D problem of determining of the intersection between two surfaces of revolution simplified to 2D problem of determining of intersect between two plane curves. In contrast to usual methods in computer graphics for the determination of the intersection between two surfaces of revolution, where surfaces are approximated by patches and because of that some mistake arises, the quality of this method solution based on descriptive geometrical aproaches is in using original surfaces and the only approximation was in the determination of intersecting points of two plane curves (Obradović 2000). We can underline that the standard problem of connecting order of 3D curve points is avoided, because in this method the intersecting curve of two surfaces of revolution is determined by the set of 3D points (Obradović 2000).

Earlier the conditions for the determination of visibility of surfaces of revolution (Štulić 1994) were defined, and in future work we can find the solution to the visibility of the space intersecting curve. Rendering of surfaces of revolution by using auxiliary spheres was analysed (Štulić 1994), and in case when we look at the intersection between two surfaces of revolution we can find the solution to the mutual blocking between two surfaces. An interesting problem is the determination of illumination dividing line in case when the centre of projecting and the centre of illumination are not coincident and painting problem is also interesting. Also, we can make better software by adding a procedure for detecting of the existence of the intersection between two surfaces of revolution.

REFERENCES

1. Anagnosti P.: *Descriptive geometry*, ″Naučna knjiga″, Beograd, 1984.
2. Dovniković L.: Technical drawing –introduction to mechanical constructions, University of Novi Sad, Novi Sad, 1990.
3. Foley J.D., van Dam A., Feiner S.K., Hughes J.F., Phillips R.L.: *Introduction to Computer Graphics*, ″Addison-Wesley Publishing Company″, United States of America, 1995.
4. Hoschek J., Lasser D.: *Fundamentals of Computer Aided Geometric Design*, ″A K Peters″, Wellesley, Massachusetts, 1993.
5. Mortenson M.E.: Computer Graphics: An Introduction to the Mathematics and Geometry, ″Heinemann Newnes″, Oxford, 1989.
6. Obradović R.: *Surface-Surface intersection: Auxiliary spheres*, Novi Sad Journal of Mathematics, Volume 29, No. 3, pp. 221-230, 1999.
7. Obradović R.: Descriptive-Geometrical Methods in Computer Graphics: Intersection Between two Surfaces of Revolution by use of Common Spheres and Common Planes, PhD Dissertation, University of Novi Sad, Faculty of Natural Sciences & Mathematics, Novi Sad, 2000.
8. Rogers D., Adams A.: *Mathematical Elements for Computer Graphics*, McGraw-Hill Publishing Company, New York, 1990.
9. Štulić R.: Descriptive Geometric Methods in Computer Graphics*: Contour Lines and Iso-illuminated Lines of Surfaces of Revolution*, University of Belgrade, Belgrade, 1994.

Acknowledgments. *The autor wish to express his gratitude to Professor PhD Lazar Dovniković (University of Novi Sad, Faculty of Technical Sciences), Professor PhD Nevena Pušić (University of Novi Sad, Faculty of Natural Sciences and Mathematics), Professor PhD Nenad Grujić (University of Belgrade, Faculty of Architecture) and Professor PhD Dragan Acketa (University of Novi Sad, Faculty of Natural Science and Mathematics) for their helpful suggestions.*
The autor wish to thank the Serbian Ministry of Science, Technology and Development (Project 1819: Synthesis and Development of Geometric Knowledge for Advancement of Visualization and Representation of Spatial Configurations) for their financial support of this research project.

ODREĐIVANJE PRESEKA DVEJU ROTACIONIH POVRŠI ČIJE SE OSE SEKU KORIŠĆENJEM POMOĆNIH LOPTI

Ratko Obradović

Metod pomoćnih lopti se u deskriptivnoj geometriji koristi kod određivanja preseka dveju rotacionih površi čije se ose seku (Dovniković 1994; Anagnosti 1984). Naime, ako se u tačku preseka osa rotacija dveju površi postavi centar svih pomoćnih lopti, tada svaka pomoćna lopta u opštem slučaju seče svaku rotacionu površ po k krugova-paralela čije su ravni upravne na osu rotacione površi (k=2m, jer je krug drugog reda a kriva koja obrazuje rotacionu površ je reda m). Za slučaj kada lopta seče obe površi po dvema paralelama, parovi paralela se seku u najviše 8 tačaka i te presečne tačke paralela su tačke prostorne presečne krive dveju površi za posmatranu pomoćnu loptu. Menjanjem prečnika pomoćnih lopti dobijaju se nove paralele i u njihovom preseku nove tačke prostorne presečne krive (Obradović 2000).

Prikazanu ideju moguće je matematički formulisati i oformiti postupak u kojem neće biti važno kojim se redosledom spajaju presečne tačke za sve pomoćne lopte, jer će se procedura realizovati pomoću računara, pa će se, sa dovoljno velikim brojem pomoćnih lopti, problem spajanja presečnih tačaka izbeći (Obradović 2000).

Ključne reči: *rotaciona površ, pomoćna lopta, deskriptivna geometrija, kompjuterska grafika.*