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ROTARY SURFACE DRAWING IN OBLIQUE AXONOMETRY

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Abstract. *In the present research a simple construction for rotary surface drawing is shown in oblique axonometry using only the vertical projection of the surface, which is the significant one. An additional vertical plane is considered and a Monge projection model is introduced. The method of the tangent cones and cylincers is used and a formula for determination of its contour generants is proved. An example of a rotary surface is shown in a cabinet projection, using the constructions.*

Key words: *rotary surface, oblique axonometry, construction*

1. Introduction

Some methods for drawing of the rotary surface contour are known, for example the method of the tangent spheres or the method of the tangent cones and cylinders in orthogonal axonometry [1]. The shadows of a rotary surface can be drawn using the same methods [2]. Simple constructions for shadow drawing of a rotary surface in orthogonal axonometry can be seen in [3]. The presented in [3] constructions cannot be used in oblique axonometry. We will show a simple method for rotary surface drawing in the case when the projection (axonometry) plane coincides with the frontal plane, especially cabinet and cavalier projection.

The problem is to draw the contour of the rotary surface, which is a curve l, containing the tangent points of the projection rays to the surface.

We shall use the idea of the tangent cones and cylinders for finding the contour, which divides the surface into visible and invisible parts. The method consists in the following: if a cone with a base- a parallel of the rotary surface, osculates the rotary surface at this parallel, then the points of this parallel, determining its contour generants, are on the contour l of the rotary surface. If we have a construction for finding these contour points on osculating cones and cylinders, then we will have points on the contour curve l fot the rotary surface.

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2. Assumptions

Let a rectangular coordinate system $Oxyz$ be given with the horizontal plane $\mu = Oxy$, the frontal plane $\nu = Oxy$ and the projection (axonometry) plane ρ, which coincides with the frontal plane: $\rho \equiv \nu$.

Let a rotary surface with the rotary axis Oz be given. We shall use as in [3] an additional projection plane γ, passing through the axis Oz and the projection ray s (see Fig. 1).

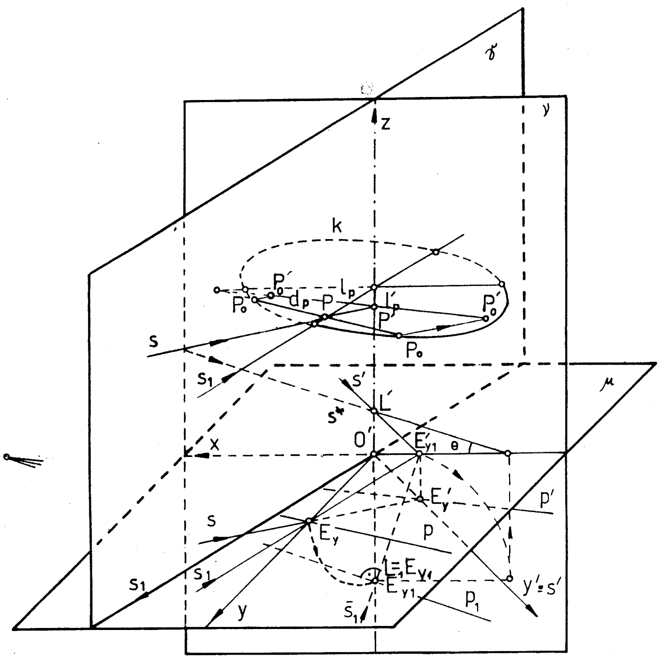

Fig. 1.

Evidently the points of l, belonging to one and the same parallel, will be symmetrical with respect to the plane γ mentioned above, and their orthogonal projections on γ will coincide. The aim is to find the projection of l on γ.

In the above part of Fig. 1 a parallel k is drawn as well as two points P_0P_0, which are symmetrical with respect to γ. Their orthogonal projections coincide with P. The axonometric projection of P on ρ will be P' and $P' \in Oz$. If we work in orthogonal axonometry, then the chord P_0P_0 is perpendicular to Oz or it is parallel to ρ, therefore it would be drawn in its real size. But in the case of oblique axonometry this is not true and we shall give some contructions in order to find the projection of the chord P_0P_0.

3. Monge projection model

Let's consider the cabinet projection with axonometric angles and parameters

$$\angle x'Oz' = \angle xOz = 90^\circ, \angle x'Oy' = 135^\circ, p = r = 2q = 1.$$

We need the orthogonal pojection of the axonometry ray s on μ and v. For this purpose we use (Fig. 1) the single point $Ey \in Oy$, which is projected in $E'_y \in Oy'$. We know that $|OE'_y| = ½|OE_y|$. Now we project the point E'_y on μ to find $E'_{y,1}$. If we take the ray $s \ni E_y$, then the orthogonal projection of s on μ is $s_1 = E_yE'_{y,1}$. Then the plane γ passes through Oz and it is parallel to s_1.

In order to find the orthogonal projection of s on $\rho \equiv v$ we use that E_y is projected in O and $E'_y \in v$ therefore $s_2 \equiv Oy'$.

In order to find the orthogonal s-projection onto the plane γ, we need the size of the angle θ betweeen s and μ : $\theta = \angle(s,\mu)$. For this purpose we rotate μ around Ox to v so that

$$E_y \to E_{y,1},\ E'_{y,1} \to E'_{y,1}$$

and $s_1 \to \bar{s}_1 = E_{y,1}E'_{y,1}$.

Through this rotation we obtain in fact a Monge projection model and we shall use its notations.

In order to find the angle θ, we rotate the ray s to a position s^* around the vertical axis, passing through the arbitrary point L, so that s^* becomes paralel to v. On Fig. 1 we choose $L_1 \equiv E_{y,1}$.

The exact constructions are shown on Fig. 2. The ray $s \ni O$ has Monge projections s_1 and $s_2 \equiv Oy'$ and L is an arbitrary point on s. By rotation of the point O around L_1 and projection to Q we obtain $s^* \equiv L_2Q$. Then $\angle\theta = \angle(x', s^*)$.

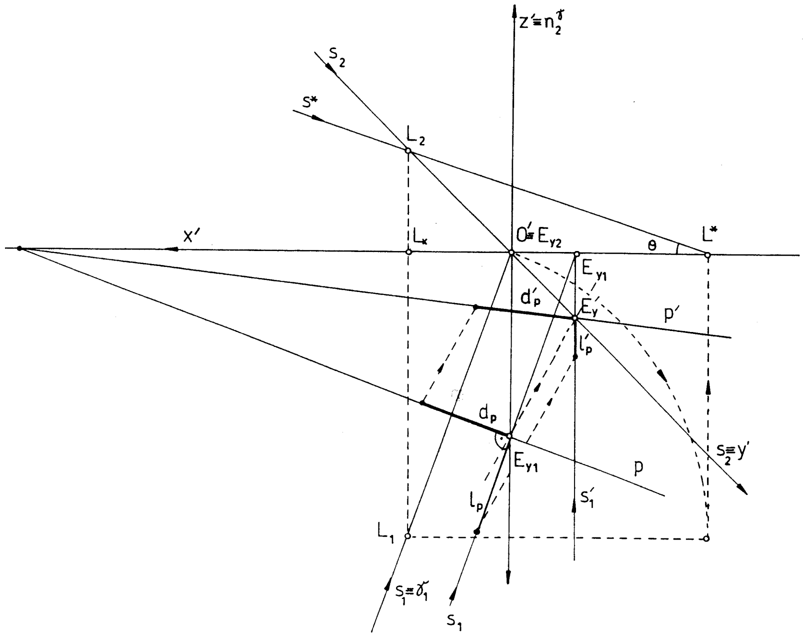

Fig. 2.

Further we need the projections of horizontal segments with direction s_1. On the Fig. 1 and Fig. 2 the segment l_p is taken on s_1 and the segment with a length d_p lies on the line

$p \perp s_1$. There exists an affinity ϕ between the first Monge projection and the axonometry projection of the points in μ after the rotation of μ to v with affinity axis Ox and with a pair of corresponding points $E_{y,1} \to E'_y$. Through the affinity $s_1 \equiv E_{y,1}E'_{y,1} \to E'_yE'_{y,1} \equiv s'_1$. Clearly $s'_1 \perp Ox'$.

On Fig. 2 p there is the rotary position of a line in μ passing through E_y and perpendicular to s_1. Through the affinity ϕ we find $p' \in E'_y$. The segment with a length d_p is projected to a segment with a length d'_p. By analogy the segment with a length l_p goes to a segment with a length l'_p.

4. Construction for contour generants of the tangent cones

If we consider a rotary surface with a verical rotary axis, then the rotary surface can be given only by its contour meridians in the frontal projection, which is the significant one (the horizontal projection consits only of parallels-circumferences). That's why we shall find a construction without using the projection on μ.

We consider a rotary cone with a radius r and an angle φ between the base and the generant. We suppose that this cone osculates the rotary surface.

We'll use Monge projection with projection planes μ and γ. On Fig. 3a the projecting ray s is parallel to γ and $\angle(s,\mu) = \angle(s^*,x) = \angle\theta$. The point V^s is the projection of V onto μ by s. The tangents, passing through V_1^s to the circle, determine the tangent points X_1, Y_1. We obtain the generants XV and YV, dividing the visible and invisible parts.

We have (Fig. 3a):

$$O_2M_2 = O_1M_1 = \frac{r^2}{O_1V_1^s} = \frac{r^2}{O_2V_2^s} = r\frac{r}{O_2V_2^s}.$$

But

$$\frac{O_2V_2^s}{h} = \cot\theta, \quad \frac{r}{h} = \cot\varphi,$$

therefore

$$O_2M_2 = r\frac{r}{h\cot\theta} = r\cot\varphi\tan\theta.$$

Using this formula, the following constructions are available, if the frontal cone projection and the ray s_2 are given (see Fig. 3b):

1. half a circle $K(O_2,r)$
2. $q \ni B_2$, $q \perp B_2V_2$
3. $Q = q \cap V_2O_2$
4. $m \ni Q$, $m \perp s_2$
5. $M_2 \equiv X_2 \equiv Y_2 = m \cap A_2B_2$.

One can easily see, after these constructions, that the above formula is performed: $O_2M_2 = r\cot\varphi\tan\theta$.

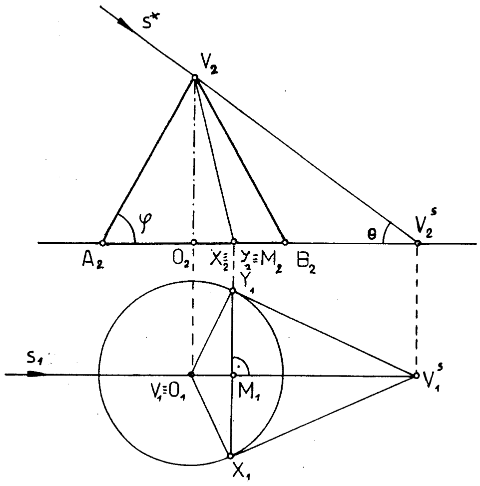

Fig. 3a.

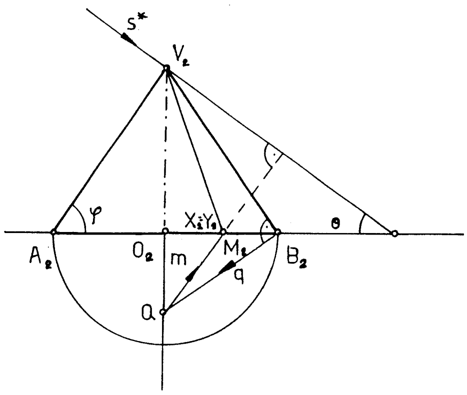

Fig. 3b.

The following cases for the point M_2 can be obtained in dependence of the angles φ and θ:

1. $M_2 \in O_2B_2$
2. $M_2 \equiv B_2$
4. $M_2 \notin O_2B_2$

In the case of $\varphi = \theta$ we obtain that $M_2 \equiv B_2$. Using such a cone one can obtain the highest and the lowest points of the orthogonal contour projection onto γ.

In the case of $\varphi = 90^o$ we have a tangent cylinder and $M_2 \equiv O_2$. This means that the projections of l, the least parallel and the axis Oz intersect.

5. AN APPLICATION OF THE ABOVE CONSTRUCTIONS FOR ROTARY SURFACE DRAWING

On Fig. 4b an application is shown - a rotary surface in cabinet projection. The rays s_1, $s_2 \equiv Oy'$ are shown. The angle θ and the ray s^* are found after the rotation around the vertical axis through the point L to a position parallel to ν through the construction described on Fig. 2.

The plane γ becomes parallel to $\nu \equiv \rho$ through the rotation around Oz. The orthogonal projection of the surface and the ray s^* are pictured in Fig. 4a. We shall project the curve l on γ.

By radii perpendicular to s^*, the points A and G are found, belonging to the projection contour in γ; the points A' and G' are on the axis Oz (that is the case of $\theta = \varphi$.)

The points D are on the least parallel ($\varphi = 90^o$) The points B, C, E, F... on the curve l are found on arbitrary parallels, using the above constructions. The tangents to l with direction s^* determine points P and Q, by which the horn points P' and Q' are found. The curve l divides the surface in two part - visible (to the left from l) and univisible (to the right from l.)

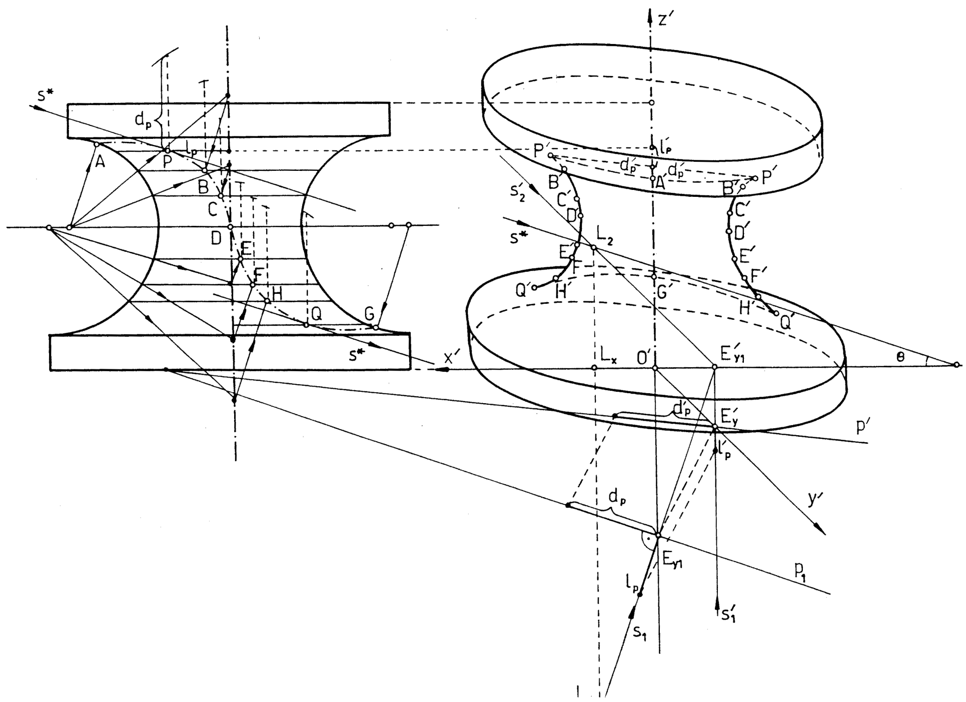

Fig. 4a. Fig. 4b.

The construction for finding the points *P′* is shown on Fig. 4a. The perpendicular through the point *P* to the segment (projection of the parallel) is drawn. We intersect this perpendicular with the corresponding radius. The segment d_p is obtained in this way. The segment l_p lies on the projection of this parallel with a left end *P* and a right end on *Oz*.

We plot these segments on p_1 and s_1 resp. (see Fig. 4b) and determine d'_p and l'_p using the affinity ϕ.The middle point of the chord *P′P′* is moved down along *Oz* by l'_p. We plot the segments, parallel and equal to d'_p to the right and to the left. One can see that all middle points belonging to the left of *Oz* on Fig. 4a, will be moved down along *Oz* and the middle points to the right will be moved up.

In conclusion, the same constructions can be used for finding shadows of rotary surfaces. The case of cavalier projection can be considered by analogy.

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CRTANJE OBRTNE POVRŠI U KOSOJ AKSONOMETRIJI

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U prezentovanom istraživanju prikazuje se jednostavna konstrukcija za crtanje obrtnih površi u kosoj aksonometriji korišćenjem samo karakteristične frontalne projekcije površi. Razmatra se i dodatna vertikalna projekcija i uvodi se model Monžove projekcije. Koristi se metoda dodirnih konusa i cilindara i dokazuje formula za određivanje generanta konture. Korišćenjem izložene konstrukcije dat je primer crtanja obrtne površi.

Ključne reči: *obrtna površ, kosa aksonometrija, konstrukcija*